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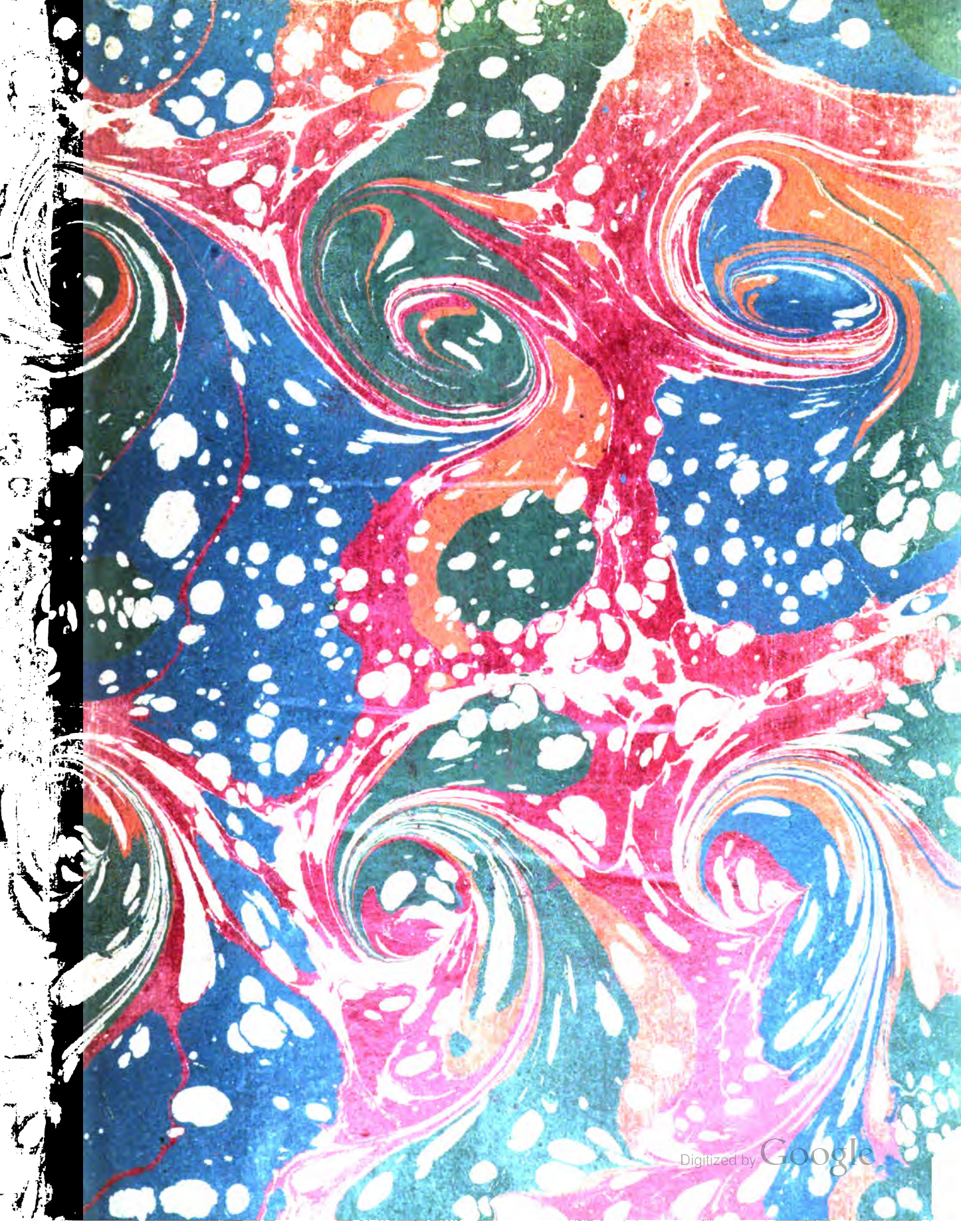
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Matth. 317.

Matth. 317

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THEORIA MOTUS LUNAE

EXHIBENS

OMNES EIUS INAEQUALITATES

IN

ADDITAMENTO

HOC IDEM ARGUMENTUM ALITER TRACTATUR

SIMULQUE OSTENDITUR

QUEMADMODUM MOTUS LUNAE CUM OMNIBUS

INAEQUALITATIBUS

INNUMERIS ALIIS MODIS

REPRAESENTARI

ATQUE AD CALCULUM REUOCARI POSSIT

AUCTORE

L. EULERO

IMPENSIS

ACADEMIAE IMPERIALIS SCIENTIARUM

PETROPOLITANAE

ANNO 1753.

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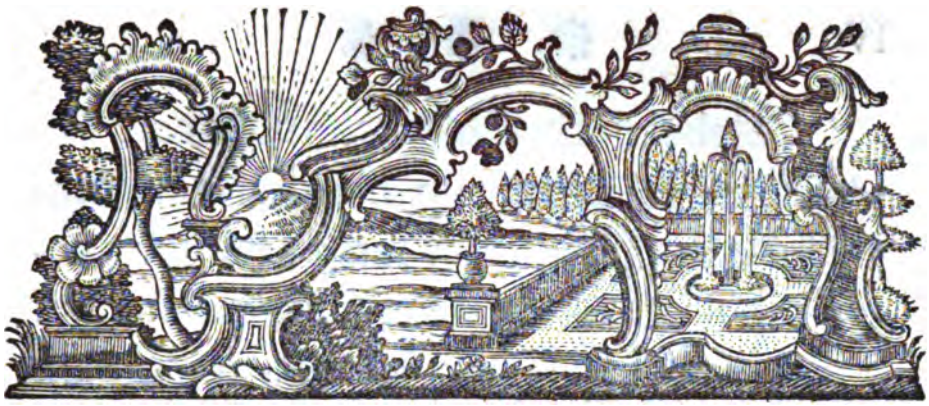

Academiam Scientiarum Petropolitanam triennio abhinc omnes, qui ingenii viribus confiſi, ad examinandam Neutonianam de motu Lunae Theoriam, animum applicare vellent, inuitaſſe, atque ei, qui in hac parte tenuiſſet primas, praemii loco propoſuiſſe nummos aureos centum; poſtmodum Celeberrimum Clairautum huius certaminis exſtitiſſe victorem, publico Academicorum, qui Petropoli agunt, iudicio diuulgatum.

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Celeberrimus Eulerus, Academiae Petropolitanae membrum honorarium, officii ſui eſſe exiſtimauit, ferre vna cum ceteris de illa Clairauti diſſertatione iudicium. Transmiſit ergo huc ad nos una cum ſua ſententia ampliſſimam de eodem argumento diſſertationem; quae quo celerius innoteſceret, viſum eſt Academiae Praeſidi, minoris Ruſſiae Hetmano Illuſtriſſimo Cyrillo Gregoridae, Comiti Raſumouio, eam tradere Academicis in ſolenni conuentu examinandam, ea fini, vt ſi ſuffragio Academicorum comprobata, dignaque iudicata foret, quae orbis eruditi proponeretur theatro, ea praelo quam maturrime ſubiiceretur; quandoquidem ille mos iam inde a principio obtinuit, vt, quae in publico coetu praeleguntur diſſertationes, eae vel ante ſolennem actum, vel paruulo intermiſſo ſpatio typis diuulgentur poſtea. Cete-

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Ceterum dissertatio illa constat magnam partem calculis, veris quidem illis et omnibus numeris absolutis, sed propter nimiam sui molem atque difficultatem auditu molestissimis: quae si recitarentur publice, periculum erat, ne auditorum animi auscultandis iis deficerent, neue Academia Summorum patientia Virorum abuti, caeterosque enicare odio videretur. Praeuidit hoc incommodum, prouiditque Illustrissimi Praesidis sapientia. Mandavit Astronomiae Professori V. C. Nicetae Popouio, cuius id temporis dicendi erat prouincia, vt prolixam illam Euleri dissertationem, omissis calculis, redigeret in compendium, et quae inde excepisset, auditorum causa recitaret publice. Quo quidem facto et auribus hominum consuluit, et tamen rerum capitum participes eos facere aequabili temperamento instituit: at-

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que dissertationem, ne quid forte naeuorum obreperet, ipso auctore coram typis excudi aequum censuit.

Iam qui illo tempore interfuerant conuentui Academicorum solenni auditores, aequis nimis auribusque auscultarunt excerpta recitantem Euleriana Popouium: est ergo quod sperare liceat, et iis, quorum interest, vt qui hoc genere studiorum maxime delectantur, factum iri satis ipsa dissertatione. Datum Petropoli Nov. 1752.

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INTRODUCTIO.

Cum nullum sit dubium, quin summi Newtoni Theoria, qua motum Planetarum felicissimo cum successu certis legibus adstrinxit, plurimum ad motum Lunae accuratius determinandum contulerit, maximi sane in Astronomia momenti est quaestio: vtrum haec Newtoni Theoria omnino sit sufficiens omnibus motus inaequalitatibus, quae in Luna obseruantur, exactissime explicandis nec ne? Quanquam autem a Newtoni asseclis plerumque affirmari solet, nullam in motu Lunae obseruari inaequalitatem, cuius ratio in ista Theoria non contineatur; tamen tantum abest, vt hic consensus a quoquam perspicue sit monstratus, vt potius applicatio huius Theoriae ad Lunam tantis implicetur calculi difficultatibus, quibus penitus euoluendis vires ingenii humani vix sufficere videntur. Plurimae quidem adhuc prodierunt Tabulae Lunares, quae ex Theoria Newtoniana deductae perhibentur, sed praeterquam quod saepius ultra 5′ ab obseruationibus discrepent, earum conuenientia cum Theoria ipsa neutiquam est euicta; quin potius pleraeque Tabulae inaequalitatum non tam Theoriae quam obseruationibus sunt superstructae. Huiusmodi ergo tabularum siue consensus siue dissensus cum obseruationibus neque ad Theoriam Newtonianam pleniſſime confirmandam, neque ad eam infringendam alle-

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gari poteſt: nam quatenus iſtae tabulae obſeruationibus ſatisfaciunt, hoc non ſoli Theoriae eſt tribuendum; quatenus autem cum obſeruationibus minus conueniunt, hoc ne Theoriae quidem imputari poterit, propterea quod iſtae Tabulae non ſoli Theoriae innituntur.

Quaeſtio itaque, cujus mentionem feci, recte elabi nequit, niſi ante eiusmodi Tabulae exhibeantur, quae ex ſola Theoria, nullis obſeruationibus in ſubſidium vocatis, ſint formatae; tum enim demum ex huiusmodi Tabularum collatione cum ingenti obſeruationum ſummo ſtudio inſtitutarum copia diiudicare licebit, vtrum Theoria omnibus obſeruationibus reſpondeat, an vero correctione quapiam indigeat. Non difficile quidem eſt ex principiis mechanicis motum Lunae aequationibus analyticis complecti; quoniam autem hae aequationes plures variabiles inter ſe permixtas continent, atque adeo differentialibus ſecundi ordinis implicantur, earum reſolutio maximis difficultatibus eſt obnoxia; et quoniam alio modo niſi per approximationem ſuſcipi non poteſt, vtcunque inſtituatur, ſemper non leue dubium remanet, vtrum partes, quae in calculo ſunt neglectae et praetermiſſae, nihil, quod in motu Lunae eſſet notabile, efficere potuerint. Hoc modo explicatio motuum Lunae tota ad ſolam Analyſin transfertur, ac difficultates, quibus premitur, inde oriuntur, quod Analyſis nondum ſatis eſt exculta.

Cum igitur Theoria Newtoniana hoc principio latiſſime patente innitatur, quod omnia corpora coeleſtia ſe mutuo attrahant in ratione reciproca duplicata diſtantiarum,

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rum, ſi motum Lunae ſecundum hanc Theoriam definire velimus, vires erunt ſpectandae omnes, quibus Luna ſollicitatur. Atque inter has vires primaria eſt ea, qua ad terram vrgetur, quae ſi ſola adeſſet, terraque quieſceret, Luna in ellipſi perfecta ſecundum regulas Keplerianas motum ſuum circa terram abſolueret. At cum Luna praeterea aeque ac terra ipſa etiam ad ſolem trahatur, hac vi motus ille regularis non mediocriter perturbabitur: atque haec vis a ſole profecta omnium difficultatum, quae in determinatione motus Lunae offenduntur, cauſa eſt exiſtimanda. Reliquae enim vires, quibus forte Luna ſecundum Theoriam Newtoni ad reliquos planetas vrgeri deberet, tam ſunt exiguae, vt effectus inde oriundus merito pro nihilo haberi queat.

Solas ergo vires ſolis ac terrae in computum duci oportet, ſi motum Lunae ſecundum Theoriam definire velimus, atque cum ex his viribus formulae analyticae fuerint erutae, quae motum Lunae complectantur, omne ſtudium in his formulis ita euoluendis erit impendendum, vt inde ad quoduis tempus propoſitum locus Lunae aſſignari, ac more apud Aſtronomos ſolito ſecundum longitudinem et latitudinem definiri queat. Hinc porro Tabulae Aſtronomicae pro motu Lunae erunt condendae, quibus omnes inaequalitates tam in longitudine quam in latitudine exhibeantur, ex quibus ſi pro cujusuis obſeruationis momento locus Lunae computetur, conſenſus vel diſſenſus calculi ab obſeruationibus Theoriam vel confirmabit, vel ejus defectum declarabit. Neque tamen Theoria ſola hujusmodi Tabulis conſtruendis ſufficit, ſed quaedam ele-

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menta extrinsecus ab obseruationibus assumi oportet, quae sunt 1°. Excentricitas orbitae Lunaris, quae salua theoria vel major vel minor esse potuisset; pendet enim a motu Lunae primitus impresso, quem Theoria non determinat, sed tanquam cognitum assumit. 2°. Locus Lunae medius pro quapiam Epocha proposita, qui pariter ex obseruationibus est concludendus. 3°. Locus Apogei orbitae Lunaris pro Epocha quadam data. 4°. Tempus periodicum Lunae secundum motum medium, quod pendet a distantia Lunae media a Terra, ideoque ex sola Theoria definiri nequit. 5°. Locus nodorum Lunae pro Epocha quadam data: et 6to denique inclinatio media orbitae Lunaris ad planum Eclipticae.

His autem sex elementis per obseruationes definitis reliqua omnia, quibus ad locum Lunae pro quouis tempore assignandum opus est, ex sola Theoria sunt petenda; quae primo ad quoduis tempus locum Apogei eiusque ideo motum verum praebere debet, vt inde ex loco Lunae medio eius anomalia media colligi queat. Deinde Theoria quoque omnes correctiones seu Prostaphaereses, quae loco Lunae medio vel additae vel sublatae eius locum verum exhibeant, suppeditare debet; atque istae correctiones, quae motus inaequalitates appellari solent, partim ab Anomalia media Lunae, partim ab eius Phasi seu elongatione a sole, partimque ab Anomalia solis media pendent, ex quo triplici fonte numerus inaequalitatum in immensum augetur. Porro etiam Theoria motum nodi eiusque omnes inaequalitates indicare tenetur, ac denique etiam pro quouis tempore orbitae Lunaris veram inclinationem

da

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ad ecliptieam, vt inde eius latitudinem veram eruere liceat.

Cum autem, vt iam innui, nemo adhuc omnes inaequalitates, quae in motu Lunae reperiuntur, ex Theoria elicuerit, vt ex iis iudicium ferri possit, quantum haec Theoria cum obseruationibus conueniat; etsi nullum est dubium quin discrimen, si quod deprehenderetur, admodum paruum sit futurum: iam pridem haec quaestio ex solo motu Apogei dirimi est coepta, dum aliis motus Apogei ex obseruationibus cognitus magnopere a Theoria discrepare est visus, alii autem etiam hoc loco pulcerrimum consensum Theoriae et veritatis iactauerunt. Mirum autem est, ipsum Newtonum nihil circa motum Apogei ex Theoria statuisse, sed eum ex solis obseruationibus in calculum transtulisse, cum tamen motum nodorum summa sagacitate ex Theoria elicuisset, atque veritati consentaneum ostendisset. Cur igitur motum Apogei plane silentio praeterierit, nulla alia ratio subesse videtur, nisi quod animaduerterit hunc motum, prouti ex Theoria prodiret, obseruationibus parum fore conformem. Ex iis enim, quae Newtonus in suo immortali opere de motu absidum in genere tradidit, non admodum difficile videtur motum apogei Lunae definire: verum hic praeter expectationem euenit, vt motus apogei annuus vix 20° superans reperiatur, cum tamen ex obseruationibus constet, Apogeum Lunae interuallo unius anni vltra 40° promoueri.

Siue autem ista motus Apogei quantitas 20° legitime sit ex Theoria deriuata, siue minus; consideratio Apogei tutissimum praebet remedium quaestionem de sufficientia

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Theoriae Newtonianae decidendi. Quamuis enim ex Theoria inaequalitas quaepiam in ipso motu Lunae aliquot minutis secundis vel etiam primis maior minorue prodiret, quam experientia monstraret, tamen tantilla differentia merito vel leui cuipiam errori in obseruationibus, vel vitio in approximatione commisso tribueretur; quandoquidem aliunde certum est Theoriam Newtonianam non admodum a veritate recedere. At longe aliter est comparata ratio motus Apogei: quodsi enim vires Lunam sollicitantes tantillum a Theoria Newtoniana discrepent, vt ex iis in ipso motu Lunae vix perceptibile discrimen nasceretur, tamen inde in motu Apogei annuo differentia plurium graduum oriri poterit. Quae tanta differentia cum nulli errori vel obseruationum vel ipsius calculi, siquidem omni cura instituatur, tribui queat, inuestigatio motus apogei certissimum suppeditat criterium iudicandi, vtrum quaepiam Theoria veritati sit consentanea nec ne?

Quodsi ergo calculo rite administrato Theoria Newtoni reperiatur tantum Apogei Lunaris motum exhibere, quantus per obseruationes deprehenditur, scilicet ultra 40° quotannis; fortius certe argumentum, quo veritas hujus Theoriae indubie demonstretur, desiderari nequit. Sin autem contra eueniat, vt progressio Apogei annua ex Theoria rite deriuata notabiliter a 40° deficiat, hinc certo erit concludendum Theoriam Newtonianam correctione quapiam indigere, neque vires, quibus Luna reuera sollicitatur, exactissime huic Theoriae esse conformes.

Verum haec ipsa quaestio, vtrum Theoria Newtoniana ad verum apogei Lunae motum perducat nec ne? est profun-

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fundissimae indaginis, atque summam in calculo circumspectionem ac sollertiam requirit. Quanquam enim ex principiis generalibus, vnde vulgo motus absidum definiri solet, satis luculenter semissis tantum pro motu Apogei Lunae eliciatur; tamen quoniam in calculo plures termini, qui in determinationem motus Lunae ingrediuntur, ob paruitatem sunt reiecti, merito dubitatio suboritur, num hi ipsi termini, si eorum ratio esset habita, non istum defectum compensare valuerint? Quin etiam non defuere Geometrae, qui consensum huius Theoriae cum vero Apogei motu demonstrare sunt conati: verum plerumque non difficile erat paralogismum in ipsorum ratiociniis deprehendere. Maximam autem hoc loco attentionem meretur iudicium profundissimi Geometrae Clairaut, qui cum primum validissimis argumentis statuisset, Newtoni Theoriam non vltra dimidium veri apogei Lunaris motus suppeditare, subito in contrariam abiit sententiam statuens hanc Theoriam, elegantissime cum veritate conspirare, neque certe tanto perspicaciae Viro a priori sententia, quam omni studio propugnauerat, recedendum esset putandus, nisi firmissimis argumentis eo esset adductus.

Cum autem omnes rationes, quae Ipsum ad hanc retractationem impulerint, nondum publice explicuerit, licet at mihi quidem, qui semper contrariae sententiae fui addictus, tantisper arduam hanc quaestionem tanquam nondum decisam spectare, donec per propriam inuestigationem inuenero, quid de ea sit statuendum. Postquam enim iam a longo temporis intervallo plurimum studii in indagatione motus Lunae consumsissem, ac variis methodis insistens

semper

semper conclusionem Theoriae Newtonianae minus fauentem esse adeptus; quam tamen pro rite demonstrata venditare non eram ausus, propterea quod approximatione essem vsus, ac semper suspicio quaedam ratione terminorum praetermissorum remaneret. Super in aliam incidi viam hanc inuestigationem suscipiendi, quae mihi multo certior videtur, ita vt per eam nulla dubitatione interiecta ad veritatem penetrare confidam. Ne autem si forte Theoriam Newtonianam minus sufficientem inuenero, calculum secundum aliam Theoriam de nouo instituere cogar, statim meam inuestigationem in latiori sensu exordiar, viresque quibus Luna ad terram sollicitatur, non exacte sed proxime tantum quadratis distantiarum reciproce proportionales assumam: deinceps scilicet innotescet, vtrum haec aberratio a regula Newtoniana locum habeat nec ne? Calculum autem ita adornabo, vt quicquid euenerit, non solum verum apogei motum assequar, sed etiam omnes Lunae inaequalitates inde elicere valeam, quas deinceps Astronomorum more tabulis complectibitur.

Primum ergo problema in latissimo significatu concipiam, vt corporis a viribus quibuscunque sollicitati motum inuestigaturus: deinde vires, quibus Luna actu vrgetur, considerabo, in calculum introducam, et aequationes Lunae motum determinantes exhibebo. Has porro aequationes variis modis in alias formas transmutabo, donec eas eo perduxero, vbi ad finem propositum maxime accommodatae videbuntur, quo cum peruenero, tandem eo motum Apogei, quam cunctas Lunae inaequalitates motus ex calculo deriuare studebo.

CAPUT

CAPUT I.

DE MOTU CORPORIS A VIRIBUS QUIBUSCUNQUE SOLLICITATI.

§. 1.

Quoniam corpus a viribus quibuscunque sollicitari ponimus, fieri potest, vt eius motus non in eodem plano absoluatur. Hinc ad ejus motum per calculum ita repraesentandum, vt ad quoduis tempus verus locus, in quo corpus versabitur, assignari queat, conueniet corporis motum ad planum quoddam fixum pro lubitu assumtum referri. Exhibeat igitur Tabula hoc planum, atque corpus iam versetur extra hoc planum in puncto L, vnde ad planum demittatur perpendiculum L M; eritque punctum M locus corporis ad planum relatus. Quod si ergo ad quoduis tempus propositum hunc corporis locum relatum M, simulque eius a plano distantiam L M indicare valeamus, verus corporis locus L ad hoc tempus innotescet. Fig. 1.

§. 2. Ad locum autem puncti M commodius determinandum, assumamus in plano rectam quandam fixam C Q pro axe, ita vt ducta ex M ad hanc rectam perpendiculari M P, locus puncti M more apud Geometras recepto per coordinatas orthogonales definiatur. Assumto ergo porro in axe puncto quodam fixo C, vnde abscissae C P computentur, erit P M applicata puncto M respondens, & ipsum punctum L determinabitur per tres coordinatas inter se normales C P, P M & M L.

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M L. Cum igitur praesenti temporis momento corpus in L versari ponatur, vocentur istae tres coordinatae eo spectantes:

$$CP = p;\ PM = q \text{ et } ML = r$$

elapso autem temporis elemento, quod per dt indicemus, coordinatae ternae tum locum corporis indicantes erunt:

$$p + dp;\ q + dq; \text{ et } r + dr.$$

§. 3. Quaecunque nunc fuerint vires, quibus corpus sollicitatur, eae semper per cognitam virium resolutionem reduci poterunt ad ternas vires secundum directiones ternarum coordinatarum vrgentes. Ponamus ergo vim $= P$, qua corpus in L secundum directionem ipsi P C parallelam trahitur: eam porro vim $= Q$, qua corpus secundum directionem ipsi M P parallelam trahitur: eamque denique vim $= R$, qua corpus secundum directionem L M sollicitatur. Has scilicet vires ita directas concipio, vt si corpus earum actioni libere obediret, valores coordinatarum p, q, r inde diminuerentur. His positis, ex principiis Mechanicae constat, si elementum temporis dt pro constanti assumatur, motum corporis his tribus formulis differentio-differentialibus determinari

I. $ddp = -\frac{1}{2}Pdt^2$; II. $ddq = -\frac{1}{2}Qdt^2$; III. $ddr = -\frac{1}{2}Rdt^2$.

§. 4. Verum hae coordinatae ad vsum astronomicum, ad quem hic potissimum respicimus, non satis sunt accommodatae. Nam si spectatorem in C constitutum assumimus, locus L, vbi corpus cernetur, commodissime

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me per quantitatem rectae C M, et angulum Q C M vna cum angulo M C L repraesentatur: atque si tabula planum eclipticae referat, rectaque C Q ad principium arietis sit directa, angulus Q C M in Astronomia vocari solet sideris longitudo, angulus M C L vero eius latitudo, et recta C M eius distantia curtata. Vocemus ergo porro:

I. Distantiam curtatam seu rectam C M $= x$

II. Longitudinem seu angulum Q C M $= \varphi$

III. Latitúdinem seu angulum M C L $= \psi$

ac posito constanter sinu toto $= 1$, erunt valores coordinatarum ante adhibitarum:

C P $= p = x \cos \varphi$; P M $= q = x \sin \varphi$ & M L $= r = x \tang \psi$

atque distantia corporis vera a puncto C erit C L $= x \sec. \psi = \frac{x}{\cos \psi}$.

§. 5. Sumtis nunc differentialibus more consueto obtinebimus:

$$dp = dx \cos \varphi - x d\varphi \sin \varphi \; ; \; dq = dx \sin \varphi + x d\varphi \cos \varphi$$

$$\text{et } dr = dx \tang \psi + \frac{x d\psi}{\cos \psi^2}$$

atque hinc demuo differentialibus sumendis reperietur,

$$ddp = ddx \cos \varphi - 2 dx d\varphi \sin \varphi - x dd\varphi \sin \varphi - x d\varphi^2 \cos \varphi$$

$$ddq = ddx \sin \varphi + 2 dx d\varphi \cos \varphi + x dd\varphi \cos \varphi - x d\varphi^2 \sin \varphi$$

$$ddr = ddx \tang \psi + \frac{2 dx d\psi}{\cos \psi^2} + \frac{x dd\psi}{\cos \psi^2} + \frac{2 x d\psi^2 \sin \psi}{\cos \psi^3}$$

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Binae priores formulae rite combinatae suppeditabunt sequentes multo concinniores

$$ddp \cos\Phi + ddq \sin\Phi = ddx - x d\Phi^2$$
$$ddq \cos\Phi - ddp \sin\Phi = 2dx d\Phi + x ddΦ$$

sicque habebitur:

$$ddx - x d\Phi^2 = -\tfrac{1}{2} dt^2 (P \cos\Phi + Q \sin\Phi)$$
$$2dx d\Phi + x ddΦ = -\tfrac{1}{2} dt^2 (Q \cos\Phi - P \sin\Phi)$$

Tertiam vero aequationem deinceps magis tractabilem efficiemus.

§. 6 Manifestum autem est formulam $P \cos\Phi + Q \sin\Phi$ praebere vim ex viribus P et Q compositam, qua corpus in L secundum directionem rectae MC vrgetur, formulam vero alteram $Q \cos\Phi - P \sin\Phi$ exprimere vim ex eadem resolutione secundum directionem MQ ad MC normalem directam. Cum igitur hae duae vires assumtis binis P et Q aequiualeant, ponamus esse

I. Vim corpus L secundum MC trahentem $= V$

II. Vim corpus L secundum MQ trahentem $= T$

manente tertia vi corpus ad planum normaliter secundum LM vrgente $= R$. Atque sequentes habebimus aequationes:

I. $2dx d\Phi + x ddΦ = -\tfrac{1}{2} T dt^2$

II. $ddx - x d\Phi^2 = -\tfrac{1}{2} V dt^2$

III. $ddx \tang\psi + \frac{2dx d\psi}{\cos\psi^2} + \frac{x ddψ}{\cos\psi^2} + \frac{2x d\psi^2 \sin\psi}{\cos\psi^3} = -\tfrac{1}{2} R dt^2$

§. 7. Quo autem effectum tertiae vis R commodius ad calculum reuocemus, more apud Astronomos recepto contemplemur planum, in quo corpus durante elemento

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elemento temporis dt mouetur, et quod simul per punctum C transeat. Hoc igitur planum cum plano assumto intersectionem alicubi formabit, quae sit recta C Ω, ac linea nodorum appellari solet; sicque erit Ω C L planum orbitae, in qua corpus L praesenti instanti mouetur, et angulus, quo hoc planum Ω C L ad planum fixum Q C M inclinatur, vocatur inclinatio orbitae ad eclipticam pro tempore praesenti. Cum igitur ex his duabus rebus latitudo sideris definiri soleat, ponamus

Longitudinem nodi ascendentis seu angulum $QC\Omega = \pi$
ac inclinationem orbitae Ω C L ad eclipticam $= \varrho$
atque loco latitudinis ψ has duas quantitates π et ϱ definire oportebit.

§. 8. Tertia ergo aequatio in duas dispertietur, ad quas inueniendas ex M et L ad lineam nodorum C Ω ducantur normales M N et L N, eritque angulus L N M [mensura inclinationis orbitae ad eclipticam]; ideoque L N M $= \varrho$. Tum vero ob angulum Ω CM $= \Phi - \pi$ et C M $= x$ erit:

$$C N = x \cos(\Phi - \pi) \text{ et } M N = x \sin(\Phi - \pi)$$

hinc elicietur $M L = x \operatorname{tang} \varrho \sin(\Phi - \pi)$, vnde prodit $\operatorname{tang} M C L = \operatorname{tang} \psi = \operatorname{tang} \varrho \sin(\Phi - \pi)$, quae formula inseruit latitudini ψ ex cognita inclinatione ϱ et loco nodi eiusue longitudine π inueniendae, si quidem iam cognita fuerit longitudo sideris Φ. Quoniam autem sidus elemento temporis dt in eodem plano manet, in differentiatione formulae $\operatorname{tang} \psi = \operatorname{tang} \varrho \sin(\Phi - \pi)$,

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quantitates π et ϱ tanquam conſtantes ſpectari poterunt, eritque idcirco.

$$\frac{d\psi}{\text{cof}\,\psi^2} = d\varphi\,\text{tang}\,\varrho\,\text{cof}\,(\varphi-\pi)$$

§. 9. Interim tamen nihil impedit, quominus in eadem differentiatione quantitates π et ϱ tanquam variabiles tractemus, quales reuera eſſe poſſunt ſucceſſu temporis; vnde orietur haec aequatio:

$$\frac{d\psi}{\text{cof}\,\psi^2} = \frac{d\varrho}{\text{cof}\,\varrho^2}\,\text{ſin}\,(\varphi-\pi) + (d\varphi-d\pi)\,\text{tang}\,\varrho\,\text{cof}\,(\varphi-\pi)$$

hicque valor ipſius $\frac{d\psi}{\text{cof}\,\psi^2}$ collatus cum praecedente praebebit hanc aequalitatem:

$$\frac{d\varrho}{\text{cof}\,\varrho^2}\,\text{ſin}\,(\varphi-\pi) = d\pi\,\text{tang}\,\varrho\,\text{cof}\,(\varphi-\pi)$$

vnde obtinemus $\frac{d\varrho}{\text{ſin}\,\varrho\,\text{cof}\,\varrho} = \frac{d\pi\,\text{cof}\,(\varphi-\pi)}{\text{ſin}\,(\varphi-\pi)} = \frac{d\pi}{\text{tang}\,(\varphi-\pi)}$.

Cum iam ſit $\frac{d\varrho}{\text{ſin}\,\varrho\,\text{cof}\,\varrho} = \frac{d.\,\text{tang}\,\varrho}{\text{tang}\,\varrho} = d.\,l\,\text{tang}\,\varrho$, erit

$$d.\,l\,\text{tang}\,\varrho = \frac{d\pi}{\text{tang}\,(\varphi-\pi)}$$

ex quo, ſi longitudo nodi iam fuerit reperta, ſine labore inclinatio ad eclipticam ϱ inveſtigari poterit.

§. 10. Differentiemus formulam primo inuentam

$$\frac{d\psi}{\text{cof}\,\psi^2} = d\varphi\,\text{tang}\,\varrho\,\text{cof}\,(\varphi-\pi)$$

iterum, et cum ſit $d.\,\text{tang}\,\varrho = \frac{d\pi\,\text{tang}\,\varrho\,\text{cof}\,(\varphi-\pi)}{\text{ſin}\,(\varphi-\pi)}$

erit:

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erit:

$$\frac{d\,d\psi}{\cos\psi^2} + \frac{2d\psi^2\sin\psi}{\cos\psi^3} = d\,d\,\varphi\,\text{tang}\,\varrho\,\cos\,(\varphi-\pi)$$
$$+\frac{d\varphi d\pi\,\text{tang}\varrho\cos(\varphi-\pi)^2}{\sin\,(\varphi-\pi)} - d\varphi(d\varphi-d\pi)\text{tang}\varrho\sin(\varphi-\pi)$$

seu $\frac{d\,d\psi}{\cos\psi^2} + \frac{2d\psi^2\sin\psi}{\cos\psi^3} = d\,d\,\varphi\,\text{tang}\,\varrho\,\cos\,(\varphi-\pi)$
$$+\frac{d\varphi d\pi\,\text{tang}\varrho}{\sin(\varphi-\pi)} - d\varphi^2\,\text{tang}\varrho\sin(\varphi-\pi)$$

qui valores pro ψ in tertia aequatione superiori substituti suppeditabunt:

$$ddx\,\text{tang}\varrho\sin(\varphi-\pi) + 2dx\,d\varphi\,\text{tang}\,\varrho\cos(\varphi-\pi) + x\,dd\varphi\,\text{tang}\,\varrho\cos(\varphi-\pi)$$
$$+\frac{x\,d\varphi d\pi\,\text{tang}\varrho}{\sin(\varphi-\pi)} - x\,d\varphi^2\,\text{tang}\,\varrho\,\sin(\varphi-\pi) = -\tfrac{1}{2}\,\mathrm{R}\,dt^2$$

quae transmutatur in hanc:

$$(ddx-x\,d\varphi^2)\text{tang}\varrho\sin(\varphi-\pi) + (2dx\,d\varphi + x\,dd\varphi)\,\text{tang}\,\varrho\cos(\varphi-\pi)$$
$$+\frac{x\,d\varphi d\pi\,\text{tang}\varrho}{\sin\,(\varphi-\pi)} = -\tfrac{1}{2}\mathrm{R}\,dt^2$$

§. 11. Commode, hic euenit vt in ista formula illae ipsae expressiones differentio-differentiales $ddx - x\,d\varphi^2$ et $2\,dx\,d\varphi + x\,dd\varphi$ occurrant, quae ex actione duarum reliquarum virium sunt enatae: vnde si formularum harum valores aequiualentes $-\frac{1}{2}\,\mathrm{V}\,dt^2$ et $-\frac{1}{2}\,\mathrm{T}\,dt^2$ substituamus, impetrabimus

$$-\tfrac{1}{2}\mathrm{V}\,dt^2\,\text{tang}\varrho\sin(\varphi-\pi) - \tfrac{1}{2}\mathrm{T}\,dt^2\,\text{tang}\varrho\cos(\varphi-\pi) \mp \frac{x\,d\varphi d\pi\,\text{tang}\varrho}{\sin\,(\varphi-\pi)} = -\tfrac{1}{2}\mathrm{R}\,dt^2$$

qua differentiale $d\pi$, quo promotio elementaris lineae nodorum indicatur, ita determinabitur, vt sit

$$d\pi = \tfrac{1}{2}\,d\,t^2 \cdot \frac{\sin(\varphi-\pi)}{x\,d\varphi}\left(\mathrm{V}\sin(\varphi-\pi) + \mathrm{T}\cos(\varphi-\pi) - \frac{\mathrm{R}}{\text{tang}\,\varrho}\right)$$

Deinde

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Deinde cum ſit $d.\,l\,\text{tang}\,\varrho = \frac{d\pi}{\text{tang}\,(\varphi-\pi)}$, erit

$$d.l\,\text{tang}\,\varrho = \tfrac{1}{2}dt^2.\,\frac{\text{cof}(\varphi-\pi)}{x\,d\varphi}\left(\text{V}\,\text{ſin}(\varphi-\pi)+\text{T}\,\text{cof}(\varphi-\pi)-\frac{\text{R}}{\text{tang}\,\varrho}\right)$$

Duas ergo has aequationes loco ſuperioris tertiae, ex qua latitudo ψ inueniri debebat, in calculum introduci conueniet; inuentis enim π et ϱ erit $\text{tang}\,\psi = \text{tang}\,\varrho\,\text{ſin}\,(\varphi-\pi)$.

§. 12. Hinc patet lineam nodorum nunquam eſſe mobilem, quin ſimul inclinatio ϱ variationi ſit obnoxia. Eadem enim vis $\text{V}\,\text{ſin}\,(\varphi-\pi) + \text{T}\,\text{cof}(\varphi-\pi) - \frac{\text{R}}{\text{tang}\,\varrho}$, quae lineae nodorum motum imprimit eius longitudinem π immutando, ſimul in inclinatione ϱ variationem generat. Nulli autem plane immutationi tam linea nodorum, quam inclinatio erunt ſubjectae, ſi vis illa euaneſcat, quod euenit ſi media directio omnium virium corpus L ſollicitantium in ipſum planum ΩCL, in quo corpus ſemel moueri coepit, perpetuo incidat; hicque eſt caſus, quo corpus continuo in eodem plano moueri pergit. Generatim ergo corporis a tribus viribus V, T, R ſollicitati motus quatuor ſequentibus aequationibus determinatur.

I. $2\,dx\,d\varphi + x\,dd\varphi = -\tfrac{1}{2}\,\text{T}\,dt^2$

II. $dd\,x - x\,d\varphi^2 = -\tfrac{1}{2}\,\text{V}\,dt^2$

III. $d\pi = \tfrac{1}{2}dt^2.\,\frac{\text{ſin}\,(\varphi-\pi)}{x\,d\varphi}\left(\text{V}\,\text{ſin}(\varphi-\pi)+\text{T}\,\text{cof}(\varphi-\pi)-\frac{\text{R}}{\text{tang}\,\varrho}\right)$

IV. $d.\,l\,\text{tang}\,\varrho = \frac{d\pi}{\text{tang}\,(\varphi-\pi)}$

quas ergo quouis caſu oblato reſolui oportet.

CAPUT

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CAPUT II.

INUESTIGATIO VIRIUM LUNAM SOLLICITANTIUM.

§. 13.

Cum Lunae motus, qualis ex centro terrae ſpectaretur, definiri debeat, ſit C terrae centrum, ad quod etiam praecipua vis, qua Luna vrgetur, directa concipitur; atque tabula exhibeat planum eclipticae, in quo nunc quidem Sol exiſtat in S, Luna vero ſupra hoc planum verſetur in L latitudinem habens borealem, vnde ad planum eclipticae perpendiculum demittatur LM. Hinc ductis rectis CL, CM, CS, itemque CQ initium arietis verſus, vnde longitudines computari ſolent, fiant ſequentes denominationes. Fig. 2.

1. Longitudo Solis ſeu angulus ACS = θ
2. Longitudo Lunae ſeu angulus ACM = ϕ
3. Latitudo Lunae ſeu angulus MCL = ψ
4. Diſtantia Solis a Terra CS = y
5. Diſtantia Lunae a Terra curtata CM = x

§. 14. Sit iam AM proiectio orbitae lunaris in planum eclipticae; ac planum, in quo Luna nunc mouetur, per centrum terrae ductum, planum eclipticae interſecet ſecundum rectam C☊, quae lineam nodorum pro tempore praeſenti exhibebit: ac terminus ☊ quidem nodum aſcendentem referet, ſiquidem lunam ſecundum regionem AM promoueri ponamus. Quod ſi ergo porro vocemus

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6. Longitudinem nodi asc: $A C \Omega = \pi$

7. Incl. orbitae Lunae ad eclipticam $= \varrho$

hinc latitudo Lunae geocentrica ita definietur, vt sit tang $\psi =$ tang. sin $(\varphi - \pi)$. Vnde incrementum latitudinis $d\psi$ commode assignabitur, cum sit, vt supra vidimus $d.$ tang $\psi = \frac{d\psi}{\text{cos}\,\psi^2} = d\varphi$ tang ϱ cos $(\varphi - \pi)$: ac praeterea ex motu nodi cognito variatio inclinationis ita definietur, vt sit $d.$ tang $\varrho = \frac{d\pi \text{ tang } \varrho}{\text{tang}(\varphi - \pi)}$, seu $d\varrho = \frac{d\pi \sin \varrho \cos \varrho}{\text{tang}(\varphi - \pi)}$.

§. 15. Cum nunc primum Luna ad centrum terrae C secundum directionem L C attrahatur, sit haec vis $=$ M. Deinde sit vis, qua Luna ad solem S vrgetur secundum L S $=$ N; atque his duabus viribus Luna proprie vrgeri censenda est. Praeterea vero cum Terra ipsa, ad quam motum Lunae referimus, in motu versetur, ut eam tanquam quiescentem considerare queamus, non solum motum Terrae, sed etiam vires, quibus Terra vrgetur, toti mundo secundum plagas oppositas imprimi concipiamus. Sit igitur vis qua Terra ad Solem vrgetur $=$ S, & vis qua ad Lunam trahitur $=$ L, his viribus contrario modo in lunam translatis, Luna sequentibus viribus sollicitata habebitur

1. Secundum directionem L C vi $=$ M $+$ L
2. Secundum directionem L S vi $=$ N
3. Secundum directionem M R ipsi S C parallelam vi $=$ S.

§. 16. Hunc

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§. 16. Nunc primo has vires ad ternas directiones ſupra aſſumtas MC, MQ et LM reducamus; ac primo quidem vis M + L dabit

pro directione MC vim $= (M + L) \cos \psi$

pro directione LM vim $= (M + L) \sin \psi$

Secunda vis N vero dabit

pro directione LM vim $= \frac{LM}{LS} . N$

pro directione MS vim $= \frac{MS}{LS} . N$

at haec vlterius reſoluta ducta Mr ipſi CS parallela dabit:

pro directione MC vim $= \frac{MC}{LS} . N$

pro directione Mr vim $= \frac{CS}{LS} . N$

Haec poſtrema a vi tertia S ſubtracta relinquet vim $S - \frac{CS}{LS} . N$, qua Luna ſecundum directionem MR ſollicitatur, quae ob angulum $CMR = SCM = \varphi - \theta$ dabit

pro directione MC vim $= (S - \frac{CS}{LS} . N) \cos (\varphi - \theta)$

pro directione Mq vim $= (S - \frac{CS}{LS} . N) \sin (\varphi - \theta)$

vbi directio Mq contraria eſt directione MQ.

§. 17. His iam viribus cum ternis initio aſſumtis V, T et R comparandis, inueniemus pro his viribus ſequentes valores:

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1. pro directione MC vim $V =$

$$(M+L)\cos\psi + \frac{MC}{LS}\cdot N + (S - \frac{CS}{LS}\cdot N)\cos(\varphi-\theta)$$

2. pro directione MQ vim $T = -(S - \frac{CS}{LS}\cdot N)\sin(\varphi-\theta)$

3. pro directione LM vim $R = (M+L)\sin\psi + \frac{LM}{LS}\cdot N$.

Cum nunc sit $CM = x$, $CS = y$, et angulus $SCM = \varphi - \theta$; erit $MS = \sqrt{(xx - 2xy\cos(\varphi-\theta) + yy)}$, et ob $LM = x\tang\psi$ erit $LS = \sqrt{(yy - 2xy\cos(\varphi-\theta) + xx\sec.\psi^2)}$, quae distantia Solis a Luna LS breuitatis gratia ponatur $= u$, vt sit $u = \sqrt{(yy - 2xy\cos(\varphi-\theta) + xx\sec.\psi^2)}$. His ergo valoribus introductis erunt vires nostrae:

1. $V = (M+L)\cos\psi + \frac{Nx}{u} + S\cos(\varphi-\theta) - \frac{Ny}{u}\cos(\varphi-\theta)$

2. $T = -S\sin(\varphi-\theta) + \frac{Ny}{u}\sin(\varphi-\theta)$

3. $R = -(M+L)\sin\psi + \frac{Nx\tang\psi}{u}$

§. 18. Quia nunc est $\tang\psi = \tang\varrho\sin(\varphi-\pi)$ et $\sin\psi = \tang\psi\cos\psi$ erit:

$$\frac{R}{\tang\varrho} = (M+L)\cos\psi\sin(\varphi-\pi) + \frac{Nx\sin(\varphi-\pi)}{u}$$

tum vero habebitur

$$V\sin(\varphi-\pi) + T\cos(\varphi-\pi) = (M+L)\cos\psi\sin(\varphi-\pi) + \frac{Nx\sin(\varphi-\pi)}{u}$$
$$+ S\cos(\varphi-\theta)\sin(\varphi-\pi) - \frac{Ny\cos(\varphi-\theta)\sin(\varphi-\pi)}{u}$$
$$- S\sin(\varphi-\theta)\cos(\varphi-\pi) + \frac{Ny\sin(\varphi-\theta)\cos(\varphi-\pi)}{u}$$

quae ob $\cos(\varphi-\theta)\sin(\varphi-\pi) - \sin(\varphi-\theta)\cos(\varphi-\pi) = \sin(\varphi-\pi)$, dat

V sin

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$$V \sin(\varphi-\pi) + T\cos(\varphi-\pi) - \frac{R}{\tang \varrho} = S\sin(\theta-\pi) - \frac{Ny}{u}\sin(\theta-\pi)$$

ex quo aequationes motum Lunae continentes erunt:

I. $2dxd\varphi + x\,dd\varphi = -\frac{1}{2}dt^2\left(\frac{Ny}{u} - S\right)\sin(\varphi-\theta)$

II. $ddx - xd\varphi^2 = -\frac{1}{2}dt^2\left((M+L)\cos\psi + \frac{Nx}{u} - \left(\frac{Ny}{u} - S\right)\cos(\varphi-\theta)\right)$

III. $d\pi = -\frac{1}{2}dt^2 \cdot \frac{\sin(\varphi-\pi)\sin(\theta-\pi)}{x\,d\varphi}\left(\frac{Ny}{u} - S\right)$

IV. $d.\, l\tang \varrho = \frac{dx}{\tang(\varphi-\pi)}$

vbi $\theta-\pi$ exprimit angulum Ω CS ſeu diſtantiam Solis a nodo aſcendente.

§. 19. Jam ſecundum Theoriam Newtoni, ſi maſſam Terrae ponamus $= \mathfrak{T}$ ac Lunae $= \mathfrak{C}$, ob diſtantiam $CL = \frac{x}{\cos\psi}$, foret vis $M = \frac{\mathfrak{T}\cos\psi^2}{x\,x}$ et vis $L = \frac{\mathfrak{C}\cos\psi^2}{x\,x}$, ſicque vis tota $M + L = (\mathfrak{T}+\mathfrak{C}) \cdot \frac{\cos\psi^2}{x\,x}$. Quo autem, ſi forte haec Theoria inſufficiens deprehendatur, rem generalius complectamur, ponamus hanc vim:

$$M + L = (\mathfrak{T}+\mathfrak{C})\cos\psi^2\left(\frac{1}{x\,x} - \frac{1}{b\,b}\right)$$

vbi terminus $\frac{1}{b\,b}$ defectum huius vis a Theoria Newtoniana exhibeat; qui cum ſit minimus, pro conſtanti haberi poterit ſaltem pro exigua variabilitate, quam diſtantia x ſubit. Vim autem Solis exacte Theoriae Newtonianae conformem aſſumere poterimus; quoniam etiamſi inde recederet, differentia non ſolum foret

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quam minima, ſed quia pro Luna aeque diſcreparet ac pro Terra, in noſtris formulis nullius plane eſſet momenti.

§. 20. Poſita ergo Solis maſſa $= \odot$, erit vis, qua Terram ad ſe attrahit $S = \frac{\odot}{yy}$, vis autem qua Lunam ad ſe trahit $N = \frac{\odot}{uu}$. His ergo valoribus virium in calculum inductis, motus lunae ex quatuor ſequentibus aequationibus determinari debet:

I. $2dxd\varphi + xdd\varphi = -\frac{1}{2}dt^2\left(\frac{\odot y}{u^3} - \frac{\odot}{yy}\right)$ ſin $(\varphi-\theta)$

II. $ddx - xd\varphi^2 = -\frac{1}{2}dt^2(\mathfrak{D}+\mathfrak{C})$ coſ $\psi^3\left(\frac{1}{xx} - \frac{1}{bb}\right)$
$$-\tfrac{1}{2}dt^2\left(\frac{\odot x}{u^3} - \frac{\odot y}{u^3}\text{ coſ }(\varphi-\theta) + \frac{\odot}{yy}\text{coſ}(\varphi-\theta)\right)$$

III. $d\pi = -\frac{1}{2}dt^2 \cdot \frac{\text{ſin }(\varphi-\pi)\text{ ſin }(\theta-\pi)}{xd\varphi}\left(\frac{\odot y}{u^3} - \frac{\odot}{yy}\right)$

IV. $d.l\text{ tang }\varrho = \frac{d\pi}{\text{tang}(\varphi-\pi)}$

Hic iam primum curandum eſt, vt elementum temporis, quod eſt quantitas heterogenea, ex calculo eliminemus; id quod commodiſſime fiet, ſi motum medium ſolis vtpote tempori proportionalem, loco temporis in calculum introducemus.

§. 21. Cum igitur etiam motus Solis in his aequationibus ſit ratio habenda, eum prius inueſtigemus: et quoniam pro terra quieſcente Sol a ſola vi $\frac{\odot}{yy}$ ad terram

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ram ſollicitari concipiendus eſt, ſi formulas pro Luna inuentas ad ſolem accommodemus, obtinebimus:

$$2\,dy\,d\theta + y\,dd\theta = 0$$

$$dd y - y\,d\theta^2 = -\tfrac{1}{2}\,dt^2 . \frac{\odot}{yy}$$

ſi iam diſtantiam Solis a terra mediam ponamus $= b$ ejusque anomaliam mediam $= q$; caſu quo excentricitas orbitae ſolaris eſſet nulla, foret ſemper $y = b$ & $d\theta = dq$: vnde altera aequatio dabit $-b\,dq^2 = -\tfrac{1}{2}\,dt^2 . \frac{\odot}{bb}$. Quare loco elementi temporis dt elementum anomaliae mediae ſolis ita in calculum introduci debet, vt vbique loco $\tfrac{1}{2}\,dt^2$ ſcribatur $\frac{b^3\,dq^2}{\odot}$, id quod tam in his formulis pro Sole, quam in ſuperioribus pro Luna fieri poterit.

§. 22. Cum iam b denotet diſtantiam ſolis a terra mediam, ſit eius vera diſtantia $y = b\,\omega$, et anomalia eius vera $= s$, erit $d\theta = ds$, quandoquidem a motu apogei ſolis animum abſtrahimus. Hinc itaque erit

$$2\,d\omega\,ds + \omega\,dd s = 0$$

$$dd\omega - \omega\,ds^2 = -\frac{dq^2}{\omega\omega},$$

quarum prior integrata dat $\omega\omega\,ds = C\,dq$ ob dq conſtans, ideoque $\omega\,ds^2 = \frac{CC\,dq^2}{\omega^3}$; qui valor in altera aequatione ſubſtitutus praebet,

$$dd\omega = \frac{CC\,dq^2}{\omega^3} - \frac{dq^2}{\omega\omega}.$$

quae

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quae per $2\,d\omega$ multiplicata et integrata dat:

$$\frac{d\omega^2}{dq^2} = D - \frac{CC}{\omega\omega} + \frac{2}{\omega}$$

vnde fit $dq = \frac{\omega\, d\omega}{\sqrt{(D\omega\omega + 2\omega - CC)}}$

ac proinde $ds = \frac{C\, d\omega}{\omega\sqrt{(D\omega\omega + 2\omega - CC)}}$

§. 23. Quanquam autem hinc valores finiti haud difficulter deduci possent, tamen alia vtar methodo, quae in motu Lunae maiorem praestabit vtilitatem. Inuento autem $\omega\omega\, ds = C\, dq$, alteram aequationem ita transformo, vt elementi constantis dq ratio non amplius habeatur:

$$dq.\, d.\frac{d\omega}{dq} - \omega\, ds^2 = -\frac{dq^2}{\omega\omega}$$

Sit nunc $\omega = \frac{1}{u}$, vt habeat $ds = C\,u\,u\,dq$, et $d\omega = -\frac{du}{uu}$, et ob $dq = \frac{ds}{C\,u\,u}$ erit $\frac{d\omega}{dq} = -\frac{C\,du}{ds}$; hinc sumto iam elemento ds constante, erit

$$\frac{-d^2s}{C\,u\,u}\cdot\frac{C\,ddu}{ds} - \frac{ds^2}{u} = -\frac{ds^2}{CC\,u\,u} \text{ seu}$$

$$ddu + u\,ds^2 = \frac{ds^2}{CC}$$

vnde statim elicitur $u = \frac{1 - e\cos s}{CC}$, vbi e excentricitatem orbitae solaris indicabit.

§. 24. Hinc porro habebitur $\omega = \frac{CC}{1-e\cos s}$, et $y = \frac{CC\,b}{1-e\cos s}$, anomalia vera s ab apogeo computata; vnde distantia apogei

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apogei a terra posito $s = 0$ erit $= \frac{CCb}{1-e}$, et distantia perigei posito $s = 180°$ prodit $= \frac{CCb}{1+e}$; sicque distantia media fiet $= \frac{CCb}{1-ee}$, quae cum per hypothesin aequalis esse debeat ipsi b, statui oportet $CC = 1-ee$: hincque erit

$$y = \frac{b(1-ee)}{1-e\cos s} \text{ et } \omega = \frac{1-ee}{1-e\cos s}$$

Porro autem aequatio $\omega\omega\, ds = C\, dq = dq\sqrt{(1-ee)}$ abibit in hanc:

$$dq = \frac{(1-ee)^{\frac{3}{2}}\, ds}{(1-e\cos s)^2} \text{ et } q = \int \frac{(1-ee)^{\frac{3}{2}}\, ds}{(1-e\cos s)^2}$$

ex qua, vti satis constat, vel data anomalia vera s inueniri potest anomalia media q, vel viciſſim. His itaque formulis motum Solis continentibus in determinatione motus Lunae vtamur.

§. 25. Primo ergo loco $\frac{1}{2}ds^2$ vbique scribamus $\frac{b^3\, dq^2}{\odot}$ et $b\omega$ loco y, quo facto nostrae aequationes fient

I. $2dx\,d\varphi + x\,dd\varphi = -b^3\, dq^2 \left(\frac{b\omega}{u^3} - \frac{1}{bb\omega\omega}\right) \sin(\varphi-\theta)$

II. $ddx - x\,d\varphi^2 = -\frac{(\mathfrak{E}+\mathfrak{C})\, b^3\, dq^2}{\odot} \cos\psi^3 \left(\frac{1}{xx} - \frac{1}{bb}\right)$
$$- b^3\, dq^2 \left(\frac{x}{u^3} - \frac{b\omega\cos(\varphi-\theta)}{u^3} + \frac{\cos(\varphi-\theta)}{bb\omega\omega}\right)$$

III. $d\pi = -b^3\, dq^2 \cdot \frac{\sin(\varphi-\pi)\sin(\theta-\pi)}{x\,d\varphi} \left(\frac{b\omega}{u^3} - \frac{1}{bb\omega\omega}\right)$

Ponatur porro $u = bv$, adque in calculum quoque introduca-

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ducatur distantia media lunae a terra, quae sit $= a$, positoque $x = az$, prodibit:

I. $2dzd\varphi + zdd\varphi = -\frac{bdq^2}{a}\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)\sin(\varphi - \theta)$

II. $ddz - zd\varphi^2 = -\frac{(\mathfrak{d} + \mathfrak{C})b^3}{\odot a^3}dq^2\cos\psi^3\left(\frac{1}{zz} - \frac{aa}{bb}\right)$
$$- \frac{zdq^2}{v^3} + \frac{b\omega dq^2\cos(\varphi-\theta)}{av^3} - \frac{bdq^2\cos(\varphi-\theta)}{a\omega\omega}$$

III. $d\pi = -\frac{bdq^2}{azd\varphi}\sin(\varphi-\pi)\sin(\theta-\pi)\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)$

§. 26. Ponamus nunc ad abbreuiandum:

$\frac{(\mathfrak{d}+\mathfrak{C})b^3}{\odot a^3} = m$; $\frac{(\mathfrak{d}+\mathfrak{C})b^3}{\odot abb} = \mu$, seu $\mu = \frac{maa}{bb}$

quarum litterarum valores m et μ per obseruationes definiri debent; tum vero sit $\frac{a}{b} = \nu$, quae est quantitas valde parua a parallaxi solis pendens. Hisque valoribus introductis, aequationes nostrae sequentes induent formas:

I. $2dzd\varphi + zdd\varphi = -\frac{1}{\nu}dq^2\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)(\sin\varphi - \theta)$

II. $ddz - zd\varphi^2 = -\frac{mdq^2\cos\psi^3}{zz} + \mu dq^2\cos\psi^3$
$$- \frac{zdq^2}{v^3} + \frac{1}{\nu}dq^2\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)\cos(\varphi - \theta)$$

III. $d\pi = -\frac{dq^2}{\nu zd\varphi}\sin(\varphi-\pi)\sin(\theta-\pi)\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)$

IV. $d.l\tan\varrho = \frac{d\pi}{\tan(\varphi-\pi)}$

§. 27.

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§. 27. Cum iam posuerimus:

$$x = az; \quad y = bw; \quad v = \frac{n}{b} \text{ et } \nu = \frac{a}{b}$$

erit $u = V(bb\omega\omega - 2ab\omega z \cos(\varphi-\theta) + aazz \sec.\psi^2)$
atque $v = V(\omega\omega - 2\nu\omega z \cos(\varphi-\theta) + \nu\nu zz \sec.\psi^2)$
vbi notandum est quantitates ω, q et s ex motu solis ita inter se pendere, vt sit

$$\omega = \frac{1-ee}{1-e\cos s} \text{ et } dq = \frac{(1-ee)^{\frac{3}{2}} ds}{(1-e\cos s)^2} = \frac{\omega\omega ds}{V(1-ee)}$$

ita vt huic ad datum quoduis tempus tam valor ipsius φ quam anomaliae verae s definiri possit. Modum autem has formulas ad calculum reuocandi hic non trado, quia eum alias fusius iam expossui: hoc solum hic notari conveniet, excentricitatis orbitae solaris valorem ex obseruationibus colligi $e = 0,01680$.

§. 28. Nunc antequam vlterius progredi queamus, valorem irrationalem ipsius v tolli conueniet, quod facile per seriem praestabitur maxime conuergentem, ob ν fractionem valde paruam; sumta enim parallaxi solis $= 12''$, quia parallaxis lunae media est $= 3380''$, erit $\frac{a}{b} = \nu = \frac{12}{3380} = \frac{1}{281}$. Hinc sufficit seriei illius conuergentis, quam reperiemus, aliquot tantum terminos ab initio assumsisse; quia reliqui ob paruitatem continuo magis crescentem tuto omitti poterunt. Cum autem angulus $\varphi-\theta$, qui distantiam solis a luna secundum longitudinem denotat, in hac resolutione frequentissime occurret, breuitatis gratia ponamus $\varphi - \theta = \eta$
ita vt pro v sequentem habeamus valorem irrationalem

$$v = V(\omega\omega - 2\nu\omega z \cos\eta + \nu\nu zz \sec.\psi^2).$$

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§. 29. Quoniam ergo in noſtris formulis occurrit $\frac{1}{v^3}$ ob $\frac{1}{\dot{v}^3} = (\omega\omega - 2v\omega z\cos\eta + vvzz\sec\psi^2)^{-\frac{3}{2}}$, nanciſcemur

$$\frac{1}{v^3} = \frac{1}{\omega^3} + \frac{3vz\cos\eta}{\omega^4} - \frac{3vvzz\sec.\psi^2}{2\omega^5} + \frac{15vvzz}{2\omega^5}\cos\eta^2$$

vbi terminos altiores ipſius v poteſtates inuoluentes ſine haeſitatione reiicere poſſumus; in ipſis aequationibus autem tantum in prima ipſius v poteſtate ſubſiſtemus. Habebimus ergo:

$$\frac{1}{v}\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right) = \frac{3z\cos\eta}{\omega^3} + \frac{3vzz}{2\omega^4}(5\cos\eta^2 - \sec.\psi^2) \text{ ſeu}$$

$$\frac{1}{v}\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right) = \frac{3z\cos\eta}{\omega^3} + \frac{3vzz}{4\omega^4}(5 + 5\cos 2\eta - 2\sec.\psi^2)$$

hincque porro:

$$\frac{1}{v}\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)\sin(\varphi-\theta) = \frac{3z\sin 2\eta}{2\omega^3} + \frac{3vzz}{8\omega^4}(5\sin\eta + 5\sin 3\eta - 4\sin\eta\sec.\psi^2)$$

$$\frac{1}{v}\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)\cos(\varphi-\theta) = \frac{3z}{2\omega^3}(1+\cos 2\eta) + \frac{3vzz}{8\omega^4}(15\cos\eta + 5\cos 3\eta - 4\cos\eta\sec.\psi^2)$$

atque $\frac{z}{v^3} = \frac{z}{\omega^3} + \frac{3vzz}{\omega^4}\cos\eta$

§. 30. Subſtituamus hos valores in noſtris aequationibus atque obtinebimus:

$$\text{I. } 2dz\,d\varphi + z\,ddz = -dq^2\left(\frac{3z\sin 2\eta}{2\omega^3} + \frac{3vzz}{8\omega^4}(5\sin\eta + 5\sin 3\eta - 4\sin\eta\sec.\psi^2)\right)$$

$$\text{II. } ddz - z\,d\varphi^2 = -\frac{m\,dq^2\cos\psi^3}{z\,z} + \mu\,dq^2\cos\psi^3 + \frac{z\,dq^2}{2\omega^3} + \frac{3z\,dq^2}{2\omega^3}\cos 2\eta$$

$$+ \frac{3vzz\,dq^2}{8\omega^4}(7\cos\eta + 5\cos 3\eta - 4\cos\eta.\sec.\psi^2)$$

III. $d\pi$

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III. $d\pi = -\frac{dq^2}{2d\Phi}\sin(\Phi-\pi)\sin(\theta-\pi)\left(\frac{3z\cos\eta}{\omega^3}+\frac{3zz}{4\omega^4}(5+5\cos 2\eta-2\sec.\psi^2)\right)$

IV. $d.l\tang\varrho = \frac{d\pi}{\tang(\Phi-\pi)}$.

Hic iam obſervare licet, cum angulus ψ nunquam fere 5° ſuperet, eiusque ſecans nonniſi in terminis tam per M multiplicatis, ac propterea reſpectu reliquorum valde parvis occurrat, fine vllius erroris ſenſibilis metu in his terminis poni poſſe $\sec.\psi = 1$.

§. 31. Deinde vt etiam ex maioribus terminis $\cos\psi$ eliminemus; conſideremus formulam $\tang\psi = \tang\varrho\sin(\Phi-\pi)$, eritque $\sec.\psi = \frac{1}{\cos\psi} = \sqrt{(1+\tang\varrho^2\sin(\Phi-\pi)^2)}$. Hinc ergo habebimus:

$$\cos\psi^3 = (1+\tang\varrho^2.\sin(\Phi-\pi)^2)^{-\frac{3}{2}}$$

et cum $\tang\varrho^2$ nunquam fere fractionem $\frac{1}{114}$ ſuperet erit ſatis exacte:

$$\cos\psi^3 = 1-\tfrac{3}{2}\tang\varrho^2.\sin(\Phi-\pi)^2 - \text{ vel etiam}$$
$$\cos\psi^3 = 1-\tfrac{3}{4}\tang\varrho^2+\tfrac{3}{4}\tang\varrho^2\cos 2(\Phi-\pi)$$

qui valor pro $\cos\psi^3$ in termino maiore $\frac{mdq^2\cos\psi^3}{zz}$ ſubſtitui poteſt: in altero autem termino $\mu dq^2\cos\psi^3$ quia per ſe eſt valde paruus, atque adeo ſecundum Theoriam Newtoni euaneſceret, nihil impedit, quo minus loco $\cos\psi^3$ ſcribamus vnitatem.

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§. 32. Hoc ergo modo si aequationes nostras a consideratione latitudinis Lunae ψ liberemus ad sequentes perueniemus aequationes:

I. $2dzd\Phi + zdd\Phi = -dq^2\left(\frac{3z\sin 2\eta}{2\omega^3} + \frac{3\gamma zz}{8\omega^4}(\sin\eta + 5\sin 3\eta)\right)$

II. $ddz - zd\Phi^2 = -\frac{mdq^2}{zz}(1 - \frac{3}{4}\tang\varrho^2 + \frac{3}{4}\tang\varrho^2\cos 2(\Phi - \pi)) + \mu dq^2$
$+ \frac{zdq^2}{2\omega^3} + \frac{3zdq^2}{2\omega^3}\cos 2\eta + \frac{3\gamma zzdq^2}{8\omega^4}(3\cos\eta + 5\cos 3\eta)$

III. $d\pi = -\frac{dq^2}{zd\Phi}\sin(\Phi - \pi)\sin(\theta - \pi)\left(\frac{3z\cos\eta}{\omega^3} + \frac{3\gamma zz}{4\omega^4}(3 + 5\cos 2\eta)\right)$

IV. $d.\,\ell\tang\varrho = \frac{d\pi}{\tang(\Phi - \pi)}$.

Nunc igitur in hoc erit incumbendum, vt ex his quatuor aequationibus omnia motus phaenomena, quae in Luna secundum Theoriam adesse debent, sollicite eruantur, atque tum cum obseruationibus conferantur.

CAPUT

CAPUT III.

INTRODUCTIO ANOMALIAE VERAE LUNAE IN PRAECEDENTES AEQUATIONES.

§. 33.

Quoniam noſtra quaeſtio circa Lunam verſatur, loco anomaliae mediae ſolis, quam pro tempore in calculum introduximus, magis conueniet motu Lunae medio vti, qui itidem tempori eſt proportionalis. Verum ex ſequentibus patebit calculum commodiorem reddi, ſi loco motus medii adhibeamus anomaliam Lunae mediam, cuius incrementa itidem tempori ſunt proportionalia. Sit itaque ad datum tempus anomalia media Lunae $= p$; et cum eius incrementum dp ad incrementum anomaliae mediae ſolis eodem tempuſculo acceptum datum ac per obſeruationes cognitam teneat rationem, ponamus $dp = n\,dq$. Tabulae autem Aſtronomicae pro interuallo 365 dierum praebent:
Motum anomaliae mediae Solis $11^s, 29^\circ, 44', 39'' = 1295079''$
Motum anomaliae mediae Lunae

$$13^{\text{Rev.}}\ 2^s, 28^\circ, 43' 13'' = 17167393''$$

vnde fit $n = \frac{17167393}{1295079} = 13,25586$

§. 34. Poſito ergo $\frac{dp}{n}$ loco dq, aequationes noſtrae erunt

I. $2dzd\varphi + zdd\varphi = -\frac{dp^2}{nn}\left(\frac{3z\sin 2\eta}{2\omega^3} + \frac{9zzz}{8\omega^4}(\sin\eta + 5\sin 3\eta)\right)$

II. ddz

II. $ddz - zd\Phi^2 = -\frac{mdp^2}{nnzz}(1 - \frac{3}{4}\tang\varrho^2 + \frac{3}{4}\tang\varrho^2\cos 2(\Phi - \pi)) + \frac{\mu dp^2}{nn}$

$$+ \frac{z\,dp^2}{2n^2\omega^3} + \frac{3z\,dp^2}{2n^2\omega^3}\cos 2\eta + \frac{3\nu zz dp^2}{8nn\omega^4}(3\cos\eta + 5\cos 3\eta)$$

III. $d\pi = \frac{-dp^2}{nnzd\Phi}\sin(\Phi - \pi)\sin(\theta - \pi)\left(\frac{3z\cos\eta}{\omega^3} + \frac{3\nu zz}{4\omega^4}(3 + 5\cos 2\eta)\right)$

IV. $d.\,l\tang\varrho = \frac{d\pi}{\tang(\Phi - \pi)}$

atque hic elementum dp assumtum est constans: simul autem patet terminos, qui per nn sunt diuisi, prae ceteris satis esse paruos, cum sit $nn = 175,71795$. Quae circumstantia sequentes approximationes non mediocriter adiuuabit.

§. 35. Nunc antequam vlterius progrediamur, aequationem primam per z multiplicemus, atque integratione in priori parte instituta obtinebimus

$$zzd\Phi = Cdp - \frac{dp}{nn}\int dp\left(\frac{3z^2\sin 2\eta}{2\omega^3} + \frac{3\nu z^3}{8\omega^4}(\sin\eta + 5\sin 3\eta)\right)$$

ponamus breuitatis gratia hoc membrum integrale

$$\int dp\left(\frac{3zz}{2\omega^4}\sin 2\eta + \frac{3\nu z^3}{8\omega^4}(\sin\eta + 5\sin 3\eta)\right) = S$$

quod integrale, ne introductio constantis incertitudinem pariat, ita capi assumo, vt nullum terminum mere constantem contineat, quippe qui iam in C esset comprehensus. Hoc ergo circa determinationem integrationis probe obseruato, erit $zzd\Phi = dp\left(C - \frac{S}{nn}\right)$: vbi terminus S aequabilem arearum descriptionem, quam Regula Keplerí in planetis primariis infert, perturbat; est enim

enim $\frac{1}{2} z z d\Phi$ elementum areae deſcriptae, quod ſi ipſi $C dp$ eſſet aequale, tempori exacte eſſet proportionale.

§. 36. Cum igitur ſit $d\Phi = \frac{dp}{zz}\left(C - \frac{S}{nn}\right)$, erit $z d\Phi^2 = \frac{dp^2}{z^3}\left(CC - \frac{2}{nn} CS + \frac{1}{n^4} SS\right)$, quo valore ſubſtituto reliquae noſtrae aequationes ſequentes induent formas:

II. $ddz = \frac{dp^2}{z^3}(CC - \frac{2}{nn} CS + \frac{1}{n^4} SS)$

$$- \frac{m dp^2}{n n z z}(1 - \tfrac{3}{4}\operatorname{tang}\varrho^2 + \tfrac{3}{4}\operatorname{tang}\varrho^2 \cos 2(\Phi - \pi)) + \frac{\mu dp^2}{nn}$$

$$+ \frac{z dp^2}{2nn\omega^3} + \frac{3z dp^2}{2nn\omega^3}\cos 2\eta + \frac{3\nu z z dp^2}{8nn\omega^4}(3\cos\eta + 5\cos 3\eta)$$

III. $d\pi = -\frac{z dp}{Cnn - S}\sin(\Phi - \pi)\sin(\theta - \pi)\left(\frac{3e\cos\eta}{\omega^3} + \frac{3\nu z z}{4\omega^4}(3 + 5\cos 2\eta)\right)$

et quarta manet $d.l\operatorname{tang}\varrho = \frac{d\pi}{\operatorname{tang}(\Phi - \pi)}$ vt ante.

Eo igitur pertigimus, vt inueſtigari oporteat quantitates z, π et ϱ, quibus inuentis obtinebitur Φ ex formula primum eruta. Cum autem ſit $d\eta = d\Phi - d\theta$, ob $d\theta = ds = \frac{dq\sqrt{(1-ee)}}{\omega\omega} = \frac{dp\sqrt{(1-ee)}}{n\omega\omega}$, erit $d\eta = \frac{dp}{zz}\left(C - \frac{S}{nn}\right) - \frac{dp\sqrt{(1-ee)}}{n\omega\omega}$. Tum vero eſt vti vidimus $\omega = \frac{1-ee}{1-e\cos s}$, vnde et huius differentiale ad dp reduci poterit.

§. 37. Si hunc calculum proſequi vellemus, tota inueſtigatio tandem eo rediret, vt definiretur quantum

longitudo Lunae vera ab eius longitudine media, quae ex anomalia media p haberetur, discreparet: hoc autem discrimen nonnunquam vltra 8 gradus exsurgere posset, ideoque correctiones admodum notabiles requireret. Vt igitur nobis quam minimae correctiones inuestigandae relinquantur, expediet differentiam inter locum Lunae verum, et locum corporis quod secundum regulas Kepleri in ellipsi circa Terram reuolueretur, ita tamen mobili, vt eius motus absidum cum motu apogei Lunae per obseruationes cognito conueniret. Seu quod eodem redit, quaeramus primo ex anomalia Lunae media p secundum regulas Kepleri anomaliam eius veram quae sit $= r$, vnde si longitudo apogei fuerit $= \pi$, quantita, $\pi + r$ nunquam multum vltra gradum a longitudine Lunae vera differet: vnde discrimen multo facilius inueniri poterit, si quidem debita orbitae lunaris excentricitas in calculum inducatur. Hinc loco anomaliae Lunae mediae p eius anomaliam veram, quae scilicet mediae pro excentricitate rite assumta conueniat, in aequationes nostras inferamus.

§. 38. Tabulae quidem astronomicae excentricitatem orbitae lunaris plerumque variabilem statuunt; sed cum hic non de vera huius orbitae excentricitate quaestio sit, quam de excentricitate illius orbitae ellipticae mobilis, in qua corpus motum proxime motum Lunae referat; huius excentricitas media erit statuenda inter maximam ac minimam, quae vulgo orbitae lunari tribuuntur: vnde ista excentricitas media colligitur $= 0,05445$. Ne autem huic conclusioni nimium fidamus genera-

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generatim hanc excentricitatem ponamus $= k$; atque anomalia vera per mediam ita determinabitur, vt sit $dp = \frac{(1-kk)^{\frac{3}{2}} dr}{(1-k\cos r)^2}$, vel sit brevitatis gratia $\frac{1-kk}{1-k\cos r} = t$, vt sit $dp = \frac{tt\,dr}{\sqrt{(1-kk)}}$. Porro autem reliqua differentialia ita ad elementum dr reuocabuntur, vt sit:

$$ds = \frac{ttdr\sqrt{(1-ee)}}{nn\omega\sqrt{(1-kk)}} = d\vartheta, \text{ et } d\eta = \frac{tt\,dr}{zz\sqrt{(1-kk)}}\left(C - \frac{S}{nn}\right) - \frac{tt\,dr\sqrt{(1-ee)}}{nn\omega\sqrt{(1-kk)}}.$$

§. 39. Si motus Lunae cum motu huius corporis, quod imaginamur, perfecte conueniret, tum vbique foret $z = \frac{1-kk}{1-k\cos r}$ seu $z = t$: quoniam autem hi duo motus inter se non conueniunt, non erit $z = t$. Ponamus ergo esse:

$$z = tu = \frac{(1-kk)u}{1-k\cos r} \text{ seu } x = \frac{(1-kk)au}{1-k\cos r}$$

vbi primum obseruo, quantitatem u valde parum ab vnitate recedere. Erit autem quantitas variabilis, quae alium terminum constantem praeter vnitatem non inuolvet: nam si alium terminum constantem contineret, is in a posset comprehendi, idque indicio esset distantiam mediam a non recte esse assumtam. Habebit ergo u huiusmodi formam $1 + Z$, vbi Z ex terminis nonnisi variabilibus constabit. Praeterea autem animaduerto, hanc quantitatem Z nullum terminum huius formae $\alpha\cos r$ complecti debere; quoniam hoc indicio esset excentricitatem k non recte esse assumtam, sed eam vel maiorem vel minorem accipi oportuisse.

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§. 40. His igitur notatis, quod quantitas u primo terminum conſtantem $= 1$ contineat, tum vero nullum terminum formae $\alpha \cos r$ inuoluat, ſtatuamus $z = tu$ ſeu $z = \frac{(1-kk)u}{1-k\cos r}$ poſito breuitatis gratia $t = \frac{1-kk}{1-k\cos r}$. Atque cum ſupra elementum dp conſtans poſuiſſemus, hac conditione exuenda erit $ddz = dp\, d.\frac{dz}{dp}$, et $\frac{ddz}{dp^2} = \frac{1}{dp}\, d.\frac{dz}{dp}$ Diuiſa ergo ſecunda aequatione per dp^2, erit:

II. $$\frac{1}{dp}\, d.\, \frac{dz}{dp} = \frac{CC}{t^3u^3} - \frac{2CS}{nnt^3u^3} + \frac{SS}{n^4t^3u^3} - \frac{m}{nnttuu}\left(1-\tfrac{3}{4}\tang\varrho^2+\tfrac{3}{4}\tang\varrho^2\cos 2(\Phi-\pi)\right)+\frac{\mu}{nn}+\frac{tu}{2nn\omega^3} + \frac{3tu\cos 2\eta}{2nn\omega^3}+\frac{3vttuu}{8nn\omega^4}(3\cos\eta+5\cos 3\eta)$$

III. $$d\pi = \frac{-tu\,dp}{Cnn-S}\sin(\Phi-\pi)\sin(\theta-\pi)\left(\frac{3tu\cos\eta}{\omega^3}+\frac{3vttuu}{4\omega^4}(3\mp 5\cos 2\eta)\right)$$

vbi nunc nullum differentiale aſſumtum eſt conſtans, ſed iam pro lubitu quoduis differentiale conſtans aſſumi poterit.

§. 41. Poſito autem $x = tu$ et $dp = \frac{ttdr}{\sqrt{(1-kk)}}$ exiſtente $t = \frac{1-kk}{1-k\cos r}$ erit primo:

$$S = \int \frac{ttdr}{\sqrt{(1-kk)}}\left(\frac{3ttuu}{2\omega^3}\sin 2\eta+\frac{3vt^3u^3}{8\omega^4}(\sin\eta+5\sin 3\eta)\right) \text{ ſeu}$$

$$S = \int \frac{dr}{\sqrt{(1-kk)}}\left(\frac{3t^4uu}{2\omega^3}\sin 2\eta+\frac{3vt^5u^3}{8\omega^4}(\sin\eta+5\sin 3\eta)\right)$$

Hinc

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Hinc fiet $d\Phi = \frac{dr}{nn\sqrt{(1-kk)}}\left(C - \frac{S}{nn}\right)$ atque

$$d\eta = \frac{dr}{uu\sqrt{(1-kk)}}\left(C - \frac{S}{nn}\right) - \frac{tt\,dr\sqrt{(1-ee)}}{n\omega\omega\sqrt{(1-kk)}}$$

Porro autem ob $dz = t\,du + u\,dt$, erit $\frac{dz}{dp} = \frac{t\,du + u\,dt}{tt\,dr}\sqrt{(1-kk)}$;

at eſt $dt = -\frac{(1-kk)k\,dr\sin r}{(1-k\cos r)^2} = -\frac{k\,tt\,dr\sin r}{1-kk}$; ſicque fiet

$\frac{dz}{dp} = \frac{du\sqrt{(1-kk)}}{t\,dr} - \frac{ku\sin r}{\sqrt{(1-kk)}}$; ac poſito elemento dr conſtante

erit $d.\frac{dz}{dp} = \frac{ddu\sqrt{(1-kk)}}{t\,dr} - \frac{du\,dt\sqrt{(1-kk)}}{tt\,dr} - \frac{k\,du\sin r}{\sqrt{(1-kk)}} - \frac{ku\,dr\cos r}{\sqrt{(1-kk)}}$

hincque ob $\frac{dt}{tt} = -\frac{k\,dr\sin r}{1-kk}$ habebitur:

$$d.\frac{dz}{dp} = \frac{ddu\sqrt{(1-kk)}}{t\,dr} - \frac{ku\,dr\cos r}{\sqrt{(1-kk)}}.$$

§. 42. Hinc iam porro obtinemus pro ſecunda aequatione

$$\frac{1}{dp}\,d.\frac{dz}{dp} = \frac{(1-kk)\,ddu}{t^3\,dr^2} - \frac{ku\cos r}{tt}$$

qui valor ſubſtitutus in aequatione per $\frac{t^3}{1-kk}$ multiplicata orietur haec aequatio:

II. $$\frac{ddu}{dr^2} - \frac{ktu\cos r}{1-kk} = \frac{CC}{(1-kk)u^3} - \frac{2CS}{(1-kk)nnu^3} + \frac{SS}{n^4(1-kk)u^3}$$
$$- \frac{m\,t}{nn(1-kk)uu}(1 - \tfrac{3}{4}\tan g\varrho^2 + \tfrac{3}{4}\tan g\varrho^2\cos 2(\Phi-\pi)) + \frac{\mu\,t^3}{nn(1-kk)} + \frac{t^4\,u}{2nn(1-kk)\omega^3}$$
$$+ \frac{3t^4\,u\cos 2\eta}{2nn(1-kk)\omega^3} + \frac{3vt^5\,un}{8nn\omega^4(1-kk)}(3\cos\eta + 5\cos 3\eta)$$

III. $$d\pi = -\frac{uudr\sin(\Phi-\pi)\sin(\theta-\pi)}{(Cnn-S)\sqrt{(1-kk)}}\left(\frac{3t^4\cos\eta}{\omega^3} + \frac{3ut^5u}{4\omega^4}(3+5\cos 3\eta)\right)$$

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Quartam aequationem $d.l\tang\varrho = \frac{d\pi}{\tang(\Phi-\pi)}$, cum nullam mutationem ſubeat, ſuperfluum foret continuo repetere.

§. 43. Conueniet autem quantitates conſtantes C et m, quarum valores nondum nouimus, ſaltem vero proxime indagare, quo facilius deinceps ipſam aequationum reſolutionem dirigere queamus. Perſpicuum autem eſt, ſi omnes quantitates a ſitu ſolis pendentes ex calculo deleantur, tum vtique fieri debere $\omega = 1$. Cum igitur primum S ab angulo η pendeat, terminos tam S quam η inuoluentes omittamus, ac pro ω quidem ſcribamus 1; quia tantum determinationem ad verum accedentem requirimus, quem in finem quoque inclinationem orbitae negligamus. Hinc aequatio ſecunda dabit:

$$-\frac{kt\cos r}{1-kk} = \frac{CC}{1-kk} - \frac{mt}{nn(1-kk)} + \frac{\mu t^3}{n^2(1-kk)} + \frac{t^4}{2nn(1-kk)}$$ ſiue

$$CC = \frac{mt}{nn} - \frac{\mu t^3}{nn} - kt\cos r - \frac{t^4}{2nn}$$

Cum autem ſit $t = \frac{1-kk}{1-k\cos r} = 1 + k\cos r$ proxime, ob k valde paruum habebitur.

$$CC = \frac{m}{nn} - \frac{\mu}{nn} - \frac{1}{2nn}$$
$$+ \frac{m}{nn}k\cos r - \frac{3\mu k}{nn}\cos r - k\cos r - \frac{2k}{nn}\cos r$$

vnde perſpicuum eſſe oportere.

$$\frac{m}{nn} = 1 + \frac{2+3\mu}{nn} \text{ et } CC = 1 + \frac{3+4\mu}{2nn}$$

§. 44. His

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§. 44. His igitur constantium $\frac{m}{nn}$ et CC valoribus proximis inuentis ponamus esse reuera:

$$\frac{m}{nn} = 1 + \frac{2+3\mu+\gamma}{nn} \text{ et } CC = 1 + \frac{3+4\mu+\delta}{2nn} = \lambda\lambda$$

scribamus enim λ pro C, quia litteris maiusculis A, B, C, D etc. deinceps in operationibus sequentibus vtemur: sicque fiet

$$S = \frac{1}{\sqrt{(1-kk)}} \int dr \left(\frac{3t^3uu}{2\omega^3} \sin 2\eta + \frac{3vt^5u^3}{8\omega^4} (\sin\eta + 5\sin 3\eta) \right)$$

$$d\Phi = \frac{dr}{uu\sqrt{(1-kk)}} \left(\lambda - \frac{S}{nn} \right)$$

$$d\eta = \frac{dr}{uu\sqrt{(1-kk)}} \left(\lambda - \frac{S}{nn} \right) - \frac{ttdr\sqrt{(1-ee)}}{nn\omega\omega\sqrt{(1-kk)}}$$

$$\text{II. } \frac{(1-kk)\,ddu}{dr^2} = ktu\cos r + \frac{\lambda\lambda}{u^3} - \frac{2\lambda S}{nnu^3} + \frac{SS}{n^4u^3} + \frac{\mu t^3}{nn} + \frac{t^4u}{2nn\omega^3}$$
$$- \frac{mt}{nnuu} \left(1 - \tfrac{3}{4}\tan\varrho^2 + \tfrac{3}{4}\tan\varrho^2\cos 2(\Phi-\pi) \right)$$
$$+ \frac{3t^4u\cos 2\eta}{2nn\omega^3} + \frac{3vt^5uu}{8nn\omega^4} (3\cos\eta + 5\cos 3\eta)$$

$$\text{III. } d\pi = -\frac{uudr\sin(\Phi-\pi)\sin(\theta-\pi)}{(\lambda nn - S)\sqrt{(1-kk)}} \left(\frac{3t^4}{\omega^3}\cos\eta + \frac{3vt^5u}{4\omega^4}(3+5\cos 2\eta) \right)$$

§. 45. Ponatur $\lambda = u\sqrt{(1-kk)}$, vt sit $uu = 1 + \frac{3+4\mu+\delta}{2nn}$ defectum enim in termino indefinito δ complecti licet, existente $m = nn + 2 + 3\mu + \gamma$; tum vero ponatur $S = (1-kk)^{\frac{3}{2}} \int R dr$, vt sit $R = \frac{dS}{dr\sqrt{(1-kk)}}$; ac si pro t et ω valores restituamus, qui erant,

$t = \frac{1-kk}{1-k\cos r}$ et $\omega = \frac{1-ee}{1-e\cos s}$ habebimus:

$$R =$$

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$$R = \frac{3(1-kk)^2(1-e\cos s)^2}{2(1-ee)^3(1-k\cos r)^4}\,uu\sin 2\eta + \frac{3\nu(1-kk)^4(1-e\cos s)^4}{8(1-ee)^4(1-k\cos r)^5}\,u^3(\sin\eta + 5\sin 3\eta)$$

$$d\Phi = \frac{dr}{uu}\left(u - \frac{1}{nn}\int R\,dr\right);\quad d\theta = ds = \frac{(1-kk)^{\frac{3}{2}}(1-e\cos s)^2}{n(1-ee)^{\frac{3}{2}}(1-k\cos r)^2}\,dr$$

$$\frac{d\eta}{dr} = \frac{u}{uu} - \frac{\int R\,dr}{nnuu} - \frac{(1-kk)^{\frac{3}{2}}(1-e\cos s)^2}{n(1-ee)^{\frac{3}{2}}(1-k\cos r)^2}$$

§. 46. Aequatio autem secunda facta hac substitutione, si per $1-kk$ diuidatur, abibit in sequentem:

II. $$\frac{ddu}{dr^2} = \frac{ku\cos r}{1-k\cos r} + \frac{uu}{n^3} - \frac{2u\int R\,dr}{nnu^3} + \frac{(\int R\,dr)^2}{n^4u^3} + \frac{u(1-kk)^2}{nn(1-k\cos r)^3} - \frac{n}{nn(1-k\cos r)uu}(1-\tfrac{3}{4}\tan g\varrho^2 + \tfrac{3}{4}\tan g\varrho^2\cos 2(\varphi-\pi)) + \frac{(1-kk)^3(1-e\cos s)^3}{2nn(1-ee)^3(1-k\cos r)^4}\,u(1+3\cos 2\eta) + \frac{3\nu(1-kk)^4(1-e\cos s)^4}{8nn(1-ee)^4(1-k\cos r)^5}\,u^2(3\cos\eta + 5\cos 3\eta)$$

III. $$dz = -\frac{dr\sin(\varphi-\pi)\sin(\theta-\pi)}{(unn - R\int dr)}\left(\frac{3(1-kk)^3(1-e\cos s)^3}{(1-ee)^3(1-k\cos r)^4}\,nu\cos\eta + \frac{3\nu(1-kk)^4(1-e\cos s)^4}{4(1-ee)^4(1-k\cos r)^5}\,u^2(3+5\cos 2\eta)\right)$$

Ac si ε denotet inclinationem mediam orbitae lunaris, quantitas $1-\frac{3}{4}\tan g\varrho^2 + \frac{3}{4}\tan g\varrho^2\cos 2(\varphi-\pi)$ in has duas partes discerpi poterit:
$(1-\frac{3}{4}\tan g\varepsilon^2) + \frac{3}{4}(\tan g\varepsilon^2 - \tan g\varrho^2 + \tan g\varrho^2\cos 2(\varphi-\pi))$
quarum illa est constans, haec vero proprie a nodo et inclinatione pendet.

§. 47.

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§. 47. Euoluamus autem producta illa ex t et ω orta, et quoniam excentricitates k et e sunt valde parvae, sufficit ad eos vsque terminos tantum progredi, qui coefficientes habeant kk, ek et ee, eosque qui per altiores potestates sint multiplicati omittere. Hinc erit:

$$\frac{1}{1-k\cos r} = 1 + \tfrac{1}{2}kk + k\cos r + \tfrac{1}{2}k^2\cos 2r$$

$$\frac{k\cos r}{1-k\cos r} = \tfrac{1}{2}kk + k\cos r + \tfrac{1}{2}k^2\cos 2r$$

$$\frac{(1-kk)^2}{(1-k\cos r)^3} = 1 + 3k\cos r, \text{ quia hic terminus per } \mu \text{ multiplicatur.}$$

$$\frac{(1-kk)^{\frac{3}{2}}}{(1-k\cos r)^2} = 1 + 2k\cos r + \tfrac{3}{2}kk\cos 2r$$

$$\frac{(1-kk)^3}{(1-k\cos r)^4} = 1 + 2kk + 4k\cos r + 5kk\cos 2r$$

$$\frac{(1-kk)^4}{(1-k\cos r)^5} = 1 + 5k\cos r, \text{ quia hic terminus iam per } \nu \text{ est multiplicatus.}$$

§. 48. Porro vero pro terminis ex ω enatis est:

$$\frac{(1-e\cos s)^2}{(1-ee)^{\frac{3}{2}}} = 1 + 2ee - 2e\cos s + \tfrac{1}{2}ee\cos 2s$$

$$\frac{(1-e\cos s)^3}{(1-ee)^3} = 1 + \tfrac{9}{2}ee - 3e\cos s + \tfrac{3}{2}ee\cos 2s$$

$$\frac{(1-e\cos s)^4}{(1-ee)^4} = 1 - 4e\cos s, \text{ quia hic factor tantum in minimis terminis occurrit.}$$

Hinc ergo colligimus:

$$\frac{(1-kk)^{\frac{3}{2}}(1-e\cos s)^2}{(1-ee)^{\frac{3}{2}}(1-k\cos r)^2} = 1 + 2ee + 2k\cos r + \tfrac{3}{2}kk\cos 2r - 2e\cos s - 2ek\cos(r+s) - 2ek\cos(r-s) + \tfrac{1}{2}ee\cos 2s$$

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$$\frac{(1-kk)^3(1-e\cos s)^3}{(1-ee)^3(1-k\cos r)^4} = 1 + 2kk + \tfrac{9}{2}ee + 4k\cos r + 5kk\cos 2r - 3e\cos s - 6ek\cos(r-s) - 6ek\cos(r+s) + \tfrac{3}{2}ee\cos 2s$$

$$\frac{(1-kk)^4(1-e\cos s)^4}{(1-ee)^4(1-k\cos r)^5} = 1 + 5k\cos r - 4e\cos s,$$

atque hinc fiet: $d\Phi = \frac{dr}{uu}(u - \frac{1}{nn}\int R\,dr)$ atque

$$\frac{ds}{dr} = \frac{1+2ee}{n} + \frac{2k}{n}\cos r - \frac{2e}{n}\cos s - \frac{2ek}{n}\cos(r-s) + \frac{3kk}{2n}\cos 2r + \frac{ee}{2n}\cos 2s - \frac{2ek}{n}\cos(r+s)$$

$$\frac{d\eta}{dr} = \frac{u}{uu} - \frac{\int R\,dr}{nnuu} - \frac{1-2ee}{n} - \frac{2k}{n}\cos r + \frac{2e}{n}\cos s + \frac{2ek}{n}\cos(r-s) - \frac{3kk}{2n}\cos 2r - \frac{ee}{2n}\cos 2s + \frac{2ek}{n}\cos(r+s)$$

§. 49. Introductis nunc his valoribus euolutis in formulas noſtras, iisque, qui per ſinum coſinumue alterius anguli ſunt multiplicati, pariter ſecundum ſimplices angulos explicatis, obtinebimus primum valorem ipſius R, qui erit:

$$R = \tfrac{3}{2}u^2\begin{cases}(1+2kk+\tfrac{9}{2}ee)\sin 2\eta + 2k\sin(2\eta-r) + 2k\sin(2\eta+r)\\ +\tfrac{5}{2}kk\sin(2\eta-2r) + \tfrac{5}{2}kk\sin(2\eta+2r)\\ -\tfrac{3}{2}e\sin(2\eta-s) - \tfrac{3}{2}e\sin(2\eta+s)\\ +\tfrac{3}{4}ee\sin(2\eta-2s) + \tfrac{3}{4}ee\sin(2\eta+2s)\\ -3ek\sin(2\eta-r+s) - 3ek\sin(2\eta+r-s)\\ -3ek\sin(2\eta-r-s) - 3ek\sin(2\eta+r+s)\end{cases}$$

$$+\tfrac{3}{8}vu^3\begin{cases}\sin\eta + \tfrac{5}{2}k\sin(\eta-r) - 2e\sin(\eta-s)\\ 5\sin 3\eta + \tfrac{5}{2}k\sin(\eta+r) - 2e\sin(\eta+s)\\ +\tfrac{25}{2}k\sin(3\eta-r) - 10e\cos(3\eta-s)\\ +\tfrac{25}{2}k\sin(3\eta+r) - 10e\cos(3\eta+s)\end{cases}$$

§. 50.

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§. 50. Aequatio autem ſecunda principalis ſequentem induet formam:

$$\text{II.}\quad \frac{ddu}{dr^2} = \frac{nn}{u^3} - \frac{2u\int R\,dr}{nnu^3} + \frac{(\int R\,dr)^2}{n^4u^3}$$
$$+ \frac{3m\,\text{tang}\,\varrho^2}{4nnuu}(1-\cos 2(\Phi-\pi))(1+k\cos r)$$
$$- \frac{m}{nnuu}(1+\tfrac{1}{2}kk+k\cos r+\tfrac{1}{2}k^2\cos 2r)+\frac{\mu}{nn}(1+3k\cos r)$$
$$+ u\left(\tfrac{1}{2}kk + k\cos r + \tfrac{1}{2}kk\cos 2r\right)$$
$$+ \frac{u}{2nn}\left\{\begin{array}{l} 1+2kk+\tfrac{9}{2}ee+4k\cos r-3e\cos s-6ek\cos(r-s) \\ \quad +5kk\cos 2r+\tfrac{3}{2}ee\cos 2s-6ek\cos(r+s)\end{array}\right.$$
$$+ \frac{3u}{2nn}\left\{\begin{array}{l} (1+2kk+\tfrac{9}{2}ee)\cos 2\eta+2k\cos(2\eta-r)+2k\cos(2\eta+r) \\ +\tfrac{5}{2}kk\cos(2\eta-2r)+\tfrac{5}{2}kk\cos(2\eta+2r) \\ -\tfrac{3}{2}e\cos 2(2\eta-s)-\tfrac{3}{2}e\cos(2\eta+s) \\ +\tfrac{3}{4}ee\cos(2\eta-2s)+\tfrac{3}{4}ee\cos(2\eta+2s) \\ -3ek\cos(2\eta-r+s)-3ek\cos(2\eta+r-s) \\ -3ek\cos(2\eta-r-s)-3ek\cos(2\eta+r+s)\end{array}\right.$$
$$+ \frac{3vuu}{8nn}\left\{\begin{array}{l} 3\cos\eta+5\cos 3\eta+\tfrac{15}{2}k\cos(\eta-r)+\tfrac{15}{2}k\cos(\eta+r) \\ -6e\cos(\eta-s)-6e\cos(\eta+s)+\tfrac{21}{2}k\cos(\eta-r)+\tfrac{21}{2}k\cos(\eta+r) \\ -10e\cos(3\eta-s)-10e\cos(3\eta+s)\end{array}\right.$$

vbi terminos, qui adhuc vlteriori euolutione indigent, primo loco poſui, et cum terminus tang ϱ^2 implicans iam ſit valde paruus, in eius multiplicatore ſecundam ipſius k poteſtatem omiſi: ſin autem alicuius momenti videantur, loco $1 + k\cos r$ ſcribi poterit $1 + \frac{1}{2}kk + k\cos r + \frac{1}{2}kk\cos 2r$.

§. 51. Pro longitudine vero nodi inuenienda aequatio ſequens prodibit reſoluenda:

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$$d\pi = -\frac{3uudr\sin(\Phi-\pi)\sin(\theta-\pi)}{unn-\int R dr}\cos\eta\,(1+2kk+\tfrac{3}{2}ee+4k\cos r$$
$$+5kk\cos 2r-3e\cos s+\tfrac{3}{2}ee\cos 2s)$$
$$-\frac{3vu^3dr\sin(\Phi-\pi)\sin(\theta-\pi)}{4(unn-\int R dr)}(3+5\cos 2\eta)(1+5k\cos r)$$

At est $\sin(\Phi-\pi)\sin(\theta-\pi) = \frac{1}{2}\cos\eta - \frac{1}{2}(\Phi+\theta-2\pi)$; vnde $\sin(\Phi-\pi)\sin(\theta-\pi)\cos\eta = \frac{1}{4}+\frac{1}{4}\cos 2\eta - \frac{1}{4}\cos 2(\Phi-\pi) - \frac{1}{4}\cos 2(\theta-\pi)$ et $\sin(\Phi-\pi)\sin(\theta-\pi)\cos 2\eta = \frac{1}{4}\cos\eta + \frac{1}{4}\cos 3\eta$
$$-\tfrac{1}{4}\cos(3\Phi-\theta-2\pi) - \tfrac{1}{4}\cos(3\theta-\Phi-2\pi)$$

Tum vero ob $\int R dr$ valde paruum prae unn, erit satis exacte

$$\frac{1}{unn-\int R\,dr} = \frac{1}{unn} + \frac{\int R dr}{uunn^4} + \frac{(\int R dr)^2}{u^3n^6}$$

vbi quidem postremus terminus tuto omitti potest.

§. 52. Praeterea vero ponatur $u = 1 + \frac{v}{nn}$, vt sit $ddu = \frac{ddv}{nn}$, et reiectis terminis per n^4 diuisis, qui iam per exiguam quantitatem sunt multiplicati, erit

$$\frac{d\Phi}{dr} = u - \frac{2uv}{nn} + \frac{3uv^2}{n^4} - \frac{\int R dr}{nn} + \frac{2v\int R dr}{n^4}$$

$$\frac{d\eta}{dr} = u - \frac{1-2ee}{n} - \frac{2k}{n}\cos r + \frac{2e}{n}\cos s - \frac{3kk}{2n}\cos 2r - \frac{ee}{2n}\cos 2s$$
$$+\frac{2ek}{n}\cos(r-s) + \frac{2ek}{n}\cos(r+s)$$
$$-\frac{2uv-\int R dr}{nn} + \frac{3uv^2+2v\int R dr}{n^4}$$ atque

$$R = \tfrac{3}{2}(1+2kk\mp\tfrac{8}{3}ee)\sin 2\eta + 3k\sin(2\eta-r) - \tfrac{2}{4}e\sin(2\eta-s)$$
$$+3k\sin(2\eta+r) - \tfrac{2}{4}e\sin(2\eta+s)$$
$$+\frac{3v}{nn}\sin 2\eta + \tfrac{15}{4}kk\sin(2\eta-2r) + \tfrac{8}{8}ee\sin(2\eta-2s)$$
$$+\tfrac{15}{4}kk\sin(2\eta+2r) + \tfrac{8}{8}ee\sin(2\eta+2s)$$
$$+\frac{3vv}{2n^4}\sin 2\eta - \tfrac{8}{2}ek\sin(2\eta-r+s) - \tfrac{8}{2}ek\sin(2\eta+r-s)$$
$$-\tfrac{8}{2}ek\sin(2\eta-r-s) - \tfrac{8}{2}ek\sin(2\eta+r+s)$$
$$+$$

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$$+\frac{6kv}{nn}\sin(2\eta-r)+\frac{6kv}{nn}\sin(2\eta+r)$$
$$-\frac{9ev}{2nn}\sin(2\eta-s)-\frac{9ev}{2nn}\sin(2\eta+s)$$
$$+\tfrac{3}{8}v\sin\eta+\tfrac{15}{8}vk\sin(\eta-r)-\tfrac{3}{4}ve\sin(\eta-s)$$
$$+\tfrac{7}{8}\tfrac{5}{8}vk\sin(3\eta-r)-\tfrac{15}{4}ve\sin(3\eta-s)$$
$$+\tfrac{15}{8}v\sin 3\eta+\tfrac{15}{8}vk\sin(\eta+r)-\tfrac{3}{4}ve\sin(\eta+s)$$
$$+\tfrac{7}{8}\tfrac{5}{8}vk\sin(3\eta+s)-\tfrac{15}{4}ve\sin(3\eta+s)$$

§. 53. Ipſa vero aequatio ſecunda per hanc ſubſtitutionem, poſtquam per nn fuerit multiplicata, in formam ſequentem abibit.

II. $$\frac{ddv}{dr^2}=uunn-3uuv+\frac{6uvv}{nn}-2u\int R\,dr$$
$$+\frac{6uv}{nn}\int R\,dr+\frac{1}{nn}\left(\int R\,dr\right)^2$$
$$+\frac{3m\tan g\varrho^2}{4}(1-\cos 2(\varphi-\pi))\left(1+\tfrac{1}{2}kk+k\cos r+\tfrac{1}{2}kk\cos 2r\right)$$
$$-\frac{3mv\tan g\varrho^2}{2nn}(1-\cos 2(\varphi-\pi))\quad(1+k\cos r)$$
$$-\frac{3mvv}{n^4}(1+k\cos r)-m\left(1+\tfrac{1}{2}kk+k\cos r+\tfrac{1}{2}kk\cos 2r\right)$$
$$+\frac{2mv}{nn}\left(1+\tfrac{1}{2}kk+k\cos r+\tfrac{1}{2}kk\cos 2r\right)+nn\left(\tfrac{1}{2}kk+k\cos r+\tfrac{1}{2}kk\cos 2r\right)$$
$$+v\left(\tfrac{1}{2}kk+k\cos r+\tfrac{1}{2}kk\cos 2r\right)+\mu(1+3k\cos r)$$
$$+\tfrac{1}{2}+kk+\tfrac{3}{4}ee+2k\cos r-\tfrac{3}{2}e\cos s-3ek\cos(r-s)$$
$$+\tfrac{5}{2}kk\cos r+\tfrac{3}{4}ee\cos 2s-3ek\cos(r+s)$$
$$+\frac{v}{2nn}(1+4k\cos r-3e\cos s)$$
$$+\tfrac{3}{2}(1+2kk+\tfrac{3}{2}ee)\cos 2\eta+3k\cos(2\eta-r)-\tfrac{3}{4}e\cos(2\eta-s)$$
$$+3\cos(2\eta+r)-\tfrac{3}{4}e\cos(2\eta+s)$$

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$$+\tfrac{15}{4}kk\cos(2\eta-2r)+\tfrac{9}{8}ee\cos(2\eta-2s)$$
$$+\tfrac{15}{4}kk\cos(2\eta+2r)+\tfrac{9}{8}ee\cos(2\eta+2s)$$
$$-\tfrac{9}{2}ek\cos(2\eta-r+s)-\tfrac{9}{2}ek\cos(2\eta+r-s)$$
$$-\tfrac{9}{2}ek\cos(2\eta-r-s)-\tfrac{9}{2}ek\cos(2\eta+r+s)$$

$$+\frac{3v}{2nn}\left\{\begin{array}{l}\cos 2\eta+2k\cos(2\eta-r)+2k\cos(2\eta+r)\\ -\tfrac{3}{2}e\cos(2\eta-s)-\tfrac{3}{2}e\cos(2\eta+s)\end{array}\right.$$

$$+\tfrac{3}{8}v\left\{\begin{array}{l}+3\cos\eta+\tfrac{15}{4}k\cos(\eta-r)-6e\cos(\eta-s)\\ +5\cos 3\eta+\tfrac{15}{2}k\cos(\eta+r)-6e\cos(\eta+s)\\ +\tfrac{25}{2}k\cos(3\eta-r)-10e\cos(3\eta-s)\\ +\tfrac{25}{2}k\cos(3\eta+r)-10e\cos(3\eta+s)\end{array}\right.$$

$$+\frac{3vv}{4nn}(3\cos\eta+5\cos 3\eta)$$

§. 54. Cum autem fit $m=nn+2+3\mu+\gamma$ et $uu=1+\frac{3+4\mu+\delta}{2nn}$, ſi hi valores ſubſtituantur, plures termini ſe mutuo deſtruent, aequatioque prodibit ſequenti forma concinniori contenta: vbi quidem in terminis per ſe minimis loco m ſcribi licebit nn, et 1 loco uu vel u.

II. AEQUATIO.

$$\frac{ddv}{dr^2}=\tfrac{1}{2}\delta-\gamma+\tfrac{9}{4}ee-\gamma k\cos r+\tfrac{3}{2}kk\cos 2r$$
$$-2\left(1+\frac{3+4\mu+\delta}{4nn}\right)\int R\,dr+\frac{1}{nn}\left(\int R\,dr\right)^2$$
$$-v\left(1-\tfrac{3}{2}kk-3k\cos r-\tfrac{3}{2}kk\cos 2r\right)$$
$$+\frac{vv}{nn}(3-3k\cos r)-\tfrac{3}{2}e\cos s+\tfrac{3}{4}ee\cos 2s-3ek\cos(r-s)$$
$$-3ek\cos(r+s)+\tfrac{3}{2}(1+2kk+\tfrac{9}{8}ee)\cos 2\eta$$
$$+$$

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$$+ 3k\cos(2\eta - r) + \tfrac{13}{4}kk\cos(2\eta - 2r) - \tfrac{3}{4}e\cos(2\eta - s)$$
$$+ 3k\cos(2\eta + r) + \tfrac{13}{4}kk\cos(2\eta + 2r) - \tfrac{3}{4}e\cos(2\eta - s)$$
$$+ \tfrac{3}{8}ee\cos(2\eta - 2s) - \tfrac{3}{2}ek\cos(2\eta - r + s) - \tfrac{3}{2}ek\cos(2\eta - r - s)$$
$$+ \tfrac{3}{8}ee\cos(2\eta + 2s) - \tfrac{3}{2}ek\cos(2\eta + r - s) - \tfrac{3}{2}ek\cos(2\eta + r + s)$$

$$+ \frac{v}{nn}\begin{cases} 2\gamma - \tfrac{3}{2}\delta + 6k\cos r + 2(3\mu + \gamma)k\cos r + 6\int R\,dr \\ - \tfrac{3}{2}e\cos s + \tfrac{3}{2}\cos 2\eta + 3k\cos(2\eta - r) + 3k\cos(2\eta + r) \\ - \tfrac{3}{4}e\cos(2\eta - s) - \tfrac{3}{4}e\cos(2\eta + s) \end{cases}$$

$$+ \tfrac{3}{8}v\begin{cases} 3\cos\eta + \tfrac{13}{2}k\cos(\eta - r) - 6e\cos(\eta - s) \\ + 5\cos 3\eta + \tfrac{13}{2}k\cos(\eta + r) - 6e\cos(\eta + s) \\ + \tfrac{25}{2}k\cos(3\eta - r) - 10e\cos(3\eta - s) \\ + \tfrac{25}{2}k\cos(3\eta + r) - 10e\cos(3\eta + s) \end{cases}$$

$$+ \frac{3vv}{4nn}(3\cos\eta + 5\cos 3\eta)$$

$$+ \tfrac{3}{4}(nn + 2 + 3\mu + \gamma)\frac{(1-2v)}{nn}\tan g\varrho^2(1 - \cos 2(\varphi - \pi))$$
$$(1 + \tfrac{3}{2}kk + k\cos r + \tfrac{3}{2}kk\cos 2r)$$

§. 55. Pro loco nodi autem inveniendo prodibit sequens aequatio.

$$\frac{d\pi}{dr} = \frac{-3}{nnn}\left(1 + \frac{2nv + \int R\,dr}{nnn}\right)(1 + 2kk + \tfrac{3}{2}ee)$$

$$\begin{cases} \quad \tfrac{3}{4} + \tfrac{3}{4}\cos 2\eta - \tfrac{3}{4}\cos 2(\Phi - \pi) - \tfrac{3}{4}\cos 2(\theta - \pi) \\ + k\cos r + \tfrac{3}{2}k\cos(2\eta - r) - \tfrac{3}{2}k\cos(2\Phi - 2\pi - r) \\ - \tfrac{3}{4}e\cos s + \tfrac{3}{2}k\cos(2\eta + r) - \tfrac{3}{2}k\cos(2\Phi - 2\pi + r) \\ \qquad - \tfrac{3}{8}e\cos(2\eta - s) - \tfrac{3}{2}k\cos(2\theta - 2\pi - r) \\ \qquad - \tfrac{3}{8}e\cos(2\eta + r) - \tfrac{3}{2}k\cos(2\theta - 2\pi + r) \end{cases}$$

$$- \frac{3v}{4nnn}\begin{cases} \tfrac{13}{4}\cos\eta + \tfrac{5}{4}\cos 3\eta - \tfrac{3}{2}\cos(3\Phi + \theta - 2\pi) \\ - \tfrac{5}{4}\cos(3\Phi - \theta - 2\pi) - \tfrac{5}{4}\cos(3\theta - \Phi - 2\pi) \end{cases}$$

At

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At pro inclinatione orbitae habebitur:

$$\frac{d.l\,\text{tang}\frac{\varrho}{2}}{dr} = \frac{-3}{nn}\left(1+\frac{2nv+\int R\,dr}{nn}\right)\left(1+2kk+\tfrac{9}{8}ee\right)$$

$$\left\{\begin{array}{l} \frac{15}{4}\sin 2(\varphi-\pi) + \frac{15}{4}\sin 2(\theta-\pi) - \frac{15}{4}\sin 2\eta \\ -\frac{15}{2}k\sin(2\eta-r) + \frac{15}{2}k\sin(2\varphi-2\pi-r) \\ -\frac{15}{2}k\sin(2\eta+r) + \frac{15}{2}k\sin(2\varphi-2\pi+r) \\ +\frac{3}{8}e\sin(2\eta-s) + \frac{15}{2}k\sin(2\theta-2\pi-r) \\ +\frac{3}{8}e\sin(2\eta+s) + \frac{15}{2}k\sin(2\theta-2\pi+r) \end{array}\right.$$

$$-\frac{3v}{4nn}\left(-\tfrac{15}{4}\sin\eta-\tfrac{5}{4}\sin 3\eta+\tfrac{3}{2}\sin(\varphi+\theta-2\pi)\right.$$
$$\left.+\tfrac{5}{4}\sin(3\varphi-\theta-2\pi)+\tfrac{5}{4}\sin(3\theta-\varphi-2\pi)\right.$$

Quomodo igitur his aequationibus ad motum Lunae cognoscendum vti conueniat, in sequentibus capitibus videamus.

CAPUT

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CAPUT IV.

INUESTIGATIO INAEQUALITATIS LUNAE ABSOLUTAE, QUAE VARIATIO DICITUR.

§. 33.

Ex his aequationibus perſpicitur in determinationem motus Lunae plurimorum angulorum vel ſinus vel coſinus ingredi, qui anguli formantur per variam combinationem ſequentium 4 angulorum:

1. ex diſtantia Solis a Luna, quem angulum poſuimus $= \eta$
2. ex anomalia Lunae vera $= r$
3. ex anomalia Solis vera $= s$
4. ex diſtantia Lunae a nodo aſcendente $= \Phi - \pi$.

Ne igitur a tanta angulorum multitudine obruamur, a caſibus ſimplicioribus ordiamur: ac primo quidem in eas tantum motus inaequalitates inquiramus, quae a ſolo angulo η pendeant, neque idcirco excentricitatem vel Solis vel Lunae implicent, neque ab orbitae lunaris inclinatione ad eclipticam afficiantur.

§. 57. Has igitur inaequalitates, quae a ſolo ſitu Solis reſpectu Lunae naſcuntur, atque ab Aſtronomis ſub nomine variationis comprehendi ſolent, ex praecedentibus aequationibus eliciemus, ſi tam excentricitatem Lunae k quam ſolis e pro nihilo habeamus, atque inclinationem orbitae lunaris ad eclipticam euanescentem ſtatuamus, ita vt ſit $k = o$, $e = o$ et tang $\varrho = o$. Sic enim obtinebimus eas inaequalitates Lunae, quae ab his elementis non pendent, ideoque tantum per angu-

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lum η determinantur; quae cum vnica tabula comprehendi queant, haec tabula variationem Lunae indicare dicitur. Interim tamen hic animaduerti oportet, partem quandam exiguam variationis quoque ab excentricitate orbitae Lunae k pendere, quam partem deinceps supplebimus, cum huius excentricitatis rationem sumus habituri.

§. 58. Reiectis ergo terminis k, e, et tang ϱ continentibus, habebimus:

$$\frac{d\varphi}{dr} = \varkappa - \frac{2\varkappa v - \int R\,dr}{nn} + \frac{3\varkappa v^2 + 2v\int R\,dr}{n^4}$$

$$\frac{d\eta}{dr} = \varkappa - \frac{1}{n} - \frac{2\varkappa v - \int R\,dr}{nn} + \frac{3\varkappa v^2 + 2v\int R\,dr}{n^4}$$

$R = \frac{3}{2}\sin 2\eta + \frac{3v}{nn}\sin 2\eta + \frac{3}{8}v\sin\eta + \frac{15}{8}v\sin 3\eta$, ac denique

$$\frac{ddv}{dr^2} = \tfrac{1}{2}\delta - \gamma - 2\left(1 + \frac{3+4\mu+\delta}{4nn}\right)\int R\,dr + \frac{1}{nn}\left(\int R\,dr\right)^2 - v + \frac{3vv}{nn}$$
$$+ \tfrac{3}{2}\cos 2\eta + \frac{v}{nn}(2\gamma - \tfrac{3}{2}\delta) + \frac{3v\cos 2\eta}{2nn} + \frac{6v}{nn}\int R\,dr$$
$$+ \tfrac{8}{8}\cos\eta + \tfrac{15}{8}v\cos 3\eta$$

Hic autem notandum est esse $\varkappa = \sqrt{\left(1 + \frac{3+4\mu+\delta}{2nn}\right)}$; quoniam vero valores litterarum μ et δ demum cum per consensum obseruationum, tum per indolem calculi definire instituimus, hic ex obseruationibus petamus valores ipsius $\varkappa$; cum enim sit $\varkappa : 1 = d\varphi : dr$, hoc est vt motus Lunae medius ad motum anomaliae, erit $\varkappa = 1,0085272$. Fieri quidem potest, vt hic valor aliquantulum a vero differat, sed errorem si quis lateat infra detegemus, facillimeque emendabimus.

§. 59.

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§. 59. Cum igitur iam supra inuenerimus esse
$n = 13,25586$ ac proinde $nn = 175,71795$
erit $\frac{1}{n} = 0,075438$, ideoque $\alpha - \frac{1}{n} = 0,933089$
Hic autem numerus, qui iam quasi medium valorem rationis $\frac{d\eta}{dr}$ exprimit, in omnibus operationibus, quae sequuntur, frequentissime occurret, hincque breuitatis gratia ponamus

$$\alpha - \frac{1}{n} = a, \text{ vt sit } \alpha + \frac{1}{n} = V(+\frac{3+4\mu+\delta}{2nn})$$

eritque ergo $a = 0,933089$, qui valor quam minime a vero discrepat, vti mox patebit. Quod autem verus ipsius a valor aliquantulum diuersus esse possit, inde primo patet, quod minutias, quae ex terminis $\frac{3\alpha v^2 + 2v\int R dr}{n^4}$ quantitati constanti accrescere potuissent, hic neglexi-mus; tum vero fieri potest, vt ratio media differentialium $d\eta$ ad dr alia sit atque quantitatum finitarum η et r.

§. 60. Si has formulas attente contemplemur, mox deprehendemus valorem integralis $\int R dr$ constare ex cosinibus angulorum 2η, η, 3η, et 4η. Quanquam enim altiora quoque multipla huius anguli ingredientur, tamen facile patet, coefficientes eorum continuo fieri minores, ita vt in quadruplo tuto subsistere possimus: similis autem erit ratio valoris ipsius v. Hinc ponamus:
$\int R dr = \mathfrak{A} \cos 2\eta + \mathfrak{B} \cos 4\eta + \mathfrak{a} v \cos \eta + \mathfrak{b} v \cos 3\eta$
$v = A \cos 2\eta + B \cos 4\eta + a v \cos \eta + b v \cos 3\eta$
atque hos valores fictitios in formulis nostris substitua-

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mus, vt inde valores istorum coefficientium assumtorum determinare possimus: quippe qui modus aptissimus videtur ad cognitionem integralium perueniendi. Quia autem est circiter $v = \frac{1}{2\delta z}$, patet terminos per v^2 multiplicatos prae reliquis tam esse exiguos, vt eos qui multo fuerint minores, sine haesitatione praetermittere possimus.

§. 61. Per hos ergo valores assumtos consequemur:

$$\frac{d\varphi}{dr} = \alpha - \frac{(2\alpha A + \mathfrak{A})}{nn}\cos 2\eta - \frac{(2\alpha B + \mathfrak{B})}{nn}\cos 4\eta + \frac{A(3\alpha A + 2\mathfrak{A})}{2n^4}\cos 2\eta + \frac{A(3\alpha A + 2\mathfrak{A})}{2n^4}\cos 4\eta - \frac{(2\alpha a + \mathfrak{a})}{nn}v\cos\eta - \frac{(2\alpha b + \mathfrak{b})}{nn}v\cos 3\eta$$

atque ob $\alpha - \frac{1}{n} = \alpha$ erit minimis terminis omissis, quia hi in operatione multo magis diminuerentur:

$$\frac{d\eta}{dr} = \alpha - \frac{(2\alpha A + \mathfrak{A})}{nn}\cos 2\eta - \frac{(2\alpha B + \mathfrak{B})}{nn}\cos 4\eta - \frac{(2\alpha a + \mathfrak{a})}{nn}v\cos\eta - \frac{(2\alpha b + \mathfrak{b})}{nn}v\cos 3\eta$$

His positis erit:

$$\frac{d.\cos 2\eta}{dr} = -\sin 2\eta.\frac{2d\eta}{dr} = -2\alpha\sin 2\eta - \frac{(2\alpha B + \mathfrak{B})}{nn}\sin 2\eta + \frac{(2\alpha A + \mathfrak{A})}{nn}\sin 4\eta + \frac{(2\alpha a + \mathfrak{a})}{nn}v\sin\eta + \frac{(2\alpha a + \mathfrak{a})}{nn}v\sin 3\eta - \frac{(2\alpha b + \mathfrak{b})}{nn}v\sin\eta$$

d.

$$\frac{d.\cos 4\eta}{dr} = -\sin 4\eta.\frac{4d\eta}{dr} = -4\alpha\sin 4\eta + \frac{2(2\varkappa A+\mathfrak{A})}{\varkappa\varkappa}\sin 2\eta$$

$$\frac{d.\cos\eta}{dr} = -\sin\eta.\frac{d\eta}{dr} = -\alpha\sin\eta$$

$$\frac{d.\cos 3\eta}{dr} = -\sin 3\eta.\frac{3d\eta}{dr} = -3\alpha\sin 3\eta$$

§. 62. Quod si iam secundum has formulas quantitas integralis $\int R\,dr$ differentietur, obtinebitur:

$$R = (-2\alpha\mathfrak{A} - \frac{\mathfrak{A}(2\varkappa B+\mathfrak{B})}{\varkappa\varkappa} + \frac{2\mathfrak{B}(2\varkappa A+\mathfrak{A})}{\varkappa\varkappa})\sin 2\eta$$
$$+ (\frac{\mathfrak{A}(2\varkappa A+\mathfrak{A})}{\varkappa\varkappa} - 4\alpha\mathfrak{B})\sin 4\eta$$
$$+ (\frac{\mathfrak{A}(2\varkappa a+\mathfrak{a})}{\varkappa\varkappa} + \frac{\mathfrak{A}(2\varkappa b+\mathfrak{b})}{\varkappa\varkappa} - \alpha\mathfrak{a})\nu\sin\eta$$
$$+ (\frac{\mathfrak{A}(2\varkappa a+\mathfrak{a})}{\varkappa\varkappa} - 3\alpha b)\nu\sin 3\eta$$

Cum iam sit per hypothesin

$$R = \tfrac{1}{2}\sin 2\eta + \frac{3A}{2\varkappa\varkappa}\sin 4\eta + \tfrac{3}{8}\nu\sin\eta + \tfrac{3}{8}\nu\sin 3\eta$$
$$- \frac{3B}{2\varkappa\varkappa} + \frac{3a}{2\varkappa\varkappa}\nu + \frac{3a}{2\varkappa\varkappa}\nu$$
$$- \frac{3b}{2\varkappa\varkappa}\nu$$

prodibit terminis homogeneis comparandis:

$$2\alpha\mathfrak{A} = -\tfrac{1}{2} - \frac{\mathfrak{A}(2\varkappa B+\mathfrak{B}) + 2\mathfrak{B}(2\varkappa A+\mathfrak{A})}{\varkappa\varkappa} + \frac{3B}{2\varkappa\varkappa}$$

$$4\alpha\mathfrak{B} = -\frac{3A}{2\varkappa\varkappa} + \frac{\mathfrak{A}(2\varkappa A+\mathfrak{A})}{\varkappa\varkappa}$$

$$\alpha\mathfrak{a} = -\tfrac{1}{8} - \frac{3(a-b)}{2\varkappa\varkappa} + \frac{\mathfrak{A}(2\varkappa a+\mathfrak{a}) - \mathfrak{A}(2\varkappa b+\mathfrak{b})}{\varkappa\varkappa}$$

$$3\alpha b = -\tfrac{3}{8} - \frac{3a}{2\varkappa\varkappa} + \frac{\mathfrak{A}(2\varkappa a+\mathfrak{a})}{\varkappa\varkappa}$$

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§. 63. Aequatio autem noſtra differentio-differentialis, ſi pro $\int R\,dr$ et v valores aſſumti ſubſtituantur, ſequentem induet formam:

$$\frac{ddv}{dr^2} = (\tfrac{1}{2}\delta-\gamma) - 2\kappa\mathfrak{A}\cos 2\eta - 2\kappa\mathfrak{B}\cos 4\eta - 2\kappa a\nu\cos\eta - 2\kappa b\nu\cos 3\eta$$

$$+\frac{\mathfrak{A}\mathfrak{A}}{2nn} \qquad +\frac{\mathfrak{A}\mathfrak{A}}{2nn}$$

$$-A \qquad -B \qquad -a\nu \qquad -b\nu$$

$$+\frac{3AA}{2nn} \qquad +\frac{3AA}{2nn}$$

$$+\tfrac{1}{2}$$

$$+\frac{(4\gamma-3\delta)}{2nn}A \quad +\frac{(4\gamma-3\delta)}{2nn}B \quad +\frac{3b}{4nn}\nu$$

$$+\frac{3A}{4nn} \quad +\frac{3B}{4nn} \quad +\frac{3A}{4nn} \quad +\frac{3a}{4nn}\nu \quad +\frac{3a}{4nn}\nu$$

$$+\frac{3A\mathfrak{A}}{nn} \qquad +\frac{3A\mathfrak{A}}{nn} \qquad +\tfrac{5}{8}\nu \qquad +\tfrac{15}{8}\nu$$

vbi quidem perſpicuum eſt, quinam termini reſpectu reliquorum tam ſint parui, vt ſine errore deleri queant.

§. 64. Quaeramus ergo primum differentiale $\frac{dv}{dr}$ ac reperietur:

$$\frac{dv}{dr} = \begin{array}{l} (-2A\alpha - \frac{A(2\kappa B+\mathfrak{B})}{nn} + \frac{2B(2\kappa A+\mathfrak{A})}{nn})\sin 2\eta \\ (-4\alpha B + \frac{A(2\kappa A+\mathfrak{A})}{nn})\sin 4\eta \\ (-a\alpha + \frac{A(2\kappa a+a)}{nn} - \frac{A(2\kappa b+b)}{nn})\nu\sin\eta \\ (-3\alpha b + \frac{A(2\kappa a+a)}{nn})\nu\sin 3\eta \end{array}$$

pona-

ponatur autem breuitatis ergo:

$$\frac{dv}{dr} = -A' \sin 2\eta - B' \sin 4\eta - a' v \sin \eta - b' v \sin 3\eta$$

vt sit:

$$A' = 2\alpha A + \frac{A(2\kappa B + \mathfrak{B})}{nn} - \frac{2B(2\kappa A + \mathfrak{A})}{nn}$$

$$B' = 4\alpha B - \frac{A(2\kappa A + \mathfrak{A})}{nn}$$

$$a' = \alpha a - \frac{A(2\kappa a + \mathfrak{a})}{nn} + \frac{A(\kappa b + \mathfrak{b})}{nn}$$

$$b' = 3\alpha b - \frac{A(2\kappa a + \mathfrak{a})}{nn}$$

§. 65. Hinc cum sit:

$$\frac{d.\sin 2\eta}{dr} = \cos 2\eta . \frac{2d\eta}{dr} = 2\alpha \cos 2\eta - \frac{(2\kappa B + \mathfrak{B})}{nn} \cos 2\eta - \frac{(2\kappa A + \mathfrak{A})}{nn} \cos 4\eta$$

$$- \frac{(2\kappa A + \mathfrak{A})}{nn} - \frac{(2\kappa a + \mathfrak{a})}{nn} v \cos \eta - \frac{(2\kappa b + \mathfrak{b})}{nn} v \cos \eta - \frac{(2\kappa a + \mathfrak{a})}{nn} v \cos 3\eta$$

$$\frac{d.\sin 4\eta}{dr} = \cos 4\eta . \frac{4d\eta}{dr} = 4\alpha \cos 4\eta - \frac{2(2\kappa A + \mathfrak{A})}{nn} \cos 2\eta - \frac{2(2\kappa B + \mathfrak{B})}{nn}$$

$$\frac{d.\sin \eta}{dr} = \cos \eta . \frac{d\eta}{dr} = \alpha \cos \eta; \text{ et } \frac{d.\sin 3\eta}{dr} = \cos 3\eta . \frac{3d\eta}{dr} = 3\alpha \cos 3\eta$$

prodibit

$$\frac{ddv}{dr^2} = + \frac{A'(2\kappa A + \mathfrak{A})}{nn} + \frac{2B'(2\kappa B + \mathfrak{B})}{nn}$$

$$\left(-2\alpha A' + \frac{A'(2\kappa B + \mathfrak{B})}{nn} + \frac{2B'(2\kappa A + \mathfrak{A})}{nn}\right) \cos 2\eta$$

$$\left(-4\alpha B' + \frac{A'(2\kappa A + \mathfrak{A})}{nn}\right) \cos 4\eta$$

$$\left(-\alpha a' + \frac{A'(2\kappa a + \mathfrak{a})}{nn}\right) v \cos \eta$$

$$\left(-3\alpha b' + \frac{A'(2\kappa a + \mathfrak{a})}{nn}\right) v \cos 3\eta$$

seu

seu substitutis superioribus valoribus:

$$+\frac{2A\alpha(2\kappa A+\mathfrak{A})}{nn}+\frac{8\alpha B(2\kappa B+\mathfrak{B})}{nn}$$

$$\frac{ddv}{dr^2}=(-4\alpha\alpha A+\frac{12\alpha B(2\kappa A+\mathfrak{A})}{nn})\cos 2\eta$$

$$(-16\alpha\alpha B+\frac{8\alpha A(2\kappa A+\mathfrak{A})}{nn})\cos 4\eta$$

$$(-\alpha\alpha a+\frac{3\alpha A(2\kappa a+\mathfrak{a})}{nn}-\frac{\alpha A(2\kappa b+\mathfrak{b})}{nn})\nu\cos\eta$$

$$(-9\alpha\alpha b+\frac{5\alpha A(2\kappa a+\mathfrak{a})}{nn})\nu\cos 3\eta$$

§ 66. Hi iam termini singulatim illis, qui §. 63. sunt exhibiti, aequales statuantur, atque sequentes prodibunt determinationes,

$$\tfrac{1}{2}\delta-\gamma+\frac{3AA+6A\mathfrak{A}+\mathfrak{A}\mathfrak{A}}{2nn}+\frac{3A}{4nn}=\frac{2\alpha A(2\kappa A+\mathfrak{A})}{nn}+\frac{8\alpha B(2\kappa B+\mathfrak{B})}{nn}$$

$$-A+\tfrac{3}{2}-2\kappa\mathfrak{A}+\frac{(4\gamma-3\delta)}{2nn}A+\frac{3B}{4nn}=-4\alpha\alpha A+\frac{12\alpha B(2\kappa A+\mathfrak{A})}{nn}$$

$$-B-2\kappa\mathfrak{B}+\frac{3AA+6A\mathfrak{A}+\mathfrak{A}\mathfrak{A}}{2nn}+\frac{3A}{4nn}+\frac{(4\gamma-3\delta)}{2nn}B=$$

$$-16\alpha\alpha B+\frac{8\alpha A(2\kappa A+\mathfrak{A})}{nn}$$

$$-a+\tfrac{8}{9}-2\kappa\mathfrak{a}+\frac{3a+3b}{4nn}=-\alpha\alpha a+\frac{3\alpha A(2\kappa a+\mathfrak{a})}{nn}-\frac{\alpha A(2\kappa b+\mathfrak{b})}{nn}$$

$$-b+\tfrac{15}{8}-2\kappa\mathfrak{b}+\frac{3a}{4nn}=-9\alpha\alpha b+\frac{5\alpha A(2\kappa a+\mathfrak{a})}{nn}$$

vnde primum quaeri debent valores vero proximi, qui sunt:

$$\mathfrak{A}=-\frac{3}{4\alpha};\quad \mathfrak{a}=-\frac{3}{8\alpha};\quad \mathfrak{b}=-\frac{5}{8\alpha};$$

$$A=-\frac{\frac{3}{2}+2\kappa\mathfrak{A}}{4\alpha\alpha-1};\quad a=\frac{\frac{8}{9}-2\kappa\mathfrak{a}}{1-\alpha\alpha};\quad b=-\frac{\frac{15}{8}+2\kappa\mathfrak{b}}{9\alpha\alpha-1}$$

§. 67.

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§. 67. Calculus ergo sequenti modo instituatur:

$\alpha = 0,933089$; $l\alpha = 9,969923$

$\varkappa = 1,008527$; $l\varkappa = 0,003687$

$l2\varkappa = 0,304717$

Iam est $l8\alpha = 0,873013$

subtr. a $\begin{cases} l3 = 0,477121 \\ l5 = 0,698970 \end{cases}$

$\mathfrak{a} = -0,402$; erit $l-\mathfrak{a} = 9,604108$

$\mathfrak{b} = -0,635$; $l-\mathfrak{b} = 9,825957$

$\mathfrak{A} = -0,804$; $l-\mathfrak{A} = 9,905138$

atque hinc conficietur:

$$A = -\frac{3,121}{4\alpha\alpha-1};\ a = +\frac{1,936}{1-\alpha\alpha};\ b = -\frac{3,156}{9\alpha\alpha-1}$$

quarum ergo litterarum valores proximi sunt

$A = -1,2583$; $a = +14,968$; $b = -0,4613$

§. 68. Quaeramus hinc primum valores litterarum $\mathfrak{B}$ et B.

$\mathfrak{A} = -0,804$; $l-\mathfrak{A} = 9,905138$

$2\varkappa A + \mathfrak{A} = -3,341$; vnde colligitur

$4\alpha\mathfrak{B} = +\frac{4,573}{nn}$ $l4,573 = 0,660201$

hinc erit $lnn = 2,244816$

$8,415385$

$l4\alpha = 0,571983$

$\mathfrak{B} = +0,00697$ $l\mathfrak{B} = 7,843402$

Deinde est

$$(16\alpha\alpha-1)B = 2\varkappa\mathfrak{B} - \frac{3A}{4nn} - \frac{3AA-6A\mathfrak{A}-\mathfrak{A}\mathfrak{A}}{2nn} + \frac{8\alpha A(2\varkappa A+\mathfrak{A})}{nn}$$

seu $B = +\frac{0,16819}{16\alpha\alpha-1}$; vnde reperitur

$B = +0,012792$ et $lB = 8,106947$

§. 69. His iam valoribus proxime veris inuentis quaerantur exacti, ac primo quidem

$$2\alpha\mathfrak{A} = -\tfrac{3}{2} + \frac{\frac{3}{2}\mathfrak{B} - \mathfrak{A}(2\varkappa\mathfrak{B}+\mathfrak{B}) + 2\mathfrak{B}(2\varkappa\mathfrak{A}+\mathfrak{A})}{nn}$$

vnde reperitur vt ante:

$$\mathfrak{A} = -0,80378 \quad . \quad . \quad . \quad l-\mathfrak{A} = 9,905138$$

$$\alpha\mathfrak{a} = -\tfrac{3}{8} - \frac{\frac{3}{2}(\alpha-\beta) + \mathfrak{A}(2\varkappa\alpha+\mathfrak{a}) - \mathfrak{A}(2\varkappa\beta+\mathfrak{b})}{nn} = -0,65361$$

$$\mathfrak{a} = -0,70048 \quad . \quad . \quad . \quad l-\mathfrak{a} = 9,845396$$

$$3\alpha\mathfrak{b} = +\tfrac{15}{8} - \frac{\frac{3}{2}\mathfrak{a} + \mathfrak{A}(2\varkappa\mathfrak{a}+\mathfrak{a})}{nn} = +2,13900$$

$$\mathfrak{b} = -0,76413 \quad . \quad . \quad . \quad l-\mathfrak{b} = 9,883167$$

$$(4\alpha\alpha-1)\mathfrak{A} = -\tfrac{3}{2} + 2\varkappa\mathfrak{A} - \frac{\frac{3}{4}\mathfrak{B} + 12\alpha\mathfrak{B}(2\varkappa\mathfrak{A}+\mathfrak{A})}{nn} = -3,12379$$

$$\mathfrak{A} = -1,25826 \quad . \quad . \quad . \quad l-\mathfrak{A} = 0,099771$$

$$(1-\alpha\alpha)\mathfrak{a} = \tfrac{9}{8} - 2\varkappa\mathfrak{a} + \frac{\frac{3}{4}(\alpha+\beta) - 3\alpha\mathfrak{A}(2\varkappa\alpha+\mathfrak{a}) + \alpha\mathfrak{A}(2\varkappa\beta+\mathfrak{b})}{nn} \text{ vel}$$

$$\left(1-\alpha\alpha-\frac{3}{4nn}+\frac{(6\alpha\varkappa\mathfrak{A})}{nn}\right)\mathfrak{a} = \tfrac{9}{8} - 2\varkappa\alpha + \frac{\frac{3}{4}\beta - 3\alpha\mathfrak{A}\alpha + \alpha\mathfrak{A}(2\varkappa\beta+\mathfrak{b})}{nn} = -2,53335$$

hinc $\alpha = +30,989$ . . et $l\alpha = 1,491207$

Vnde patet valorem ipsius $\mathfrak{a}$ ante inuentum non satis esse exactum, exactior ergo prodibit ex hac formula

$$\left(\alpha-\frac{\mathfrak{A}}{nn}\right)\mathfrak{a} = -\tfrac{3}{8} - \frac{\frac{3}{2}(\alpha-\beta) + \mathfrak{A}(2\varkappa\alpha-2\varkappa\beta-\mathfrak{b})}{nn} = -0,93709$$

hinc $\mathfrak{a} = -0,99939$ . . et $l-\mathfrak{a} = 9,999735$

vnde etiam exactius valor ipsius α reperitur, ex quo denuo

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denuo valor ipsius a corrigetur, neque tandem satis exacte obtinebitur.

$a = -1,2537 \ldots 1-a = 0,098200$
$\alpha = +44,48 \ldots l\alpha = 1,648165$
$b = -0,95003 \ldots l-b = 9,977736$

§. 70. Hinc iam accuratius quaeramus valorem ipsius b

$$(9\alpha\alpha - 1)b = -\tfrac{1}{8} + 2\alpha b - \frac{\frac{3}{4}\alpha + 5\alpha A(2\alpha\alpha + a)}{nn}$$

$b = -1,0146 \ldots l-b = 0,006314$

vnde si denuo praecedentes valores corrigantur, fiet

$a = -1,2630 \ldots l-a = 0,101403$
$b = -0,9500 \ldots l-b = 9,977736$
$\alpha = +44,525 \ldots l\alpha = 1,648604$
$b = -1,015 \ldots l-b = 0,006400$
$\mathfrak{A} = -0,80378 \ldots l-\mathfrak{A} = 9,905138$
$\mathfrak{B} = +0,00697 \ldots l\mathfrak{B} = 7,843402$
$A = -1,25826 \ldots l-A = 0,099771$
$B = +0,01279 \ldots lB = 8,106947$

His autem valoribus inuentis colligitur fore

$\frac{1}{2}\delta - \gamma = +0,01742$

Hic autem valor partem insuper accipit cum ab excentricitate vtriusque orbitae, tum ab inclinatione oriundam, quam deinceps determinabimus.

§. 71. Ex his ergo valoribus habebimus:

$\int R dr = -0,80378 \cos 2\eta + 0,00697 \cos 4\eta$
$9,905138 \qquad 7,843402$
$-1,2630\, v \cos\eta + 0,9500\, v \cos 3\eta$
$0,101403 \qquad 9,977736$

$$\begin{array}{rrrr} \varkappa = & -1,25826 \cos 2\eta & + 0,01279 \cos 4\eta & \\ & 0,099771 & 8,106947 & \\ & +44,525\,\nu \cos\eta & -1,015\,\nu\cos 3\eta & \\ & 1,648604 & 0,006400 & \end{array}$$

hincque porro

$$\begin{array}{rrrr} \frac{d\Phi}{dr} = & 1 & + 0,019015 \cos 2\eta & - 0,0000762 \cos 4\eta \\ & & 8,279096 & 5,881955 \\ & +0,0001103 & & \\ & - & 0,50381\,\nu\cos\eta & + 0,017068\,\nu\cos 3\eta \\ & & 9,702270 & 8,232184 \end{array}$$

at est $\frac{d\eta}{dr} = \frac{d\Phi}{dr} - \frac{1}{n}$, posuimusque $\varkappa - \frac{1}{n} = \alpha$, existente $\varkappa = V\left(1 + \frac{3+4\mu+\delta}{2nn}\right)$

§. 72. Ponatur breuitatis gratia

$$\frac{d\Phi}{dr} = \mathfrak{O} + \mathfrak{P}\cos 2\eta - \mathfrak{Q}\cos 4\eta - \mathfrak{R}\nu\cos\eta + \mathfrak{S}\nu\cos 3\eta$$

erit $\frac{d\eta}{dr} = \alpha + \mathfrak{P}\cos 2\eta - \mathfrak{Q}\cos 4\eta - \mathfrak{R}\nu\cos\eta + \mathfrak{S}\nu\cos 3\eta$

sitque ad integrandum:

$$\Omega = \upsilon r + \mathfrak{p}\sin 2\eta - \mathfrak{q}\sin 4\eta - \mathfrak{r}\nu\sin\eta + \mathfrak{s}\nu\sin 3\eta$$

vnde per differentiationem elicitur:

$$\begin{array}{l} \frac{d\Omega}{dr} = \upsilon + 2\alpha\mathfrak{p}\cos 2\eta - 4\alpha\mathfrak{q}\cos 4\eta - \alpha\mathfrak{r}\nu\cos\eta + 3\alpha\mathfrak{s}\nu\cos 3\eta \\ \quad \mathfrak{P}\mathfrak{p} - \mathfrak{Q}\mathfrak{p}\cos 2\eta + \mathfrak{P}\mathfrak{p}\cos 4\eta - \mathfrak{R}\mathfrak{p}\nu\cos\eta - \mathfrak{R}\mathfrak{r}\nu\cos 3\eta \\ + 2\mathfrak{Q}\mathfrak{q} - 2\mathfrak{P}\mathfrak{q}\cos 2\eta \qquad\qquad + 2\mathfrak{R}\mathfrak{q}\nu\cos 3\eta \end{array}$$

hinc ergo fit:

$$\upsilon = \mathfrak{O} - \mathfrak{P}\mathfrak{p} - 2\mathfrak{Q}\mathfrak{q} = \varkappa + 0,0001103 - \mathfrak{P}\mathfrak{p} - 2\mathfrak{Q}\mathfrak{q}$$

$$(2\alpha - \mathfrak{Q})\mathfrak{p} = \mathfrak{P} + 2\mathfrak{P}\mathfrak{q}; \quad 4\alpha\mathfrak{q} = \mathfrak{Q} + \mathfrak{P}\mathfrak{p};$$

$$\alpha\mathfrak{r} = \mathfrak{R} - \mathfrak{R}\mathfrak{p}; \quad 3\alpha\mathfrak{s} = \mathfrak{S} + \mathfrak{R}(\mathfrak{p} - 2\mathfrak{q})$$

Ergo

Ergo $p = 0{,}010191$ . . . $lp = 8{,}008208$
$q = 0{,}000072$ . . . $lq = 5{,}859381$
$r = 0{,}53453$ . . . $lr = 9{,}727977$
$s = 0{,}00790$ . . . $ls = 7{,}897466$
$v = u - 0{,}000080$

§. 73. Longitudo igitur lunae Φ quatenus pendet a ſola diſtantia lunae a ſole erit

$$\Phi = (u - 0{,}000080)r + 0{,}010191 \sin 2\eta - 0{,}53453\, v \sin \eta - 0{,}000072 \sin 4\eta + 0{,}00790\, v \sin 3\eta$$

Simili modo cum diſtantia lunae a terra poſita ſit $= \frac{a(1-kk)u}{1-k\cos r}$, ob $u = 1 + \frac{v}{nn}$, quatenus valor ipſius u a ſola phaſi lunae pendet, erit

$$u = 1 - 0{,}00716 \cos 2\eta + 0{,}00287\, v \cos \eta + 0{,}00007 \cos 4\eta - 0{,}00009\, v \cos 3\eta$$

Verum tamen hic valor litterae a ac praecipue ipſius u non admodum certus videtur, cum a terminis neglectis licet minimis inſignem mutationem perpeti queat. Hic enim pro u non ſolum $u - \frac{1}{n}$ ſed $u - \frac{1}{n} + 0{,}0001103$ accipi debuiſſet; quare cum valorem ipſius a propius cognoſcimus, hanc determinationem repeti conueniet.

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CAPUT V.

INUESTIGATIO INAEQUALITATUM LUNAE AB EIUS EXCENTRICITATE SIMPLICI SOLUM PEDENTIUM.

§. 74.

Quemadmodum in praecedenti capite inaequalitas abſoluta ſeu variatio duabus partibus conſtans eſt inuenta, quarum poſterior a littera v ſeu a parallaxi ſolis pendebat, ac maiorem curam requirebat; ita etiam inaequalitates, quas hoc capite ſcrutamur, partes continent ab eadem parallaxi ſolis pendentes; quarum indagatio quoque accuratiorem cognitionem quorundam elementorum exigit. Hancobrem et praecedentis capitis et huius partes, quae litteram v inuoluunt deinceps, cum reliquas inaequalitates, a parallaxi ſolis non pendentes determinauerimus, ſeorſim inueſtigabimus, atque titulo inaequalitatum parallacticarum complectemur.

§. 75. In hoc ergo capite ac ſequentibus, donec ad parallaxin ſolis perueniamus, terminos formularum noſtrarum per v. multiplicatos tantisper remouebimus; et quoniam hoc loco tantum propoſitum eſt in motus lunae inaequalitates a ſola excentricitate orbitae lunaris ortas inquirere, eos terminos qui vel excentricitatem ſolis e vel inclinationem ϱ continent, praetermittemus. Cum autem in formulis noſtris duplicis generis termini relinquantur, quorum alteri per k, alteri per kk ſunt affecti, inaequalitates ab excentricitate lunae k pendentes

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in duas partes distribui conueniet; quarum altera excentricitatem tantum simplicem k implicet, cui hoc caput destinatur, altera vero excentricitatis huius quadrato kk afficiatur, de quo in sequenti capite agemus.

§. 76. Verum tam in huius generis inaequalitates, quam in sequentes, omnes inaequalitates absolutae in praecedenti capite erutae praecipue ingrediuntur; ex quo eas quoque in calculum introduci oportebit. Retinendae ergo erunt in calculo litterae $\mathfrak{A}$, $\mathfrak{B}$ et A, B, quarum valores cum iam constent, calculus vehementer contrahetur: imprimis autem quia valores litterarum $\mathfrak{B}$ et B per se sunt admodum parui, quatenus illi in valores sequentium terminorum influunt, effectum pro nihilo habendum praestabunt. Inuestigationem ergo nostram ita incipiemus, vt pro $\int R\,dr$ et v valores fictos assumamus, et quoniam $\int R\,dr$ nullum terminum constantem, v vero neque constantem neque terminum huius formae $a \cos r$ continere debet, ponamus:

$$\begin{aligned}\int R\,dr = {} & \mathfrak{A}\cos 2\eta + \mathfrak{B}\cos 4\eta + \mathfrak{C}k\cos r \\ & + \mathfrak{D}k\cos(2\eta - r) + \mathfrak{F}k\cos(4\eta - r) \\ & + \mathfrak{E}k\cos(2\eta + r) + \mathfrak{G}k\cos(4\eta + r)\end{aligned}$$

$$\begin{aligned}v = {} & A\cos 2\eta + B\cos 4\eta \\ & + Dk\cos(2\eta + r) + Fk\cos(4\eta - r) \\ & + Ek\cos(2\eta + r) + Gk\cos(4\eta + r)\end{aligned}$$

vbi quidem facile colligere licet, coefficientes $\mathfrak{F}$, $\mathfrak{G}$, F et G fore minimos.

§. 77.

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§. 77. Ex his autem valoribus assumtis obtinebimus ex (§. 52.) sequentes expressiones.

$$\frac{d\Phi}{dr} = \begin{array}{l} \kappa + \frac{A(3\kappa A+2\mathfrak{A})}{2n^4} - \frac{(2\kappa A+\mathfrak{A})}{nn}\cos 2\eta \\ \left(-\frac{(2\kappa B+\mathfrak{B})}{nn} + \frac{A(3\kappa A+2\mathfrak{A})}{2n^4}\right)\cos 4\eta \\ \left(-\frac{\mathfrak{C}}{nn} + \frac{3\kappa AD}{n^4} + \frac{A\mathfrak{D}+\mathfrak{A}D}{n^4}\right)k\cos r \\ -\frac{(2\kappa D+\mathfrak{D})}{nn}k\cos(2\eta-r) - \frac{(2\kappa E+\mathfrak{E})}{nn}k\cos(2\eta+r) \\ -\frac{(2\kappa F+\mathfrak{F})}{nn}k\cos(4\eta-r) - \frac{(2\kappa G+\mathfrak{G})}{nn}k\cos(4\eta+r) \\ +\frac{(3\kappa AD+A\mathfrak{D}+\mathfrak{A}D)}{n^4}k\cos(4\eta-r) \end{array}$$

Patebit enim ex valoribus qui inuenientur, litteras D et $\mathfrak{D}$ tantum prae reliquis fore notabiles, vnde terminos ex combinatione reliquarum litterarum oriundos tuto omittere licet. Pro valore autem ipsius $\frac{d\eta}{dr}$ etiam hi termini ex combinatione orti omitti poterunt. Posito ergo $\kappa + \frac{A(3\kappa A+2\mathfrak{A})}{2n^4} - \frac{1}{n}$ seu $\kappa + 0,000103 - \frac{1}{n} = \alpha$ erit:

$$\frac{d\eta}{dr} = \alpha - \frac{(2\kappa A+\mathfrak{A})}{nn}\cos 2\eta - \frac{2k}{n}\cos r - \frac{(2\kappa D+\mathfrak{D})}{nn}k\cos(2\eta-r) - \frac{(2\kappa E+\mathfrak{E})}{nn}k\cos(2\eta+r)$$

Cum enim haec formula differentiationibus instituendis inseruiat, reliqui termini post primum cum aliis angulis combinantur, sicque tanto minores terminos produ-

cunt;

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cunt, qui ex calculo ſine errore expungi poterunt: atque ob hanc cauſam in expreſſione valoris $\frac{d\eta}{dr}$, ſtatim terminos prae reliquis admodum paruos praetermittere viſum eſt.

§. 78. Valorem autem ipſius R atque $\frac{ddv}{dr^2}$ accuratiſſime exhiberi oportet, propterea quod his expreſſionibus totus calculus praecipue innititur, dum valor $\frac{d\eta}{dr}$ formulam tantum ſubſidiariam ſuppeditat. Erit ergo

$$R = \tfrac{3}{2}\sin 2\eta + \frac{3A}{2nn}\sin 4\eta + 3k\sin(2\eta - r) + 3k\sin(2\eta + r)$$
$$+ \frac{3A}{nn}k\sin(4\eta - r) + \frac{3A}{nn}k\sin(4\eta + r)$$
$$+ \frac{3D}{2nn}k\sin r - \frac{3E}{2nn}k\sin r$$
$$+ \frac{3D}{2nn}k\sin(4\eta - r) + \frac{3E}{2nn}k\sin(4\eta + r)$$
$$- \frac{3A}{nn}k\sin r + \frac{3A}{nn}k\sin r$$

vbi quidem terminos ab k non pendentes omittere poſſumus, quia illorum iam habuimus rationem, ita vt ſit

$$R = \ldots + \frac{3(D-E)}{2nn}k\sin r + 3k\sin(2\eta - r) + 3k\sin(2\eta + r)$$
$$+ \frac{3D}{2nn}k\sin(4\eta - r) + \frac{3E}{2nn}k\sin(4\eta + r)$$
$$+ \frac{3A}{nn}k\sin(4\eta - r) + \frac{3A}{nn}k\sin(4\eta + r)$$

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§. 79. Simili modo terminis a k non pendentibus omittendis habebitur:

$$\frac{ddv}{dr^2} = -\gamma k\mathfrak{cf}r + 3k\mathfrak{cf}(2\eta - r) + 3k\mathfrak{cf}(2\eta + r) - 2u\mathfrak{F}k\mathfrak{cf}(4\eta - r) - 2u\mathfrak{G}k\mathfrak{cf}(4\eta + r)$$

$$\begin{array}{lllll}
-2u\mathfrak{C} & -2u\mathfrak{D} & -2u\mathfrak{E} & +\frac{\mathfrak{AD}}{2nn} & +\frac{\mathfrak{AE}}{2nn} \\
+\frac{\mathfrak{AD}}{2nn} & +\frac{\mathfrak{AE}}{2nn} & +\frac{\mathfrak{AE}}{2nn} & & \\
+\frac{\mathfrak{AE}}{2nn} & -D & -E & -F & -G \\
+\frac{3AD}{nn} & +\frac{3}{2}A & +\frac{3}{2}A & +\frac{3}{2}B & +\frac{3}{2}B \\
+\frac{3AE}{nn} & +\frac{(2\gamma-\frac{3}{2}\delta)}{nn}D & +\frac{(2\gamma-\frac{3}{2}\delta)}{nn}E & +\frac{3AD}{nn} & +\frac{3AE}{nn} \\
-\frac{3AA}{2nn} & +\frac{(3+3\mu+\gamma)}{nn}A & +\frac{(3+3\mu+\gamma)}{nn}A & -\frac{3AA}{4nn} & -\frac{3AA}{4nn} \\
+\frac{3AD}{nn} & & & +\frac{3\mathfrak{AD}}{nn} & +\frac{3\mathfrak{AE}}{nn} \\
+\frac{3AE}{nn} & & & +\frac{3\mathfrak{AD}}{nn} & +\frac{3\mathfrak{AE}}{nn} \\
+\frac{3\mathfrak{AD}}{nn} & & & & \\
+\frac{3\mathfrak{AE}}{nn} & & & &
\end{array}$$

Hic ſcilicet plures terminos, qui nullius futuri eſſent momenti, omiſimus, ne calculus nimium implicaretur: notandum autem eſt eſſe $u = V\left(1+\frac{3+4\mu+\delta}{2nn}\right) = 1+\frac{3+4\mu+\delta}{4nn}$ proxime; vnde $\mu = (u-1)nn - \frac{3-\delta}{4}$ et $\frac{3+3\mu+\gamma}{4nn} = 3(u-1) + \frac{3-3\delta+4\gamma}{4nn}$.

§. 80.

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§. 80. Quaeramus nunc quoque ex forma pro $\int R\,dr$ fícta valorem ipſius R, atque excluſis terminis ab k non pendentibus reperiemus:

$$\left(-\frac{\mathfrak{A}(2\varkappa\mathfrak{E}+\mathfrak{E})}{nn}+\frac{\mathfrak{A}(2\varkappa\mathfrak{D}+\mathfrak{D})}{nn}-\mathfrak{C}-\frac{\mathfrak{D}(2\varkappa\mathfrak{A}+\mathfrak{A})}{nn}+\frac{\mathfrak{E}(2\varkappa\mathfrak{A}+\mathfrak{A})}{nn}\right)k\sin r$$

$$\left(+\frac{2\mathfrak{A}}{n}-(2\alpha-1)\mathfrak{D}\right)k\sin(2\eta-r)$$

$$R=\left(+\frac{2\mathfrak{A}}{n}-(2\alpha+1)\mathfrak{E}\right)k\sin(2\eta+r)$$

$$\left(+\frac{\mathfrak{A}(2\varkappa\mathfrak{D}+\mathfrak{D})}{nn}+\frac{4\mathfrak{B}}{n}+\frac{\mathfrak{D}(2\varkappa\mathfrak{A}+\mathfrak{A})}{nn}-(4\alpha-1)\mathfrak{F}\right)k\sin(4\eta-r)$$

$$\left(+\frac{\mathfrak{A}(2\varkappa\mathfrak{E}+\mathfrak{E})}{nn}+\frac{4\mathfrak{B}}{n}+\frac{\mathfrak{E}(2\varkappa\mathfrak{A}+\mathfrak{A})}{nn}-(4\alpha+1)\mathfrak{G}\right)k\sin(4\eta+r)$$

atque inſtituta comparatione inuenietur:

$$\mathfrak{C}=\frac{\mathfrak{A}(2\varkappa\mathfrak{D}+\mathfrak{D})-(\mathfrak{D}-\mathfrak{E})(2\varkappa\mathfrak{A}+\mathfrak{A})-\frac{3}{2}(D-E)-\mathfrak{A}(2\varkappa\mathfrak{E}+\mathfrak{E})}{nn}$$

$$(2\alpha-1)\,\mathfrak{D}=\frac{2\mathfrak{A}}{n}-3:\quad(2\alpha+1)\,\mathfrak{E}=\frac{2\mathfrak{A}}{n}-3;$$

$$(4\alpha-1)\,\mathfrak{F}=\frac{4\mathfrak{B}}{n}+\frac{\mathfrak{A}(2\varkappa\mathfrak{D}+\mathfrak{D})+\mathfrak{D}(2\varkappa\mathfrak{A}+\mathfrak{A})-\frac{3}{2}(2A+D)}{nn}$$

$$(4\alpha+1)\,\mathfrak{G}=\frac{4\mathfrak{B}}{n}+\frac{\mathfrak{A}(\varkappa\mathfrak{E}+\mathfrak{E})-\frac{3}{2}(2A+E)+\mathfrak{E}(2\varkappa\mathfrak{A}+\mathfrak{A})}{nn}$$

§. 81. Pro differentiali $\frac{dv}{dr}$ inueniendo, praeter terminos ſupra inventos habebimus:

$$\frac{dv}{dr}=-A'\sin 2\eta-B'\sin 4\eta$$

$$\left(+\frac{A(2\varkappa D+D)}{nn}-\frac{A(2\varkappa\mathfrak{E}+\mathfrak{E})}{nn}-\frac{D(2\varkappa A+\mathfrak{A})}{nn}+\frac{E(2\varkappa A+\mathfrak{A})}{nn}\right)k\sin r$$

$$\left(+\frac{2A}{n}-(2\alpha-1)D\right)k\sin(2\eta-r)+\left(\frac{2A}{n}-(2\alpha+1)E\right)k\sin(2\eta+r)$$

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$$(+\frac{A(2\varkappa D+\mathfrak{D})}{n}+\frac{4B}{n}+\frac{D(2\varkappa A+\mathfrak{A})}{nn}-(4\alpha-1)F)\,k\sin(4\eta-r)$$

$$(+\frac{A(2\varkappa E+\mathfrak{E})}{nn}+\frac{4B}{n}+\frac{E(2\varkappa A+\mathfrak{A})}{nn}-(4\alpha+1)G)\,k\sin(4\eta+r)$$

Ponatur autem breuitatis gratia:

$$\frac{dv}{dr}=-A'\sin 2\eta-B'\sin 4\eta-C'k\sin r-D'k\sin(2\eta-r)$$
$$-E'k\sin(2\eta+r)-F'k\sin(4\eta-r)-G'k\sin(4\eta+r)$$

vt sit:

$$A'=2A\alpha+\frac{A(2\varkappa B+\mathfrak{B})-2B(2\varkappa A+\mathfrak{A})}{nn};\ B'=4\alpha B-\frac{A(2\varkappa A+\mathfrak{A})}{nn}$$

$$C'=\frac{-A(2\varkappa D+\mathfrak{D})+A(2\varkappa E+\mathfrak{E})+(D-E)(2\varkappa A+\mathfrak{A})}{nn}$$

sive $C'=-\frac{A(\mathfrak{D}-\mathfrak{E})+\mathfrak{A}(D-E)}{nn}$

$$D'=(2\alpha-1)D-\frac{2A}{n};\quad E'=(2\alpha+1)E-\frac{2A}{n}$$

$$F'=(4\alpha-1)F-\frac{4B}{n}-\frac{A(2\varkappa D+\mathfrak{D})-D(2\varkappa A+\mathfrak{A})}{nn}$$

$$G'=(4\alpha+1)G-\frac{4B}{n}-\frac{A(2\varkappa E+\mathfrak{E})-E(2\varkappa A+\mathfrak{A})}{nn}$$

§. 82. Hinc denuo differentiando obtinebitur terminis tantum per k multiplicatis scribendis:

$$\frac{ddv}{dr^2}=(-C'+\frac{A'(2\varkappa D+\mathfrak{D})}{nn}+\frac{A'(2\varkappa E+\mathfrak{E})}{nn}+\frac{D'(2\varkappa A+\mathfrak{A})}{nn}+\frac{E'(2\varkappa A+\mathfrak{A})}{nn})\,k\cos r$$

$$(+\frac{2A'}{n}-(2\alpha-1)D')\,k\cos(2\eta-r)$$

$$(+\frac{2A'}{n}-(2\alpha+1)E')\,k\cos(2\eta+r)$$

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$$(+\frac{4B'}{n}+\frac{A'(2\varkappa D+\mathfrak{D})}{nn}+\frac{D'(2\varkappa A+\mathfrak{A})}{nn}-(4\alpha-1)F')k\cos(4\eta-r)$$

$$(+\frac{4B'}{n}+\frac{A'(2\varkappa E+\mathfrak{E})}{nn}+\frac{E'(2\varkappa A+\mathfrak{A})}{nn}-(4\alpha+1)G')k\cos(4\eta+r)$$

vnde comparatione instituta orietur:

$$\gamma=-2\varkappa\mathfrak{C}-\frac{\frac{3}{2}AA+3A(D+E)+2A(\mathfrak{D}+2\mathfrak{E})+2\mathfrak{A}(2D+E)+\frac{1}{2}\mathfrak{A}(\mathfrak{D}+\mathfrak{E})}{nn}$$
$$-\frac{A'(2\varkappa D+\mathfrak{D})-A'(2\varkappa E+\mathfrak{E})-(D'+E')(2\varkappa A+\mathfrak{A})}{nn}$$

$$\left.\begin{array}{l}(2\alpha-1)^2D-\frac{2(2\alpha-1)}{n}A-\frac{2A'}{n}+3-2\varkappa\mathfrak{D}-D\\+\frac{3}{2}A+\frac{\frac{1}{2}\mathfrak{A}\mathfrak{C}+(2\gamma-\frac{3}{2}\delta)D+(3+3\mu+\gamma)A}{nn}\end{array}\right\}=0$$

$$\left.\begin{array}{l}(2\alpha+1)^2E-\frac{2(2\alpha+1)}{n}A-\frac{2A'}{n}+3-2\varkappa\mathfrak{E}-E\\+\frac{3}{2}A+\frac{\frac{1}{2}\mathfrak{A}\mathfrak{C}+(2\gamma-\frac{3}{2}\delta)E+(3+3\mu+\gamma)A}{nn}\end{array}\right\}=0$$

$$\left.\begin{array}{l}(4\alpha-1)^2F-\frac{4(4\alpha-1)}{n}B-\frac{(4\alpha-1)A(2\varkappa D+\mathfrak{D})-(4\alpha-1)D(2\varkappa A+\mathfrak{A})}{nn}\\-\frac{4B'}{n}-\frac{A'(2\varkappa D+\mathfrak{D})-D'(2\varkappa A+\mathfrak{A})}{nn}-2\varkappa\mathfrak{F}-F\\+\frac{3}{2}B+\frac{\frac{1}{2}\mathfrak{A}\mathfrak{D}+3AD-\frac{1}{4}AA+3A\mathfrak{D}+3\mathfrak{A}D}{nn}\end{array}\right\}=0$$

$$\left.\begin{array}{l}(4\alpha+1)^2G-\frac{4(4\alpha+1)}{n}B-\frac{(4\alpha+1)A(2\varkappa E+\mathfrak{E})-(4\alpha+1)E(2\varkappa A+\mathfrak{A})}{nn}\\-\frac{4B'}{n}-\frac{A'(2\varkappa E+\mathfrak{E})-E'(2\varkappa A+\mathfrak{A})}{nn}-2\varkappa\mathfrak{G}-G\\+\frac{3}{2}B+\frac{\frac{1}{2}\mathfrak{A}\mathfrak{E}+3AE-\frac{1}{4}AA+3A\mathfrak{E}+3\mathfrak{A}E}{nn}\end{array}\right\}=0$$

§. 83. Incipiamus a coefficientibus $\mathfrak{D}$, $\mathfrak{E}$, et D,E; et quia $\mathfrak{C}$ est quantitas admodum exigua, erit:

$$(2\alpha-1)\mathfrak{D} = -3 + \frac{2\mathfrak{A}}{n}; \quad (2\alpha+1)\mathfrak{E} = -3 + \frac{2\mathfrak{A}}{n}$$

$$\left((2\alpha-1)^2 - 1 + \frac{2\gamma-\frac{3}{2}\delta}{nn}\right) D = -3 - \tfrac{3}{2}A + 2\varkappa\mathfrak{D} + 2\frac{(2\alpha-1)}{n}A + \frac{2A'}{n} - (3\varkappa - 3 + \frac{3-3\delta+4\gamma}{4nn})A$$

$$\left((2\alpha+1)^2 - 1 + \frac{2\gamma-\frac{3}{2}\delta}{nn}\right) E = -3 - \tfrac{3}{2}A + 2\varkappa\mathfrak{E} + 2\frac{(2\alpha+1)}{n}A + \frac{2A'}{n} - (3\varkappa - 3 + \frac{3-3\delta+4\gamma}{4nn})A$$

vnde reperitur:

$\mathfrak{D} = -3,6035 \ldots l-\mathfrak{D} = 0,556724$

$\mathfrak{E} = -1,0890 \ldots l-\mathfrak{E} = 0,037028$

ac porro

$$\left.\begin{array}{l}(-0,24973 + \frac{(2\gamma-\frac{3}{2}\delta)}{nn})D = -1,40048 - 7,4315 \\ (+7,21497 + \frac{(2\gamma-\frac{3}{2}\delta)}{nn})E = -1,40048 - 2,7403\end{array}\right\} + \frac{(2\gamma-\frac{3}{2}\delta)}{nn}.0,629$$

§. 84. Quoniam autem valorem ipsius $\frac{2\gamma-\frac{3}{2}\delta}{nn}$ nondum nouimus, hunc terminum, cum certo sit valde parvus, reiiciamus. Postmodum vero cum istum terminum cognouerimus, facile erit correctionem inde oriundam, si operae pretium videbitur, inuenire.

$D = +35,3662 \ldots lD = 1,548588$

$E = -0,5739 \ldots l-E = 9,758848$

Porro autem litterae $\mathfrak{F}$ et $\mathfrak{G}$ ita elicientur, vt sit.

$(4\alpha-1)\mathfrak{F} = 0,000529 - 0,30976 + 0,06852 - 0,28042$

$(4\alpha+1)\mathfrak{G} = 0,000529 + 0,01028 + 0,02071 + 0,02638$

$\mathfrak{F} =$

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$\mathfrak{F} = -0,1907 \ldots l-\mathfrak{F} = 9,280416$

$\mathfrak{G} = +0,0122 \ldots l\mathfrak{G} = 8,087607$

Praeterea autem colligimus fore

$\mathfrak{C} = -0,67465 \ldots l-\mathfrak{C} = 9,829072$

vnde erit proxime $\frac{\mathfrak{AC}}{2mn} = 0,00154$, ex quo accuratius concluditur fore

$D = +35,3724 \ldots l\,D = 1,548664$

$E = -0,5741 \ldots l-E = 9,758988$

§. 85. Reliquae aequationes nobis praebebunt

$$6,4655\,F + 3,67820 = 0$$

$$21,3946\,G - 0,29574 = 0$$

vnde obtinebitur

$F = -0,56890$; $l-F = 9,755033$

$G = +0,01382$. $l\,G = 8,140620$

ac denique $\gamma = 1,40673$.

Supra autem iam inuenimus $\frac{3}{2}\delta - \gamma = 0,01742$, vnde ambas iſtas quantitates γ et δ, quas initio ad veros valores conſtantium litterarum m et n determinandos aſſumſimus, nunc cognitas habemus, erit enim:

$\delta = 2,84830$, et $2\gamma - \frac{3}{2}\delta = -1,45899$

ac propterea particulae illius $\frac{2\gamma - \frac{3}{2}\delta}{nn}$ hactenus neglectae valor erit $\frac{2\gamma - \frac{3}{2}\delta}{nn} = -0,00832$, cuius ope iam litterae D et E accuratius definiri poterunt.

§. 86.

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§. 86. Hinc autem potissimum valor ipsius D mutationem patitur, fiet enim re vera

$$-0,25805\,D = -8,83698 \quad \text{seu}$$
$$D = 34,24520 \quad\ldots\quad l\,D = 1,534600$$
$$7,20665\,E = -4,14578 \quad \text{seu}$$
$$E = -0,57527 \quad\ldots\quad l\text{-}E = 9,759874$$

et quoniam D parte sua tricesima diminuitur, in eadem fere ratione diminuentur valores litterarum $\mathfrak{C}$ et γ, ita vt exactius sit:

$$\mathfrak{C} = -0,65217 \quad\ldots\quad l\text{-}\mathfrak{C} = 9,814361$$
$$\gamma = +1,35984 \quad\ldots\quad l\,\gamma = 0,133490$$
$$\delta = +2,75336 \quad\ldots\quad l\,\delta = 0,439863$$
$$\text{et } \frac{2\gamma - \frac{3}{2}\delta}{nn} = -0,00804$$

Deinceps autem operae erit pretium in hos valores adhuc diligentius inquirere.

§. 87. Cum igitur finxerimus sequentes valores:

$$\int R\,dr = \mathfrak{A}\cos 2\eta + \mathfrak{B}\cos 4\eta + \mathfrak{C}\,k\cos r + \mathfrak{D}\,k\cos(2\eta - r) + \mathfrak{F}\,k\cos(4\eta - r) + \mathfrak{E}\,k\cos(2\eta + r) + \mathfrak{G}\,k\cos(4\eta + r)$$
$$v = A\cos 2\eta + B\cos 4\eta + D\,k\cos(2\eta - r) + F\,k\cos(4\eta - r) + E\,k\cos(2\eta + r) + G\,k\cos(4\eta + r)$$

horum coefficientium valores sunt.

$\mathfrak{A} = -0,80378$	$A = -1,25826$
$\mathfrak{B} = +0,00697$	$B = -0,01279$
$\mathfrak{C} = -0,65217$	. .
$\mathfrak{D} = -3,60350$	$D = +34,24520$
$\mathfrak{E} = -1,08900$	$E = -0,57527$
$\mathfrak{F} = -0,19070$	$F = -0,56890$
$\mathfrak{G} = +0,01220$	$G = +0,01382$

vnde

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vnde pro distantia lunae a terra $x = \frac{(1-kk)au}{1-k\cos r}$ fit

$$u = 1 - 0,007161\cos 2\eta + 0,000073\cos 4\eta + 0,194888k\cos(2\eta-r) - 0,003274k\cos(2\eta+r) - 0,003238k\cos(4\eta-r) + 0,000078k\cos(4\eta+r)$$

§. 88. His valoribus in §. 77. substitutis obtinebimus:

$$\frac{d\Phi}{dr} = u + 0,019015\cos 2\eta - 0,001255k\cos r + 0,00001103 - 0,000076\cos 4\eta - 0,38410k\cos(2\eta-r) + 0,01278k\cos(2\eta+r) + 0,002647k\cos(4\eta-r) - 0,000229k\cos(4\eta+r)$$

ad cuius integrale inueniendum ponamus:

$$\Phi = \mathfrak{O}r + \mathfrak{A}'\sin 2\eta + \mathfrak{B}'\sin 4\eta + \mathfrak{C}'k\sin r + \mathfrak{D}'k\cos(2\eta-r) + \mathfrak{E}'k\cos(2\eta+r) + \mathfrak{F}'k\cos(4\eta-r) + \mathfrak{G}'k\cos(4\eta+r)$$

eritque differentiando et terminis iam cognitis omittendis.

$$\frac{d\Phi}{dr} = (\mathfrak{C}' - \frac{\mathfrak{A}'(2u\mathfrak{D}+\mathfrak{D})}{nn} - \frac{\mathfrak{A}'(2u\mathfrak{E}+\mathfrak{E})}{nn} - \frac{\mathfrak{D}'(2u\mathfrak{A}+\mathfrak{A})}{nn} - \frac{\mathfrak{E}'(2u\mathfrak{A}+\mathfrak{A})}{nn})k\cos r$$
$$(-\frac{2\mathfrak{A}'}{n} + (2u-1)\mathfrak{D}')k\cos(2\eta-r)$$
$$(-\frac{2\mathfrak{A}'}{n} + (2u+1)\mathfrak{E}')k\cos(2\eta+r)$$
$$(-\frac{4\mathfrak{B}'}{n} - \frac{\mathfrak{A}'(2u\mathfrak{D}+\mathfrak{D})}{nn} - \frac{\mathfrak{D}'(2u\mathfrak{A}+\mathfrak{A})}{nn} + (4u-1)\mathfrak{F}')k\cos(4\eta-r)$$
$$(-\frac{4\mathfrak{B}'}{n} - \frac{\mathfrak{A}'(2u\mathfrak{E}+\mathfrak{E})}{nn} - \frac{\mathfrak{E}'(2u\mathfrak{A}+\mathfrak{A})}{nn} + (4u+1)\mathfrak{G}')k\cos(4\eta+r)$$

Pro terminis autem iam inuentis est

$$\mathfrak{O} = u - 0,000080;\quad \mathfrak{A}' = 0,010191;\quad \mathfrak{B}' = -0,000072$$
$$l\mathfrak{A}' = 8,008208;\quad l\mathfrak{B}' = -5,859381$$

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§. 89. Comparatione iam instituta fiet:

$$(2\alpha-1)\mathfrak{D}' = -0{,}38410 + \frac{2\mathfrak{A}'}{n}\,;\quad (2\alpha+1)\mathfrak{E}' = +0{,}01278 + \frac{2\mathfrak{A}'}{n}$$

$$(4\alpha-1)\mathfrak{F}' = +0{,}002647 + \frac{4\mathfrak{B}'}{n} + \frac{\mathfrak{A}'(2\varkappa\mathrm{D}+\mathfrak{D})+\mathfrak{D}'(2\varkappa\mathrm{A}+\mathfrak{A})}{nn}$$

$$(4\alpha+1)\mathfrak{G}' = -0{,}000229 + \frac{4\mathfrak{B}'}{n} + \frac{\mathfrak{A}'(2\varkappa\mathrm{E}+\mathfrak{E})+\mathfrak{E}'(2\varkappa\mathrm{A}+\mathfrak{A})}{nn}$$

$$\mathfrak{C}' = -0{,}001255 + \frac{\mathfrak{A}'(2\varkappa\,\mathrm{D}+\mathfrak{D})+\mathfrak{D}'(2\varkappa\,\mathrm{A}+\mathfrak{A})}{nn} + \frac{\mathfrak{A}'(2\varkappa\,\mathrm{E}+\mathfrak{E})+\mathfrak{E}'(2\varkappa\,\mathrm{A}+\mathfrak{A})}{nn}$$

vnde colligitur fore

$$\mathfrak{C}' = +0{,}01083$$
$$\mathfrak{D}' = -0{,}44167 \quad . \ . \ . \quad l-\mathfrak{D}' = 9{,}645092$$
$$\mathfrak{E}' = +0{,}00499 \quad . \ . \ . \quad l\,\mathfrak{E}' = 7{,}698640$$
$$\mathfrak{F}' = +0{,}00546 \quad . \ . \ . \quad l\,\mathfrak{F}' = 7{,}737733$$
$$\mathfrak{G}' = -0{,}00010 \quad . \ . \ . \quad l-\mathfrak{G}' = 6{,}002537$$

ita vt sit

$$\varphi = (\varkappa - 0{,}000080\,r) + 0{,}010191 \sin 2\eta + 0{,}01083\,k \sin r - 0{,}000072 \sin 4\eta$$
$$- 0{,}44167\,k \sin(2\eta - r) + 0{,}00499\,k \sin(2\eta + r)$$
$$+ 0{,}00546\,k \sin(4\eta - r) - 0{,}00010\,k \sin(4\eta + r)$$

vnde ex comparatione motus medii ad modum anomaliae erit $\varkappa = 1{,}008607$, et $\alpha = 0{,}933279$, qui valores iam propius ad veritatem accedunt, quam hactenus vsurpati.

CAPUT

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CAPUT VI.

INUESTIGATIO INAEQUALITATUM LUNAE A QUADRATO EXCENTRICITATIS IPSIUS ORTARUM,

§. 90.

Peruenimus nunc ad alteram partem inaequalitatum in motu Lunae, quae ab eius excentricitate k pendent, eiusque quadratum inuoluunt, ita vt hic nonnisi eos terminos simus contemplaturi, qui per quadratum excentricitatis lunae kk sunt multiplicati. Hic autem tam in valorem ipsius $\int R dr$, quam ipsius v termini formae $kk \cos 2\eta$ et $kk \cos 4\eta$ ingredientur, qui postquam fuerint inuenti, terminis huius generis iam ante inuentis adiici debent; praeterea vero vtrinque etiam termini formae $kk \cos 2r$ accedent. Hinc ponamus:

$$\begin{aligned}\int R dr = {} & \mathfrak{A} \cos 2\eta + \mathfrak{a}\, kk \cos 2\eta + \mathfrak{B} \cos 4\eta + \mathfrak{b}\, kk \cos 4\eta \\ & + \mathfrak{C}\, k \cos r + \mathfrak{D}\, k \cos(2\eta - r) + \mathfrak{E}\, k \cos(2\eta + r) \\ & + \mathfrak{F}\, k \cos(4\eta - r) + \mathfrak{G}\, k \cos(4\eta + r) \\ & + \mathfrak{H}\, kk \cos 2r + \mathfrak{J}\, kk \cos(2\eta - 2r) + \mathfrak{K}\, kk \cos(2\eta + 2r) \\ & + \mathfrak{L}\, kk \cos(4\eta - 2r) + \mathfrak{M}\, kk \cos(4\eta + 2r)\end{aligned}$$

$$\begin{aligned}v = {} & A \cos 2\eta + a\, kk \cos 2\eta + B \cos 4\eta + b\, kk \cos 4\eta \\ & + D\, k \cos(2\eta - r) + E\, k \cos(2\eta + r) \\ & + F\, k \cos(4\eta - r) + G\, k \cos(4\eta + r) \\ & + H\, kk \cos 2r + J\, kk \cos(2\eta - 2r) + K\, kk \cos(2\eta + 2r) \\ & + L\, kk \cos(4\eta - 2r) + M\, kk \cos(4\eta + 2r)\end{aligned}$$

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§. 91. Nunc ad terminos, quibus ante valorem ipsius $\frac{d\Phi}{dr}$ exprimi inuenimus, insuper sequentes per kk multiplicati accedent:

$$\frac{d\Phi}{dr} = \ldots + \frac{D(3\nu D + 2\mathfrak{D})}{2n^4}k^2 + \left(-\frac{(2\nu a + \mathfrak{a})}{nn} + \frac{\mathfrak{C}D}{n^4}\right)k^2\cos 2\eta$$

$$\left(-\frac{(2\nu b + \mathfrak{b})}{nn} + \frac{D(3\nu E + 2\mathfrak{E})}{2n^4} + \frac{E(3\nu D + 2\mathfrak{D})}{2n^4}\right)k^2\cos 4\eta$$

$$\left\{\begin{array}{l} -\frac{(2\nu H + \mathfrak{H})}{nn} + \frac{D(3\nu E + 2\mathfrak{E})}{2n^4} + \frac{E(3\nu D + 2\mathfrak{D})}{2n^4} \\ \qquad + \frac{A(3\nu J + 2\mathfrak{J})}{2n^4} + \frac{J(3\nu A + 2\mathfrak{A})}{2n^4} \end{array}\right\}k^2\cos 2r$$

$$\left(-\frac{(2\nu J + \mathfrak{J})}{nn} + \frac{A(3\nu H + 2\mathfrak{H})}{2n^4} + \frac{H(3\nu A + 2\mathfrak{A})}{2n^4} + \frac{\mathfrak{C}D}{n^4}\right)k^2\cos(2\eta - 2r)$$

$$\left(-\frac{(2\nu K + \mathfrak{K})}{nn} + \frac{A(3\nu H + 2\mathfrak{H})}{2n^4} + \frac{H(3\nu A + 2\mathfrak{A})}{2n^2}\right)k\cos(4\eta + 2r)$$

$$\left(-\frac{(2\nu L + \mathfrak{L})}{nn} + \frac{A(3\nu J + 2\mathfrak{J})}{2n^4} + \frac{J(3\nu A + 2\mathfrak{A})}{2n^4} + \frac{D(3\nu D + 2\mathfrak{D})}{2n^4}\right)k^2\cos(4\eta - 2r)$$

$$-\frac{(2\nu M + \mathfrak{M})}{nn}k^2\cos(4\eta + 2r)$$

vbi quidem terminos, quos minimos fore facile est praeuidere, omisimus.

§. 92. Terminus autem constans $\frac{D(3\nu D + 2\mathfrak{D})kk}{2n^4}$ reperitur $= 0,000175$, vnde posito $\nu + 0,000285 = \frac{1}{n} = \alpha$, quoniam valorem ipsius $\frac{d\eta}{dr}$ non opus est tam exacte nosse, sumamus:

$$\frac{d\eta}{dr}$$

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$$\frac{d\eta}{dr} = \alpha - \frac{(2\varkappa A + \mathfrak{A})}{nn}\cos 2\eta - \left(\frac{2k}{n} + \frac{\mathfrak{C}k}{nn}\right)\cos r$$
$$- \frac{(2\varkappa D + \mathfrak{D})}{nn}k\cos(2\eta - r) - \left(\frac{3kk}{2n} + \frac{(2\varkappa H + \mathfrak{H})}{nn}\right)\cos 2r$$
$$- \frac{(2\varkappa J + \mathfrak{J})}{nn}k^2\cos(2\eta - 2r)$$

Deinde vero praeter terminos iam tractatos habebitur:

$$R = \ldots 3k^2\sin 2\eta + \frac{3D}{nn}k^2\sin 4\eta + \frac{3(2D+J)}{2nn}k^2\sin 2r$$
$$+ (\tfrac{15}{4}k^2 + \frac{3(H-L)}{2nn})\sin(2\eta - 2r) + (\tfrac{15}{4}k^2 + \frac{3H}{2nn})\sin(2\eta + 2r)$$
$$+ \frac{3(2D+J)}{2nn}k^2\sin(4\eta - 2r)$$

atque simili modo:

$$\frac{ddv}{dr^2} = \tfrac{1}{2}\delta - \gamma + \frac{\mathfrak{A}a + \frac{1}{2}\mathfrak{D}D + 3D\mathfrak{D} + 3DD + 3Aa + 3\mathfrak{A}a + 3Aa}{nn}kk$$
$$+ (3 + \tfrac{3}{2}(A + D + E) - a - 2\varkappa a)\,kk\cos 2\eta$$
$$+ (\tfrac{3}{2}(B + F + G) - b - 2\varkappa b)\,kk\cos 4\eta$$
$$+ (\tfrac{3}{2} - 2\varkappa\mathfrak{H} - H + \frac{\mathfrak{A}\mathfrak{J} + 3\mathfrak{A}J + 3A\mathfrak{J} + 3AJ)}{nn})\,kk\cos 2r$$
$$+ (\tfrac{15}{4} - 2\varkappa\mathfrak{J} - J + \tfrac{3}{4}A + \tfrac{3}{2}D)\,kk\cos(2\eta - 2r)$$
$$+ (\tfrac{15}{4} - 2\varkappa\mathfrak{K} - K + \tfrac{3}{4}A + \tfrac{3}{2}E)\,k^2\cos(2\eta + 2r)$$
$$+ (-2\varkappa\mathfrak{L} - L + \tfrac{3}{4}B + \tfrac{3}{2}F)\,kk\cos(4\eta - 2r)$$
$$+ (-2\varkappa\mathfrak{M} - M + \tfrac{3}{4}B + \tfrac{3}{2}G)\,k^2\cos(4\eta + 2r)$$
$$+ \frac{\mathfrak{A}\mathfrak{J} + 3\mathfrak{A}J + 3A\mathfrak{J} + 3AJ + \frac{1}{2}\mathfrak{D}D + 3D\mathfrak{D} + 3DD)}{nn}kk\cos(4\eta - 2r)$$

§. 93. Eliciamus nunc quoque valorem ipsius R per differentiationem ex formula $\int R\,dr$, ac terminis apte dispositis habebimus

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$$R =$$

$kk \sin 2\eta$	$kk \sin 4\eta$	$kk \sin 2r$	$k^2 \sin(2\eta - 2r)$
$-2\alpha\alpha$	$+\frac{\alpha(2\varkappa\mathfrak{A}+\mathfrak{A})}{nn}$	$+\frac{\mathfrak{A}(2\varkappa\mathfrak{J}+\mathfrak{J})}{nn}$	$+\frac{3\mathfrak{A}}{2n}$
$+\frac{2\mathfrak{b}(2\varkappa\mathfrak{A}+\mathfrak{A})}{nn}$	$-4\alpha\mathfrak{b}$	$-2\mathfrak{H}$	$+\frac{\mathfrak{A}(2\varkappa\mathfrak{H}+\mathfrak{H})}{nn}$
$+\frac{\mathfrak{D}(2n+\mathfrak{C})}{nn}$	$+\frac{\mathfrak{E}(2\varkappa\mathfrak{D}+\mathfrak{D})}{nn}$	$+\frac{\mathfrak{C}(2\varkappa\mathfrak{D}+\mathfrak{D})}{nn}$	$+\frac{\mathfrak{D}(2n+\mathfrak{C})}{nn}$
$+\frac{\mathfrak{E}(2n+\mathfrak{C})}{nn}$	$+\frac{2\mathfrak{F}(2n+\mathfrak{C})}{nn}$	$+\frac{\mathfrak{J}(2\varkappa\mathfrak{A}+\mathfrak{A})}{nn}$	$-2(\alpha-1)\mathfrak{J}$
$+\frac{2\mathfrak{F}(2\varkappa\mathfrak{D}+\mathfrak{D})}{nn}$	$+\frac{2\mathfrak{G}(2n+\mathfrak{C})}{nn}$	$+\frac{\mathfrak{K}(2\varkappa\mathfrak{A}+\mathfrak{A})}{nn}$	$+\frac{2\mathfrak{L}(2\varkappa\mathfrak{A}+\mathfrak{A})}{nn}$

$kk \sin(2\eta+2r)$	$kk \sin(4\eta-2r)$	$kk \sin(4\eta+2r)$
$+\frac{3\mathfrak{A}}{2n}$	$+\frac{\mathfrak{A}(2\varkappa\mathfrak{J}+\mathfrak{J})}{nn}$	
$+\frac{\mathfrak{A}(2\varkappa\mathfrak{H}+\mathfrak{H})}{nn}$	$+\frac{3\mathfrak{B}}{n}$	$+\frac{3\mathfrak{B}}{n}$
$+\frac{2\mathfrak{B}(2\varkappa\mathfrak{J}+\mathfrak{J})}{nn}$	$+\frac{2\mathfrak{B}(2\varkappa\mathfrak{H}+\mathfrak{H})}{nn}$	$+\frac{2\mathfrak{B}(2\varkappa\mathfrak{H}+\mathfrak{H})}{nn}$
$+\frac{\mathfrak{E}(2n+\mathfrak{C})}{nn}$	$+\frac{\mathfrak{D}(2\varkappa\mathfrak{D}+\mathfrak{D})}{nn}$	$+\frac{\mathfrak{G}(2n+\mathfrak{C})}{nn}$
$+\frac{2\mathfrak{G}(2\varkappa\mathfrak{D}+\mathfrak{D})}{nn}$	$+\frac{2\mathfrak{F}(2n+\mathfrak{C})}{nn}$	$+\frac{\mathfrak{K}(2\varkappa\mathfrak{A}+\mathfrak{A})}{nn}$
$-2(\alpha+1)\mathfrak{K}$	$+\frac{\mathfrak{J}(2\varkappa\mathfrak{A}+\mathfrak{A})}{nn}$	$-2(2\alpha+1)\mathfrak{M}$
$+\frac{2\mathfrak{M}(2\varkappa\mathfrak{A}+\mathfrak{A})}{nn}$	$-2(2\alpha-1)\mathfrak{L}$	

vnde

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vnde oriuntur sequentes determinationes:

$$3 = -2\alpha a + \frac{2b(2nA+\mathfrak{A})+(\mathfrak{D}+\mathfrak{E})(2n+\mathfrak{C})+2\mathfrak{F}(2nD+\mathfrak{D})}{nn}$$

$$\frac{3D}{nn} = -4\alpha b + \frac{a(2nA+\mathfrak{A})+\mathfrak{E}(2nD+\mathfrak{D})+2(\mathfrak{F}+\mathfrak{G})(2n+\mathfrak{C})}{nn}$$

$$\frac{3(2D+J)}{2nn} = -2\mathfrak{H} + \frac{\mathfrak{A}(2nJ+\mathfrak{J})+\mathfrak{E}(2nD+\mathfrak{D})-(\mathfrak{J}-\mathfrak{K})(2nA+\mathfrak{A})}{nn}$$

$$\tfrac{15}{4} + \frac{3(H-L)}{2nn} = -2(\alpha-1)\mathfrak{J} + \frac{3\mathfrak{A}}{nn} + \frac{\mathfrak{A}(2nH+\mathfrak{H})}{nn} + \frac{\mathfrak{D}(2n+\mathfrak{C})+2\mathfrak{L}(2nA+\mathfrak{A})}{nn}$$

$$\tfrac{15}{4} + \frac{3H}{2nn} = 2(\alpha+1)\mathfrak{K} + \frac{3\mathfrak{A}}{2n} + \frac{\mathfrak{A}(2nH+\mathfrak{H})}{nn} + \frac{2\mathfrak{B}(2nJ+\mathfrak{J})+2\mathfrak{G}(2nD+\mathfrak{D})+\mathfrak{E}(2n+\mathfrak{C})+2\mathfrak{M}(2nA+\mathfrak{A})}{nn}$$

$$+\frac{3(2D+J)}{2nn} = -2(2\alpha-1)\mathfrak{L} + \frac{3\mathfrak{B}}{n} + \frac{\mathfrak{A}(2nJ+\mathfrak{J})}{nn} + \frac{2\mathfrak{B}(2nH+\mathfrak{H})+\mathfrak{D}(2nD+\mathfrak{D})+2\mathfrak{F}(2n+\mathfrak{C})+\mathfrak{J}(2nA+\mathfrak{A})}{nn}$$

$$0 = -2(2\alpha+1)\mathfrak{M} + \frac{3\mathfrak{B}}{n} + \frac{2\mathfrak{B}(2nH+\mathfrak{H})+2\mathfrak{G}(2n+\mathfrak{C})}{nn} + \frac{\mathfrak{K}(2nA+\mathfrak{A})}{nn}$$

§. 94. Deinde simili modo si ponatur:

$$\frac{dv}{dr} = -A'\sin 2\eta - a'k^2\sin 2\eta - B'\sin 4\eta - b'k^2\sin 4\eta$$
$$-C'k\sin r - D'k\sin(2\eta-r) - F'k\sin(4\eta-r)$$
$$-E'k\sin(2\eta+r) - G'k\sin(2\eta+r)$$
$$-H'k^2\sin 2r - J'k^2\sin(2\eta-2r) - L'k^2\sin(4\eta-2r)$$
$$-K'k^2\sin(2\eta+2r) - M'k^2\sin(4\eta+2r)$$

erit

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erit praeter valores §. 81. datos:

$$a' = 2\alpha a - \frac{2b(2nA+\mathfrak{A}) - (D+E)(2n+\mathfrak{C}) - 2F(2nD+\mathfrak{D})}{nn}$$

$$b' = 4\alpha b - \frac{a(2nA+\mathfrak{A}) - E(2nD+\mathfrak{D})}{nn} - \frac{2(F+G)(2n+\mathfrak{C}) - D(2nE+\mathfrak{E})}{nn}$$

$$H' = 2H - \frac{A(2nJ+\mathfrak{J}) - E(2nD+\mathfrak{D})}{nn} + \frac{(J-K)(2nA+\mathfrak{A}) + D(2nE+\mathfrak{E})}{nn}$$

$$J' = 2(\alpha-1)J - \frac{3A}{2n} - \frac{A(2nH+\mathfrak{H}) - D(2n+\mathfrak{C}) - 2L(2nA+\mathfrak{A})}{nn}$$

$$K' = 2(\alpha+1)K - \frac{3A}{2n} - \frac{A(2nH+\mathfrak{H}) - 2B(2nJ+\mathfrak{J}) - 2G(2nD+\mathfrak{D})}{nn} - \frac{E(2n+\mathfrak{C}) - 2M(2nA+\mathfrak{A})}{nn}$$

$$L' = 2(2\alpha-1)L - \frac{3B}{n} - \frac{A(2nJ+\mathfrak{J}) - 2B(2nH+\mathfrak{H}) - D(2nD+\mathfrak{D})}{nn} - \frac{2F(2n+\mathfrak{C}) - J(2nA+\mathfrak{A})}{nn}$$

$$M' = 2(2\alpha+1)M - \frac{3B}{n} - \frac{2B(2nH+\mathfrak{H}) - G(2n+\mathfrak{C}) - K(2nA+\mathfrak{A})}{nn}$$

vbi quidem plures terminos, quos admodum paruos fore praeuidimus, omisimus.

§. 95.

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§. 95. Hinc autem denuo differentiando obtinemus

valorem ipsius $\frac{ddv}{dr^2}$ =

kk	$kk\cos 2\eta$	$kk\cos 4\eta$	$kk\cos 2r$
$+\frac{\alpha'(2nA+\mathfrak{A})}{nn}$	$-2\alpha\alpha'$	$+\frac{\alpha'(2nA+\mathfrak{A})}{nn}$	$+\frac{A'(2nJ+\mathfrak{J})}{nn}$
$+\frac{D'(2nD+\mathfrak{D})}{nn}$	$+\frac{2b'(2nA+\mathfrak{A})}{nn}$	$-4\alpha b'$	$+\frac{E'(2nD+\mathfrak{D})}{nn}$
	$+\frac{D'(2n+\mathfrak{C})}{nn}$	$+\frac{E'(2nD+\mathfrak{D})}{nn}$	$-2H'$
	$+\frac{E'(2n+\mathfrak{C})}{nn}$	$+\frac{2F'(2n+\mathfrak{C})}{nn}$	$+\frac{J'(2nA+\mathfrak{A})}{nn}$
	$+\frac{2F'(2nD+\mathfrak{D})}{nn}$	$+\frac{2G'(2n+\mathfrak{C})}{nn}$	$+\frac{K'(2nA+\mathfrak{A})}{nn}$

$kk\cos(2\eta-2r)$	$kk\cos(2\eta+2r)$	$kk\cos(4\eta-2r)$	$kk\cos(4\eta+2r)$
$+\frac{3A'}{2n}$	$+\frac{3A'}{2n}$	$+\frac{A'(2nJ+\mathfrak{J})}{nn}$	
$+\frac{A'(2nH+\mathfrak{H})}{nn}$	$+\frac{A'(2nH+\mathfrak{H})}{nn}$	$+\frac{3B'}{n}$	$+\frac{3B'}{n}$
$+\frac{D'(2n+\mathfrak{C})}{nn}$	$+\frac{2G'(2nD+\mathfrak{D})}{nn}$	$+\frac{2B'(2nH+\mathfrak{H})}{nn}$	$+\frac{2B'(2nH+\mathfrak{H})}{nn}$
$-2(\alpha-1)J'$	$-2(\alpha+1)K'$	$+\frac{D'(2nD+\mathfrak{D})}{nn}$	$+\frac{2G'(2n+\mathfrak{C})}{nn}$
$+\frac{2L'(2nA+\mathfrak{A})}{nn}$	$+\frac{2M'(2nA+\mathfrak{A})}{nn}$	$+\frac{2F'(2n+\mathfrak{C})}{nn}$	$-2(2\alpha+1)M'$
		$-2(2\alpha-1)L'$	$+\frac{K'(2nA+\mathfrak{A})}{nn}$
		$+\frac{J'(2nA+\mathfrak{A})}{nn}$	

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vnde tandem nanciſcimur has determinationes:

$$\tfrac{1}{2}\delta-\gamma+\frac{\mathfrak{A}a+\frac{1}{2}\mathfrak{D}\mathfrak{D}+3D\mathfrak{D}+\frac{3}{2}DD+3Aa+3\mathfrak{A}a+3Aa}{nn}kk$$
$$=\frac{a'(2nA+\mathfrak{A})+D'(2nD+\mathfrak{D})}{nn}kk$$

$$3+\tfrac{3}{2}(A+D+E)-a-2na=-2aa'+\frac{2b'(2nA+\mathfrak{A})}{nn}$$
$$+\frac{(D'+E')(2n+\mathfrak{C})+2F'(2nD+\mathfrak{D})}{nn}$$

$$\tfrac{3}{2}(B+F+G)-b-2nb=-4ab'+\frac{a'(2nA+\mathfrak{A})}{nn}$$
$$+\frac{E'(2nD+\mathfrak{D})+2(F'+G')(2n+\mathfrak{C})}{nn}$$

$$\tfrac{3}{2}-2n\mathfrak{H}-H+\frac{\mathfrak{A}\mathfrak{J}+3\mathfrak{A}J+3A\mathfrak{J}+3AJ}{nn}=-2H'+\frac{A'(2nJ+\mathfrak{J})}{nn}$$
$$+\frac{E'(2nD+\mathfrak{D})+(J'+K')(2nA+\mathfrak{A})}{nn}$$

$$\tfrac{3}{4}\gamma-2n\mathfrak{J}-J+\tfrac{3}{4}A+\tfrac{3}{2}D=-2(a-1)J'+\frac{3A'}{2n}+\frac{A'(2nH+\mathfrak{H})}{nn}$$
$$+\frac{D'(2n+\mathfrak{C})+2L'(2nA+\mathfrak{A})}{nn}$$

$$\tfrac{3}{4}\varepsilon-2n\mathfrak{K}-K+\tfrac{3}{4}A+\tfrac{3}{2}E=-2(a+1)K'+\frac{3A'}{2n}+\frac{A'(2nH+\mathfrak{H})}{nn}$$
$$+\frac{2G'(2nD+\mathfrak{D})+2M'(2aA+\mathfrak{A})}{nn}$$

$$-2n\mathfrak{L}-L+\tfrac{3}{4}B+\tfrac{3}{2}F+\frac{\mathfrak{A}\mathfrak{J}+3\mathfrak{A}J+3A\mathfrak{J}+3AJ+\frac{1}{2}\mathfrak{D}\mathfrak{D}+3D\mathfrak{D}+3DD}{nn}=$$
$$-2(2a-1)L'+\frac{3B'}{n}+\frac{A'(2nJ+\mathfrak{J})+2B'(2nH+\mathfrak{H})}{nn}$$
$$+\frac{D'(2nD+\mathfrak{D})+2F'(2n+\mathfrak{C})+J'(2nA+\mathfrak{A})}{nn}$$

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$$-2\varkappa\mathfrak{M}-M+\tfrac{1}{4}B+\tfrac{1}{2}G=-2(2\alpha+1)M'+\frac{3B'}{n}+\frac{2B'(2\varkappa H+\mathfrak{H})}{nn}+\frac{2G'(2n+\mathfrak{E})+K'(2\varkappa A+\mathfrak{A})}{nn}$$

§. 96. Primum autem valoribus iam cognitis substituendis, reperitur:

$2\alpha\alpha=-3,837-0,038\,b$ et $4\alpha b=-1,073-0,019\,a$
hincque $a=-2,051$ et $b=-0,277$
ex quibus porro elicimus: $\alpha=-12,595$ et $b=-0,086$ et $\alpha'=-23,510$. Deinde pro reliquis litteris

$\mathfrak{J}=+32,663$ - - - $l\mathfrak{J}=1,514059$
$\mathfrak{K}=+1,035$ $l\mathfrak{K}=0,014776$

$J'=-2(1-\alpha)J-5,060;$
$K'=2(1+\alpha)K+0,227;$

$J=-15,555$ $lJ=1,191891$
$K=-0,370$ $lK=9,568589$

Porro $\mathfrak{L}=-1,453$ - - - $l\mathfrak{L}=0,162070$
$\mathfrak{M}=-0,000$ - - -

$L'=2(2\alpha-1)L-12,786$
$M'=2(2\alpha+1)M$

$L=+6,252$ $lL=0,796019$
$M=-0,001$ $lM=7,000000$

Denique $\mathfrak{H}=-0,123$ et $H=-1,033$
atque $\frac{1}{2}\delta-\gamma=-7,459$ kk

§. 97. His igitur valoribus inuentis innotescet primum distantia Lunae a terra curtata, quatenus a sola excen-

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excentricitate orbitae lunaris k pendet. Cum enim haec distantia posita sit $x = \frac{(1-kk)au}{1-k\cos r}$ ob $u = 1 + \frac{v}{nn}$, erit

$$\begin{aligned} u = 1 &- 0,007161\cos 2\eta &&- 0,0719\,kk\cos 2\eta \\ &+ 0,000073\cos 4\eta &&- 0,0005\,kk\cos 4\eta \\ &+ 0,194888\,k\cos(2\eta - r) &&- 0,003274\,k\cos(2\eta + r) \\ &- 0,003238\,k\cos(4\eta - r) &&+ 0,000078\,k\cos(4\eta + r) \\ &- 0,0059\,kk\cos 2r \\ &- 0,0889\,kk\cos(2\eta - 2r) &&- 0,0021\,kk\cos(2\eta + 2r) \\ & &&+ 0,0357\,kk\cos(4\eta - 2r) \end{aligned}$$

At pro longitudine Lunae, quatenus a sola excentricitate k pendet, prodibit $\frac{d\Phi}{dr} =$

$$\begin{aligned} u &+ 0,000285 + 0,019015\cos 2\eta + 0,000076\cos 4\eta \\ &+ 0,1562\,kk\cos 2\eta + 0,0008\,kk\cos 4\eta \\ &- 0,001255\,k\cos r - 0,38410\,k\cos(2\eta - r) + 0,002647\,k\cos(4\eta - r) \\ &+ 0,01278\,k\cos(2\eta + r) - 0,000229\,k\cos(4\eta + r) \\ &+ 0,0118\,kk\cos 2r - 0,0081\,kk\cos(2\eta - 2r) - 0,0076\,kk\cos(4\eta - 2r) \\ &+ 0,0102\,kk\cos(2\eta + 2r) \end{aligned}$$

§. 98. Ponatur nunc:

$$\begin{aligned} \Phi = 0r &+ \mathfrak{A}'\sin 2\eta + a'kk\sin 2\eta + \mathfrak{B}'\sin 4\eta + b'kk\sin 4\eta \\ &+ \mathfrak{C}'k\sin r + \mathfrak{D}'k\sin(2\eta - r) + \mathfrak{F}'k\sin(4\eta - r) \\ &+ \mathfrak{E}'k\sin(2\eta + r) + \mathfrak{G}'k\sin(4\eta + r) \\ &+ \mathfrak{H}'kk\sin 2r + \mathfrak{J}'kk\sin(2\eta - 2r) + \mathfrak{L}'kk\sin(4\eta - 2r) \\ &+ \mathfrak{K}'kk\sin(2\eta + 2r) + \mathfrak{M}'kk\sin(4\eta + 2r) \end{aligned}$$

atque

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atque differentiando orientur sequentes comparationes

$$\alpha + 0{,}000285 = \Omega - \frac{\mathfrak{a}'(2\alpha\mathfrak{A}+\mathfrak{A}) - \mathfrak{D}'(2\mathfrak{a}\mathfrak{D}+\mathfrak{D})}{\alpha\alpha} kk + 0{,}000199$$

$$+ 0{,}1562 = 2\alpha\mathfrak{a}' - \frac{(\mathfrak{D}'+\mathfrak{E}')(2\alpha+\mathfrak{C}) - 2\mathfrak{F}'(2\alpha\mathfrak{D}+\mathfrak{D})}{\alpha\alpha}$$

$$+ 0{,}0008 = 4\alpha\mathfrak{b}' - \frac{\mathfrak{a}'(2\alpha\mathfrak{A}+\mathfrak{A}) - \mathfrak{E})2\alpha\mathfrak{D}+\mathfrak{D}) - 2(\mathfrak{F}'+\mathfrak{G}')(2\alpha+\mathfrak{C})}{\alpha\alpha}$$

$$+ 0{,}0118 = 2\mathfrak{H}' - \frac{\mathfrak{G}'(2\alpha\mathfrak{D}+\mathfrak{D}) - \mathfrak{A}'(2\alpha\mathfrak{J}+\mathfrak{J}) - (\mathfrak{J}'+\mathfrak{K}')(2\alpha\mathfrak{A}+\mathfrak{A})}{\alpha\alpha}$$

$$- 0{,}0081 = 2(\alpha-1)\mathfrak{J}' - \frac{3\mathfrak{A}'}{2\alpha} - \frac{\mathfrak{D}'(2\alpha+\mathfrak{C}) - \mathfrak{A}'(2\alpha\mathfrak{H}+\mathfrak{H})}{\alpha\alpha} - \frac{2\mathfrak{L}'(2\alpha\mathfrak{A}+\mathfrak{A})}{\alpha\alpha}$$

$$+ 0{,}0102 = 2(\alpha+1)\mathfrak{K}' - \frac{3\mathfrak{A}'}{2\alpha} - \frac{\mathfrak{A}'(2\alpha\mathfrak{H}+\mathfrak{H}) - 2\mathfrak{G}'(2\alpha\mathfrak{D}+\mathfrak{D})}{\alpha\alpha}$$

$$- 0{,}0076 = 2(2\alpha-1)\mathfrak{L}' - \frac{3\mathfrak{B}'}{2\alpha} - \frac{\mathfrak{A}'(2\alpha\mathfrak{J}+\mathfrak{J}) - \mathfrak{J}'(2\alpha\mathfrak{A}+\mathfrak{A})}{\alpha\alpha}$$

$$- \frac{\mathfrak{D}'(2\alpha\mathfrak{D}+\mathfrak{D}) - 2\mathfrak{H}'(2\alpha+\mathfrak{C})}{\alpha\alpha}$$

$$\bullet = 2(2\alpha+1)\mathfrak{M}' - \frac{3\mathfrak{B}'}{\alpha} - \frac{2\mathfrak{G}'(2\alpha+\mathfrak{C})}{\alpha\alpha}$$

§. 99. Ex his comparationibus elicimus:

$\mathfrak{a}' = +0{,}0509$; $\mathfrak{b}' = 0{,}0008$

$\mathfrak{J}' = +0{,}5385$; $\mathfrak{K}' = 0{,}0028$

$\mathfrak{L}' = -0{,}1055$; $\mathfrak{M}' = 0{,}0000$

$\mathfrak{H}' = +0{,}0021$; et $\Omega = \alpha - 0{,}000429$

posito $k = 0,05445$. Hinc autem erit $\frac{1}{2}\delta - \gamma = -0,02302$
Cum autem iam ante inuentum esset $\frac{1}{2}\delta - \gamma = -0,01742$
erit reuera $\frac{1}{2}\delta - \gamma = -0,00560$. Tum vero inueniemus:
$\gamma = 1,40673$, vnde erit $\frac{1}{2}\delta = 1,40113$, et $\delta = 2,80226$
hincque $2\gamma - \frac{3}{2}\delta = -1,38993$ et $\frac{2\gamma - \frac{1}{2}\delta}{nn} = -0,00794$.
Verum ex cognita ratione motus medii ad motum anomaliae est $O = 1,0085272$, vnde $\varkappa = 1,0089562$
Verum esse debet $\varkappa = 1 + \frac{3 + 4\mu + \delta}{4nn}$; vnde foret
$0,0089562 = 0,008289 + \frac{\mu}{nn}$; ideoque $\frac{\mu}{nn} = 0,000667$
qui valor cum sit tam exiguus, merito dubitamus, num μ non prorsus sit $= 0$.

CAPUT

CAPUT VII.

CORRECTIO INAEQUALITATUM LUNAE, ANTE INUENTARUM.

§. 100.

Quoniam nunc quidem valores litterarum γ et δ ita inuenimus, vt eos pro proxime veris habere queamus, ex iis coefficientes terminorum, quibus inaequalitates lunae continentur, accuratius definire poterimus. Cum enim sit $\gamma = 1,40673$ et $\delta = 2,80226$, colligamus hic in vnum omnes formulas, quas hactenus pro inueniendis coefficientibus assumtis elicuimus. Posueramus autem:

$$\begin{aligned}\int R\,dv = {} & \mathfrak{A}\cos 2\eta + \mathfrak{a}\,kk\cos 2\eta + \mathfrak{B}\cos 4\eta + \mathfrak{b}\,kk\cos 4\eta \\ & + \mathfrak{C}\,k\cos r + \mathfrak{D}\,k\cos(2\eta - r) + \mathfrak{F}\,k\cos(4\eta - r) \\ & + \mathfrak{E}\,k\cos(2\eta + r) + \mathfrak{G}\,k\cos(4\eta + r) \\ & + \mathfrak{H}\,kk\cos 2r + \mathfrak{J}\,k^2\cos(2\eta - 2r) + \mathfrak{L}\,k^2\cos(4\eta - 2r) \\ & + \mathfrak{K}\,k^2\cos(2\eta + 2r) + \mathfrak{M}\,k^2\cos(4\eta + 2r)\end{aligned}$$

$$\begin{aligned}v = {} & A\cos 2\eta + a\,kk\cos 2\eta + B\cos 4\eta + b\,kk\cos 4\eta \\ & + D\,k\cos(2\eta - r) + F\,k\cos(4\eta - r) \\ & + E\,k\cos(2\eta + r) + G\,k\cos(4\eta + r) \\ & + H\,kk\cos 2r + J\,kk\cos(2\eta - 2r) + L\,kk\cos(4\eta - 2r) \\ & + K\,kk\cos(2\eta + 2r) + M\,kk\cos(4\eta + 2r)\end{aligned}$$

§. 101.

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§. 101. Hinc posito $u = V\left(1 + \frac{\mathfrak{Z} + 4\mathfrak{n} + \delta}{2nn}\right)$ collegimus fore

$$\frac{d\Phi}{dr} = u + \frac{A(3uA+2\mathfrak{A})}{2n^4} + \frac{D(3uD+2\mathfrak{D})}{2n^4}kk + \frac{A(3ua+\mathfrak{a})}{2n^4}kk$$
$$+ \frac{a(3uA+2\mathfrak{A})}{2n^4}kk - \frac{(2uA+\mathfrak{A})}{nn}\cos 2\eta - \frac{(2uB+\mathfrak{B})}{nn}\cos 4\eta$$
$$+ \frac{A(3uA+2\mathfrak{A})}{2n^4}\cos 4\eta - \frac{\mathfrak{C}}{nn}k\cos r - \frac{(2ua+\mathfrak{a})}{nn}k^2\cos 2\eta$$
$$- \frac{(2ub+\mathfrak{b})}{nn}k^2\cos 4\eta + \frac{D(3uA+2\mathfrak{A})+A(3uD+2\mathfrak{D})}{2n^4}k\cos r$$
$$- \frac{(2uD+\mathfrak{D})}{nn}k\cos(2\eta - r) - \frac{(2uF+\mathfrak{F})}{nn}k\cos(4\eta - r)$$
$$+ \frac{D(3uA+2\mathfrak{A})+A(3uD+2\mathfrak{D})}{2n^4}k\cos(4\eta - r)$$
$$+ \frac{(2uE+\mathfrak{E})}{nn}k\cos(2\eta + r) - \frac{(2uG+\mathfrak{G})}{nn}k\cos(4\eta + r)$$
$$- \frac{(2uJ+\mathfrak{J})}{nn}kk\cos(2\eta - 2r) - \frac{(2uH+\mathfrak{H})}{nn}kk\cos 2r$$
$$+ \frac{J(3uA+2\mathfrak{A})+A(3uJ+2\mathfrak{J})}{2n^4}k^2\cos 2r$$
$$- \frac{(2uK+\mathfrak{K})}{nn}kk\cos(2\eta + 2r) - \frac{(2uL+\mathfrak{L})}{nn}k^2\cos(4\eta - 2r)$$
$$+ \frac{D(3uD+2\mathfrak{D})+J(3uA+2\mathfrak{A})+A(3uJ+2\mathfrak{J})}{2n^4}k^2\cos(4\eta - 2r)$$
$$- \frac{(2uM+\mathfrak{M})}{nn}k^2\cos(4\eta + 2r)$$

§. 102.

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§. 102. Si iam ponamus

$$\alpha + \frac{A(3\kappa A + 2\mathfrak{A})}{2n^4} + \frac{D(3\kappa D + 2\mathfrak{D})}{2n^4} kk$$
$$+ \frac{A(3\kappa\alpha + 2\mathfrak{a})}{2n^4} kk + \frac{\alpha(3\kappa A + 2\mathfrak{A})}{2n^4} kk$$
$$- \frac{1}{n} = \alpha;$$ vt fit neglectis terminis admodum exiguis

$$\frac{d\eta}{dr} = \alpha - \frac{(2\kappa A + \mathfrak{A})}{nn} \cos 2\eta - \frac{(2\kappa + \mathfrak{C})}{nn} k \cos r$$
$$- \frac{(2\kappa D + \mathfrak{D})}{nn} k \cos(2\eta - r) - \frac{(2\kappa E + \mathfrak{E})}{nn} k^2 \cos(2\eta + r)$$

ex superioribus capitibus repetimus has determinationes:

$$2\alpha\mathfrak{A} = -\tfrac{3}{2}$$
$$4\alpha\mathfrak{B} = -\frac{3A}{2nn} + \frac{\mathfrak{A}(3\kappa A + \mathfrak{A})}{nn}$$
$$\mathfrak{C} = \frac{\mathfrak{A}(2\kappa D + \mathfrak{D})}{nn} - \frac{(\mathfrak{D} - \mathfrak{E})(2\kappa A + \mathfrak{A})}{nn} - \frac{\mathfrak{A}(2\kappa E + \mathfrak{E})}{nn} - \frac{3(D - E)}{2nn}$$
$$(2\alpha - 1)\mathfrak{D} = -3 + \frac{(2\kappa + \mathfrak{C})}{nn}\mathfrak{A}$$
$$(2\alpha + 1)\mathfrak{E} = -3 + \frac{(2\kappa + \mathfrak{C})}{nn}\mathfrak{A}$$
$$(4\alpha - 1)\mathfrak{F} = \frac{2(2\kappa + \mathfrak{C})}{nn}\mathfrak{B} + \frac{\mathfrak{A}(2\kappa D + \mathfrak{D})}{nn} + \frac{\mathfrak{D}(2\kappa A + \mathfrak{A})}{nn} - \frac{3(2A + D)}{2nn}$$
$$(4\alpha + 1)\mathfrak{G} = \frac{2(2\kappa + \mathfrak{C})}{nn}\mathfrak{B} + \frac{\mathfrak{A}(2\kappa E + \mathfrak{E})}{nn} + \frac{\mathfrak{E}(2\kappa A + \mathfrak{A})}{nn} - \frac{3(2A + E)}{2nn}$$
$$2\alpha\mathfrak{a} = -\tfrac{15}{4} + \frac{(\mathfrak{D} + \mathfrak{E})(2\kappa + \mathfrak{C})}{nn} + \frac{2\mathfrak{B}(2\kappa A + \mathfrak{A})}{nn}$$
$$+ \frac{2\mathfrak{F}(2\kappa D + \mathfrak{D})}{nn} + \frac{\mathfrak{D}(2\kappa F + \mathfrak{F})}{nn}$$

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$$4ab = -\frac{3D}{nn} + \frac{2(\mathfrak{F}+\mathfrak{G})(2n+\mathfrak{C})}{nn} + \frac{\mathfrak{C}(2\varkappa D+\mathfrak{D})}{nn} + \frac{\mathfrak{a}(2\varkappa A+\mathfrak{A})}{nn} + \frac{\mathfrak{D}(2\varkappa E+\mathfrak{E})}{nn}$$

$$2\mathfrak{H} = -\frac{3(2D+J)}{2nn} + \frac{\mathfrak{E}(2\varkappa D+\mathfrak{D})}{nn} + \frac{\mathfrak{A}(2\varkappa J+\mathfrak{J})}{nn} - \frac{(\mathfrak{J}-\mathfrak{K})(2\varkappa A+\mathfrak{A})}{nn}$$

$$2(a-1)\mathfrak{J} = -\tfrac{15}{4} - \frac{3(H-L)}{2nn} + \frac{3\mathfrak{A}}{2n} + \frac{\mathfrak{A}(2\varkappa H+\mathfrak{H})}{nn} + \frac{\mathfrak{D}(2n+\mathfrak{C})}{nn} + \frac{2\mathfrak{L}(2\varkappa A+\mathfrak{A})}{nn}$$

$$2(a+1)\mathfrak{K} = -\tfrac{15}{4} - \frac{3H}{2nn} + \frac{3\mathfrak{A}}{2n} + \frac{\mathfrak{A}(2\varkappa H+\mathfrak{H})}{nn} + \frac{\mathfrak{E}(2n+\mathfrak{C})}{nn} + \frac{2\mathfrak{G}(2\varkappa D+\mathfrak{D})}{nn}$$

$$2(2a-1)\mathfrak{L} = -\frac{3(2D+J)}{2nn} + \frac{3\mathfrak{B}}{n} + \frac{2\mathfrak{B}(2\varkappa H+\mathfrak{H})}{nn} + \frac{2\mathfrak{F}(2n+\mathfrak{C})}{nn} + \frac{\mathfrak{A}(2\varkappa J+\mathfrak{J})}{nn} + \frac{\mathfrak{D}(2\varkappa D+\mathfrak{D})}{nn} + \frac{\mathfrak{J}(2\varkappa A+\mathfrak{A})}{nn}$$

$$2(2a+1)\mathfrak{M} = \ldots \frac{3\mathfrak{B}}{n} + \frac{2\mathfrak{B}(2\varkappa H+\mathfrak{H})}{nn} + \frac{2\mathfrak{G}(2n+\mathfrak{C})}{nn} + \frac{\mathfrak{K}(2\varkappa A+\mathfrak{A})}{nn}$$

§. 103. Antequam vlterius progrediamur, ſequentes notandae ſunt nouae denominationes

$$A' = 2aA + \frac{A(2\varkappa B+\mathfrak{B})}{nn} - \frac{B(2\varkappa A+\mathfrak{A})}{nn}$$

$$B' = 4aB - \frac{A(2\varkappa A+\mathfrak{A})}{nn}$$

$$C' = \frac{\mathfrak{A}(D-E) - A(\mathfrak{D}-\mathfrak{E})}{nn}$$

$$D' = (2a-1)D - \frac{(2n+\mathfrak{C})}{nn}A$$

$$E' =$$

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$$E' = (2\alpha+1)\,E - \frac{(2n+\mathfrak{C})}{nn}\,A$$

$$F' = (4\alpha-1)\,F - \frac{4B}{n} - \frac{A(2\varkappa D+\mathfrak{D}) - D(2\varkappa A+\mathfrak{A})}{nn}$$

$$G' = (4\alpha+1)\,G - \frac{4B}{n} - \frac{A(2\varkappa E+\mathfrak{E}) - E(2\varkappa A+\mathfrak{A})}{nn}$$

$$a' = 2\alpha a - \frac{2b(2\varkappa A+\mathfrak{A})}{nn} - \frac{(D+E)(2n+\mathfrak{C})}{nn} - \frac{2F(2\varkappa D+\mathfrak{D})}{nn} + \frac{D(2\varkappa F+\mathfrak{F})}{nn}$$

$$b' = 4\alpha b - \frac{a(2\varkappa A+\mathfrak{A})}{nn} - \frac{A(2\varkappa a+\mathfrak{a})}{nn} - \frac{D(2\varkappa E+\mathfrak{E})}{nn} - \frac{E(2\varkappa D+\mathfrak{D})}{nn} - \frac{2(F+G)(2n+\mathfrak{C})}{nn}$$

$$H' = 2H + \frac{D\mathfrak{C} - \mathfrak{D}E - A(\mathfrak{J}-\mathfrak{K}) + \mathfrak{A}(J-K)}{nn}$$

$$J' = 2(\alpha-1)\,J + \frac{3A}{2n} - \frac{A(2\varkappa H+\mathfrak{H})}{nn} - \frac{D(2n+\mathfrak{C})}{nn} - \frac{2\mathfrak{L}(2\varkappa A+\mathfrak{A})}{nn} + \frac{A(2\varkappa L+\mathfrak{L})}{nn}$$

$$K' = 2(\alpha+1)K - \frac{3A}{n} - \frac{A(2\varkappa H+\mathfrak{H})}{nn} - \frac{E(2n+\mathfrak{C})}{nn} - \frac{2G(2\varkappa D+\mathfrak{D})}{nn}$$

$$L' = (2\alpha-1)\,L - \frac{3B}{n} - \frac{2B(2\varkappa H+\mathfrak{H})}{nn} - \frac{2F(2n+\mathfrak{C})}{nn} - \frac{D(2\varkappa D+\mathfrak{D})}{nn} - \frac{A(2\varkappa J+\mathfrak{J})}{nn} - \frac{J(2\varkappa A+\mathfrak{A})}{nn}$$

$$M' = 2(2\alpha+1)\,M - \frac{3B}{n} - \frac{2B(2\varkappa H+\mathfrak{H})}{nn} - \frac{2G(2n+\mathfrak{C})}{nn} - \frac{E(2\varkappa E+\mathfrak{E})}{nn}$$

§. 104. Nunc vt terminos completos obtineamus, saltem eos qui angulos 2η et r inuoluunt, notandum

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est in nostris aequationibus sin 2η et cos 2η non per $\frac{3}{2}(1+2kk)$ sed per $\frac{3}{2}(1+2kk+\frac{9}{2}ee)$ esse multiplicatos. Hinc cum sit fere $\frac{9}{4}ee = \frac{3}{4}kk$, loco kk hic scribi oportebit $\frac{5}{4}kk$, vnde in valore ipsius α pro 3 scripsi $3\cdot\frac{5}{4}$ seu $\frac{15}{4}$. Deinde vt in his terminis quoque rationem habeamus inclinationis orbitae, cuius medius valor sit $= \varepsilon$, ponamus $\frac{3}{4}(nn+2+3\mu+\gamma)\,\mathrm{tang}\,\varepsilon^2 = f = \frac{3}{4}(\frac{3}{2}nn\,nn - \frac{5}{2}nn - \frac{5}{4}+\gamma-\frac{3}{4}\delta)\,\mathrm{tang}\,\varepsilon^2$ ob $\mu = \frac{5}{2}(nn-1)nn - \frac{3}{4} - \frac{1}{4}\delta$, eritque nostra aequatio:

$$\frac{ddv}{dr^2} = \tfrac{1}{2}\delta-\gamma+\tfrac{3}{4}kk-\gamma k\cos r+\tfrac{3}{2}kk\cos 2r+\tfrac{3}{2}\cos 2\eta+\tfrac{15}{4}kk\cos 2\eta$$
$$+f+\tfrac{1}{2}fkk+fk\cos r+\tfrac{1}{2}fkk\cos 2r$$
$$+3k\cos(2\eta-r)+3k\cos(2\eta+r)+\tfrac{15}{4}kk\cos(2\eta-2r)+\tfrac{15}{4}kk\cos(2\eta+2r)$$
$$-2n\int R\,dr - v\left(1-\tfrac{3}{2}kk+\frac{2f}{nn}+\frac{fkk}{nn}-\frac{(2\gamma-\frac{3}{2}\delta)}{nn}\right)$$
$$+v\left(3nn+\frac{3}{2nn}-\frac{2f}{nn}+\frac{(2\gamma-\frac{3}{2}\delta)}{nn}\right)k\cos r$$
$$+v\left(\tfrac{3}{2}-\frac{f}{nn}\right)k^2\cos 2r$$
$$+v\left(\frac{3}{2nn}\cos 2\eta+\frac{3k}{nn}\cos(2\eta-r)+\frac{3k}{nn}\cos(2\eta+r)\right)$$
$$+\frac{1}{nn}\left(\int R\,dr\right)^2+\frac{6v}{nn}\int R\,dr+\frac{3vv}{nn}-\frac{3vv}{nn}k\cos r$$

§. 105. Sit breuitatis gratia:

$$1+\frac{2f}{nn}-\frac{(2\gamma-\frac{3}{2}\delta)}{nn}=g;\quad 3nn+\frac{3}{2nn}-\frac{2f}{nn}+\frac{(2\gamma-\frac{3}{2}\delta)}{nn}=h$$

et $1-\frac{3}{2}kk+\frac{2f}{nn}+\frac{fkk}{nn}-\frac{(2\gamma-\frac{3}{2}\delta)}{nn}=6$, quo termino

termino in angulis ex 2η et r compositis vtemur: eritque $\frac{ddv}{dr^2} =$

$$\tfrac{1}{2}\delta - \gamma + \tfrac{1}{4}kk + f + \tfrac{1}{2}fkk + \frac{3A}{4nn} + \frac{3\alpha kk}{4nn}$$
$$+ \frac{3Dkk}{2nn} + \frac{3Ekk}{2nn} + \frac{\mathfrak{A}\mathfrak{A}}{2nn} + \frac{\mathfrak{C}\mathfrak{C}kk}{2nn} + \frac{\mathfrak{D}\mathfrak{D}kk}{2nn} + \frac{\mathfrak{E}\mathfrak{E}kk}{2nn} - \frac{3ADkk}{2nn}$$
$$+ \frac{3A\mathfrak{A}}{nn} + \frac{3D\mathfrak{D}kk}{nn} + \frac{3AA}{2nn} + \frac{3DDkk}{2nn} + \frac{3EEkk}{2nn}$$
$$+ \cos 2\eta \left(\tfrac{1}{2} - 2\alpha\mathfrak{A} - \mathfrak{G}A \right)$$
$$+ kk\cos 2\eta \left\{ \tfrac{15}{4} - 2\alpha\mathfrak{a} - \mathfrak{G}\alpha + \tfrac{1}{2}bD + \tfrac{1}{2}bE + \frac{\mathfrak{C}D}{nn} + \frac{3\mathfrak{C}E}{nn} \right.$$
$$+ \cos 4\eta \left\{ - 2\alpha\mathfrak{B} - \mathfrak{G}B + \frac{3A}{4nn} + \frac{\mathfrak{A}\mathfrak{A}}{2nn} + \frac{3A\mathfrak{A}}{nn} + \frac{3AA}{2nn} \right.$$
$$+ kk\cos 4\eta \left\{ \begin{array}{l} - 2\alpha b - \mathfrak{G}b + \tfrac{1}{2}bF + \tfrac{1}{2}bG + \frac{3E}{2nn} + \frac{3D}{2nn} \\ + \frac{\mathfrak{A}\alpha}{nn} + \frac{\mathfrak{D}\mathfrak{E}}{nn} + \frac{3DE}{nn} + \frac{3A\alpha}{nn} + \frac{3A\alpha}{nn} + \frac{3\mathfrak{A}\alpha}{nn} \end{array} \right.$$
$$+ k\cos r \left\{ \begin{array}{l} - \gamma + f - 2\alpha\mathfrak{C} + \frac{3D}{4nn} + \frac{3E}{4nn} + \frac{3A}{nn} + \frac{\mathfrak{A}D}{nn} + \frac{\mathfrak{A}\mathfrak{E}}{nn} \\ + \frac{3A\mathfrak{D}}{nn} + \frac{3A\mathfrak{E}}{nn} + \frac{3\mathfrak{A}D}{nn} + \frac{3\mathfrak{A}E}{nn} + \frac{3AD}{nn} + \frac{3AE}{nn} - \frac{3AA}{2nn} \end{array} \right.$$
$$+ k\cos(2\eta - r) \left\{ 3 - 2\alpha\mathfrak{D} - \mathfrak{G}D + \tfrac{1}{2}bA + \frac{3F}{4nn} + \frac{\mathfrak{A}\mathfrak{E}}{nn} + \frac{3A\mathfrak{E}}{nn} \right.$$
$$+ k\cos(2\eta + r) \left\{ 3 - 2\alpha\mathfrak{E} - \mathfrak{G}E + \tfrac{1}{2}bA + \frac{3G}{4nn} + \frac{\mathfrak{A}\mathfrak{E}}{nn} + \frac{3A\mathfrak{E}}{nn} \right.$$

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$$+kk\cos 2r\left\{\begin{array}{l}\frac{3}{2}+\frac{1}{2}f-2\kappa\mathfrak{H}-6H+\frac{3J}{4nn}+\frac{3K}{4nn}+\frac{3E}{2nn}+\frac{3D}{2nn}\\ +\frac{\mathfrak{C}\mathfrak{C}}{nn}+\frac{\mathfrak{D}\mathfrak{E}}{nn}+\frac{3D\mathfrak{E}}{nn}+\frac{3\mathfrak{D}E}{nn}+\frac{3DE}{nn}-\frac{3A\mathfrak{D}}{2nn}\end{array}\right.$$

$$+kk\cos(2\eta-2r)\left\{\begin{array}{l}\frac{15}{4}-2\kappa\mathfrak{J}-6J+\frac{1}{2}bD+\left(\frac{3}{4}-\frac{f}{2nn}\right)A\\ +\frac{3H}{4nn}+\frac{\mathfrak{A}\mathfrak{H}}{nn}+\frac{3A\mathfrak{H}}{nn}+\frac{3\mathfrak{C}D}{nn}\end{array}\right.$$

$$+kk\cos(2\eta+2r)\left\{\begin{array}{l}\frac{15}{4}-2\kappa\mathfrak{K}-6K+\frac{1}{2}bE+\left(\frac{3}{4}-\frac{f}{2nn}\right)A\\ +\frac{3H}{4nn}+\frac{\mathfrak{A}\mathfrak{H}}{nn}+\frac{3A\mathfrak{H}}{nn}+\frac{3\mathfrak{C}E}{nn}\end{array}\right.$$

$$+kk\cos(4\eta-2r)\left\{\begin{array}{l}-2\kappa\mathfrak{L}-6L+\left(\frac{3}{4}-\frac{f}{2nn}\right)B\\ +\frac{3J}{4nn}+\frac{3D}{2nn}+\frac{\mathfrak{D}\mathfrak{D}}{2nn}+\frac{3D\mathfrak{D}}{2nn}+\frac{1}{2}bF\end{array}\right.$$

$$+kk\cos(\eta 4+2r)\left\{\begin{array}{l}-2\kappa\mathfrak{M}-6M+\left(\frac{3}{4}-\frac{f}{2nn}\right)B\\ +\frac{3K}{4nn}+\frac{3E}{2nn}+\frac{\mathfrak{E}\mathfrak{E}}{2nn}+\frac{3E\mathfrak{E}}{2nn}+\frac{1}{2}bG\end{array}\right.$$

$$+k\cos(4\eta-r)\left\{\begin{array}{l}-2\kappa\mathfrak{F}-6F+\frac{1}{2}hB+\frac{3D}{4nn}+\frac{3A}{2nn}\\ +\frac{\mathfrak{A}\mathfrak{D}}{nn}+\frac{3A\mathfrak{D}}{nn}+\frac{3\mathfrak{A}D}{nn}+\frac{3AD}{nn}-\frac{3AA}{4nn}\end{array}\right.$$

$$+k\cos(4\eta+r)\left\{\begin{array}{l}-2\kappa\mathfrak{G}-6G+\frac{1}{2}bB+\frac{3E}{4nn}+\frac{3A}{2nn}\\ +\frac{\mathfrak{A}\mathfrak{E}}{nn}+\frac{3A\mathfrak{E}}{nn}+\frac{3\mathfrak{A}E}{nn}+\frac{3AE}{nn}-\frac{3AA}{4nn}\end{array}\right.$$

§. 106.

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§. 106. Hinc denique nascentur sequentes aequalitates.

I. $$\tfrac{1}{2}\delta - \gamma + \tfrac{1}{4}kk + \tfrac{1}{2}fkk + \frac{3A}{4nn} + \frac{\mathfrak{A}\mathfrak{A}}{2nn} + \frac{3A\mathfrak{A}}{nn} + \frac{3\cdot AA}{2nn}$$
$$+ \frac{3(D+E)}{nn}kk - \frac{3AD\,kk}{2nn} + \frac{\mathfrak{C}\mathfrak{C}kk}{2nn} + \frac{(\mathfrak{D}\mathfrak{D}+\mathfrak{E}\mathfrak{E})}{2nn}kk$$
$$+ \frac{3D\mathfrak{D}}{nn}kk + \frac{3(DD+EE)}{2nn}kk + \frac{\mathfrak{A}a}{nn}kk$$
$$+ \frac{3Aa}{nn}kk + \frac{3\mathfrak{A}a}{nn}kk + \frac{3Aa}{nn}kk = \frac{A'(2nA+\mathfrak{A})}{nn} + \frac{a'(2nA+\mathfrak{A})}{nn}kk$$
$$+ \frac{D'(2nD+\mathfrak{D})}{nn}kk + \frac{A'(2na+a)}{nn}kk$$

II. $$\tfrac{3}{2} - 2n\mathfrak{A} - 6A = -2aA' + \frac{A'(2nB+\mathfrak{B})}{nn} + \frac{2B'(2nA+\mathfrak{A})}{nn}$$

III. $$-2n\mathfrak{B} - 6B + \frac{3A}{4nn} + \frac{\mathfrak{A}\mathfrak{A}}{2nn} + \frac{3A\mathfrak{A}}{nn} + \frac{3AA}{2nn}$$
$$= -4aB' + \frac{A'(2nA+\mathfrak{A})}{nn}$$

IV. $$\tfrac{15}{4} - 2na - 6a + \tfrac{1}{2}b(D+E) + \frac{3\mathfrak{C}(D+E)}{nn} = -2aa'$$
$$+ \frac{(D'+E')(2n+\mathfrak{C})}{nn} + \frac{2b'(2nA+\mathfrak{A})}{nn} + \frac{2F'(2nD+\mathfrak{D})}{nn}$$

V. $$-2nb - 6b + \tfrac{1}{2}b(F+G) + \frac{3(D+E)}{nn} + \frac{3D(E+J)}{nn} + \frac{3Aa}{nn} = -4ab'$$
$$+ \frac{a'(2nA+\mathfrak{A})}{nn} + \frac{E'(2nD+\mathfrak{D})}{nn} + \frac{2(F'+G')(2n+\mathfrak{C})}{nn} + \frac{D'(2nE+\mathfrak{E})}{nn}$$

VI.

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VI. $$-\gamma+f-2\alpha\mathfrak{C}+\frac{3(\mathfrak{D}+\mathfrak{E})}{4nn}+\frac{3A}{nn}-\frac{3AA}{2nn}+\frac{\mathfrak{A}(\mathfrak{D}+\mathfrak{E})}{nn}$$
$$+\frac{3A(\mathfrak{D}+\mathfrak{E})}{nn}+\frac{3A(D+E)}{nn}+\frac{3\mathfrak{A}(D+E)}{nn}=$$
$$-C'+\frac{A'(2\alpha D+\mathfrak{D})}{nn}+\frac{A'(2\alpha E+\mathfrak{E})}{nn}+\frac{(D'+E')(2\alpha A+\mathfrak{A})}{nn}$$

VII. $$3-2\alpha\mathfrak{D}-\mathfrak{F}D+\tfrac{1}{2}bA+\frac{3F}{4nn}+\frac{\mathfrak{C}(\mathfrak{A}+3A)}{nn}=$$
$$-(2\alpha-1)D'+\frac{A'(2\alpha+\mathfrak{C})}{nn}$$

VIII. $$3-2\alpha\mathfrak{E}-\mathfrak{F}E+\tfrac{1}{2}bA+\frac{3G}{4nn}+\frac{\mathfrak{C}(\mathfrak{A}+3A)}{nn}=$$
$$-(2\alpha+1)E'+\frac{A'(2\alpha+\mathfrak{C})}{nn}$$

IX. $$-3\alpha\mathfrak{F}-\mathfrak{F}F+\tfrac{1}{2}bB+\frac{3A}{2nn}+\frac{3D}{4nn}+\frac{\mathfrak{A}D}{nn}+\frac{3AD}{nn}$$
$$+\frac{3\mathfrak{A}D}{nn}+\frac{3AD}{nn}-\frac{3AA}{4nn}=-(4\alpha-1)F+\frac{4B'}{n}$$
$$+\frac{A'(2\alpha D+\mathfrak{D})}{nn}+\frac{D'(2\alpha A+\mathfrak{A})}{nn}$$

X. $$-2\alpha\mathfrak{G}-\mathfrak{F}G+\tfrac{1}{2}bB+\frac{3A}{2nn}+\frac{3E}{4nn}+\frac{\mathfrak{A}\mathfrak{E}}{nn}$$
$$+\frac{3A\mathfrak{E}}{nn}+\frac{3\mathfrak{A}E}{nn}+\frac{3AE}{nn}-\frac{3AA}{4nn}=-(4\alpha+1)G'+\frac{4B'}{n}$$
$$+\frac{A'(2\alpha E+\mathfrak{E})}{nn}+\frac{E'(2\alpha A+\mathfrak{A})}{nn}$$

XI.

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XI. $\frac{1}{8} + \frac{1}{2}f - 2x\mathfrak{H} - 6H + \frac{3(D+E)}{2nn} + \frac{3(J+K)}{4nn}$
$- \frac{3AD}{2nn} + \frac{\mathfrak{C}\mathfrak{C}}{2nn} - \frac{3AE}{2nn} + \frac{\mathfrak{D}\mathfrak{E}}{nn} + \frac{3\mathfrak{D}E}{nn} + \frac{3D\mathfrak{E}}{nn} + \frac{3DE}{nn}$
$+ \frac{\mathfrak{A}\mathfrak{J}}{nn} + \frac{3\mathfrak{A}J}{nn} + \frac{3A\mathfrak{J}}{nn} + \frac{3AJ}{nn} = -2H' + \frac{A'(2xJ+\mathfrak{J})}{nn}$
$+ \frac{E'(2xD+\mathfrak{D})}{nn} + \frac{D'(2xE+\mathfrak{E})}{nn} + \frac{(J'+K')(2xA+\mathfrak{A})}{nn}$

XII. $\frac{15}{4} - 2x\mathfrak{J} - 6J + \frac{1}{2}bD + \left(\frac{3}{4} - \frac{f}{2nn}\right)A + \frac{3H}{4nn}$
$+ \frac{\mathfrak{H}(\mathfrak{A}+3A)}{nn} + \frac{3\mathfrak{C}D}{nn} = -2(\alpha-1)J' + \frac{3A'}{2n}$
$+ \frac{D'(2n+\mathfrak{C})}{nn} + \frac{A'(2xH+\mathfrak{H})}{nn} + \frac{2L'(2xA+\mathfrak{A})}{nn}$

XIII. $\frac{15}{4} - 2x\mathfrak{K} - 6K + \frac{1}{2}bE + \left(\frac{3}{4} - \frac{f}{2nn}\right)A + \frac{3H}{4nn}$
$+ \frac{\mathfrak{H}(\mathfrak{A}+3A)}{nn} + \frac{3\mathfrak{C}E}{nn} = -2(\alpha+1)K' + \frac{3A'}{2n}$
$+ \frac{E'(2n+\mathfrak{C})}{nn} + \frac{A'(2xH+\mathfrak{H})}{nn} + \frac{2G'(2xD+\mathfrak{D})}{nn}$

XIV. $-2x\mathfrak{C} - 6L + \frac{1}{2}bF + \left(\frac{3}{4} - \frac{f}{2nn}\right)B - \frac{3AD}{2nn}$
$+ \frac{3D}{2nn} + \frac{3J}{4nn} + \frac{\mathfrak{D}\mathfrak{D}}{2nn} + \frac{3\mathfrak{D}D}{nn} + \frac{3D\mathfrak{D}}{2nn} = -2(2\alpha-1)L' + \frac{3B'}{n}$
$+ \frac{2F'(2n+\mathfrak{C})}{nn} + \frac{A'(2xJ+\mathfrak{J})}{nn} + \frac{J'(2xA+\mathfrak{A})}{nn} + \frac{D'(2xD+\mathfrak{D})}{nn}$

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$$\text{XV.}\quad -2\varkappa\mathfrak{M} - 6M + \tfrac{1}{2}bG + \left(\tfrac{1}{4} - \frac{f}{2nn}\right)B - \frac{3AE}{2nn} + \frac{3E}{2nn} + \frac{3K}{4nn} + \frac{\mathfrak{E}\mathfrak{E}}{2nn} + \frac{3\mathfrak{E}E}{nn} + \frac{3EE}{2nn} = -2(2\alpha+1)M' + \frac{3B'}{n} + \frac{2G'(2n+\mathfrak{E})}{nn}.$$

§. 107. Nunc antequam hos valores inuenire queamus, verus valor ipsius α inuestigari debet: quod fiet ex valore integrali ipsius Φ, qui si vti §. 98. ponatur $\Phi = O\eta + \mathfrak{A}'\sin 2\eta + \mathfrak{a}'kk\sin 2\eta +$ etc. obtinebitur.

$$\varkappa + \frac{A(3\varkappa A+2\mathfrak{A})}{2n^4} + \frac{D(3\varkappa D+2\mathfrak{D})}{2n^4}kk + \frac{A(3\varkappa\alpha+\mathfrak{a})}{2n^4}kk + \frac{\alpha(3\varkappa A+2\mathfrak{A})}{2n^4}kk = O - (\mathfrak{A}'+\mathfrak{a}'kk)\frac{(2\varkappa A+\mathfrak{A})}{nn} - \frac{\mathfrak{D}'(2\varkappa D+\mathfrak{D})}{nn}kk = \alpha + \frac{1}{n}$$

vbi ex obseruationibus constat esse $O = 1,0085272$ Proxime autem esse supra inuenimus esse:

$\mathfrak{A}' = 0,01$	$\mathfrak{A} = -0,80$	$A = -1,25$
$\mathfrak{a}' = 0,05$	$\mathfrak{a} = -2,05$	$\alpha = -12,60$
$\mathfrak{D}' = -0,44$	$\mathfrak{D} = -3,60$	$D = 34,25$

atque $\varkappa = 1,0085$; $kk = 0,003$; $nn = 175,71795$ vnde inuenimus $O + 0,000649 = \alpha + \frac{1}{n} = \varkappa + 0,000285$

§. 108. Cum nunc sit $O = 1,0085272$, erit $\alpha + \frac{1}{n} = 1,009176$, et ob $\frac{1}{n} = 0,075438$, habebitur vrues valor:

$$\alpha = 0,933738 \quad \text{et} \quad l\alpha = 9,9702255$$

atque $\varkappa = 1,008991$ et $l\varkappa = 0,0038874$

Hinc

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Hinc iam primo obtinemus:

$\mathfrak{A} = -0,80313 \qquad l-\mathfrak{A} = 9,9047898$

Deinde cum sit satis prope $\mathfrak{C} = -0,67465$, erit

$\frac{(2n+\mathfrak{C})}{nn} = 0,147037$ et $l\frac{2n+\mathfrak{C}}{nn} = 9,1674260$

$\mathfrak{D} = -3,593620 \ldots l-\mathfrak{D} = 0,5555310$

$\mathfrak{E} = -1,087320 \ldots l-\mathfrak{E} = 0,0363580$

atque porro ex valore ipsius A proxime cognito erit

$\mathfrak{B} = +0,006967 \ldots l\,\mathfrak{B} = 7,8430540$

et quia est satis prope $B = 0,0128$, erit $A' = 2\alpha A - 0,000720$ et $B' = 4\alpha B - 0,023926$, vnde fit;

$$\tfrac{1}{2} + 1,62172 - 6A = -4\alpha\alpha A + 0,00144\alpha + 0,000374\alpha A - 0,152\,\alpha B + 0,00091$$

$$+ 0,01247 - 6B = -16\alpha\alpha B + 0,09570\alpha - 0,038932\alpha A + 0,000014$$

§. 109. Nunc primum quaeri debent valores litterarum f, b et b: et cum sit $\varphi = 5°, 9'$ et $2\gamma - \frac{1}{2}\delta = -1,3899$ proxime, reperietur

$f = 1,093757$ et $lf = 0,0389208$

$b = 3,0423 \ldots lb = 0,4832020$

$b = 1,01591 \ldots lb = 0,0068560$

hincque erit

$2,4720\ A = -3,11947 - 0,142\ B$

$12,9369\ B = +0,07684 - 0,0355\ A$

unde concluditur fore:

$A = -1,262463 \ldots l-A = 0,1012186$

$B = +0,009404 \ldots l\,B = 7,9733114$

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Porro vero eſt

$$D' = 0,867676\ D + 0,185628$$
$$E' = 2,867676\ E + 0,185628$$

et $A' = -2,35859$ . . $l-A' = 0,3726530$

§. 110. Ex his valoribus aequationes VII et VIII induent has formas,

$$3 + 7,25185 - 6D - 1,92040 - 0,00244 + 0,01704 = -0,75286\ D - 0,16106 - 0,34680$$
$$3 + 2,19420 - 6E - 1,92040 + 0,00006 + 0,01704 = -8,22357\ E - 0,53232 - 0,34680$$

vnde prodibit

$$D = +33,6600 \ . \ . \ . \ l\,D = 1,5271130$$
$$E = -0,5785 \ . \ . \ . \ l-E = 9,7623410$$

ergo $D' = 29,39153$

$E' = -1,47347$

Ex his nanciſcemur ſequentes formulas pro calculo ſequenti

$$\frac{2\varkappa A + \mathfrak{A}}{nn} = -0,01884 \qquad l-\frac{(2\varkappa A + \mathfrak{A})}{nn} = 8,275051$$
$$\frac{2\varkappa B + \mathfrak{B}}{nn} = +0,000148 \qquad l\,\frac{2\varkappa B + \mathfrak{B}}{nn} = 6,170262$$
$$\frac{2\varkappa D + \mathfrak{D}}{nn} = +0,36611 \qquad l\,\frac{2\varkappa D + \mathfrak{D}}{nn} = 9,563604$$
$$\frac{2\varkappa E + \mathfrak{E}}{nn} = -0,01283 \qquad l-\frac{(2\varkappa E + \mathfrak{E})}{nn} = 8,108292$$

§. III.

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§. 111. Ex his iam porro inuenitur

$\mathfrak{C} = -0,64383 \ldots l-\mathfrak{C} = 9,808771$

atque $C' = -0,13847$

Porro valores litterarum $\mathfrak{F}$ et $\mathfrak{G}$ determinabuntur per has aequationes

$(4\alpha-1)\ \mathfrak{F} = 0,002049 - 0,294032 + 0,067699 + 0,021554 - 0,287335$

$(4\alpha+1)\ \mathfrak{G} = 0,002049 + 0,010306 + 0,020484 + 0,021554 + 0,004939$

ex quibus reperitur

$\mathfrak{F} = -0,17957 \ldots l-\mathfrak{F} = 9,254241$

$\mathfrak{G} = +0,01253 \ldots l\,\mathfrak{G} = 8,097936$

atque

$F' = (4\alpha-1)F - 0,00283 + 0,46219 + 0,63411 = (4\alpha-1)F + 1,09347$

$G' = (4\alpha+1)G - 0,00283 - 0,01620 - 0,01090 = (4\alpha+1)G - 0,02993$

vnde aequationes IX et X prodibunt.

$+0,36238 - 1,01591F + 0,00353 - 0,00680 - 1,00349 = -(4\alpha-1)^2 F - 2,98415 - 0,86349 + 0,00337 - 0,55370$

$-0,02528 - 1,01591G + 0,00353 - 0,00680 + 0,04634 = -(4\alpha+1)^2 G + 0,14473 + 0,03027 + 0,00337 + 0,02776$

seu

$6,43486\ F = -3,75359$

$21,40769\ G = +0,18534$

§. 110. Hinc prodeunt sequentes valores correcti pro F et G,

$F = -0,58360 \ldots l-F = 9,766112$

$G = +0,00866 \ldots l\,G = 7,937400$

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Ex formula autem sexta hinc leui calculo colligitur fore:

$$\gamma - f = 1,58161 \quad \text{et} \quad \gamma = 2,67537$$

Valores autem ex F et G deriuati erunt

$$F' = -0,49919 \quad \text{et} \quad G' = 0,01107$$

$$\frac{2\varkappa F + \mathfrak{F}}{nn} = -0,00772 \qquad l\frac{(2\varkappa F + \mathfrak{F})}{nn} = 7,887828$$

$$\frac{2\varkappa G + \mathfrak{G}}{nn} = 0,00017 \qquad l\frac{2\varkappa G + \mathfrak{G}}{nn} = 6,232305$$

§. 113. Nunc procedamus ad valores litterarum $\mathfrak{a}$ et $\mathfrak{b}$ qui erunt

$$1,867676\,\mathfrak{a} = -3,75000 - 0,68827 - 0,13148 - 0,03764\,\mathfrak{b} + 0,02776$$

$$3,735352\,\mathfrak{b} = -0,57467 - 0,04913 - 0,39807 - 0,01884\,\mathfrak{a} + 0,04611$$

vnde reperitur:

$$\mathfrak{a} = -2,42686 \ldots l\mathfrak{a} = 0,385044$$

$$\mathfrak{b} = -0,24899 \ldots l\mathfrak{b} = 9,396182$$

huncque porro

$$a' = 2\mathfrak{a}a + 0,03768\,b - 4,86420 + 0,16048$$

$$b' = 4\mathfrak{a}b + 0,01884\,a + 0,16887 + 0,64373 + 0,01450\,a - 0,01744$$

seu $a' = 2\mathfrak{a}a + 0,03768\,b - 4,70372$

$$b' = 4\mathfrak{a}b + 0,03334\,a + 0,79516$$

§. 114. Aequationes IV et V hinc induent sequentes formas:

IV.

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IV. $+3,75000-1,01591\,a+50,32200-0,36363=-4aaa$
$+4,89730)$
$-0,07034\,b+8,78034+4,10500-0,36551-0,00125\,a$
$-0,14067\,b \qquad -0,02996$

V. $+0,50246-1,01591\,b-0,87350+0,28239-0,95732$
$-0,02155\,a$
$=-16aab-0,12447\,a-2,96860-0,00142\,b$
$-0,03517\,a+0,17723$
$-0,14354$
$-0,91659$

Hinc fit

$2,47355\,a=-0,21101\,b-46,11580$
$12,93836\,b=-0,13809\,a-2,80553$

et $a=-18,64200 \ldots l-a=1,270493$
$b=-0,01794 \ldots l-b=8,253822$

ex quibus oriuntur:

$a'=-39,52164 \ldots b'=+0,10663$

et $\frac{2na+a}{nn}=-0,22790 \quad l-\frac{(2na+a)}{nn}=9,357744$

valor autem ipsius $\frac{2nb+b}{nn}$ nullius plane erit momenti, vnde eum praetermittimus.

§. 115. Ex prima autem aequatione §. 106. colligitur

$$\tfrac{1}{2}\delta=\gamma-f+0,02285$$

supra autem inuenimus esse $\gamma-f=1,58161$, sicque erit $\frac{1}{2}\delta=1,60446$ atque

$\delta=3,20892 \ldots l\delta=0,506358$

Nunc cum sit proxime: $\mathfrak{H}=-0,123$; $H=-1,033$ ideoque $\frac{2nH+\mathfrak{H}}{nn}=-0,0126$; ob $\mathfrak{L}=-1,453$ et $L=+6,252$; habebimus $-0,$

$$-0,132324\,\mathfrak{J} = -3,75000 + 0,00882 - 0,09088$$
$$+0,05336 - 0,01012$$
$$-0,52839 + 0,05474$$
$$+3,867676\,\mathfrak{K} = -3,75000 + 0,00882 - 0,09088$$
$$+0,01012$$
$$-0,15987 + 0,00917$$

Hinc reperitur

$$\mathfrak{J} = 32,05945 \ldots l\,\mathfrak{J} = 1,505956$$
$$\mathfrak{K} = -1,02714 \ldots l\text{-}\mathfrak{K} = 0,011629$$

§. 116. Hinc vlterius progrediendo habebimus.

$$J' = 2(\alpha-1)J + 0,28571 - 4,94924 + 0,23502 =$$
$$-0,01591 \qquad -0,08015$$
$$2(\alpha-1)J - 4,52457$$
$$K' = 2(\alpha+1)K + 0,28571 + 0,08507 - 0,00317 =$$
$$-0,01591$$
$$2(\alpha+1)K + 0,35170$$

vnde aequationes XII et XIII fiunt

$$+3,75000 - 64,69550 - 1,01591\,J + 51,28180$$
$$- 0,94292 + 0,00321 - 0,36999 =$$
$$- 0,00441)$$
$$-4(\alpha-1)^2 J - 0,59871 - 0,26690 + 4,32164$$
$$+0,02972 - 0,47004$$

$$+3,75000 + 2,07275 - 1,01591\,K - 0,88006$$
$$- 0,94292 + 0,00321 + 0,00636 =$$
$$- 0,00441)$$
$$-4(\alpha+1)^2 K - 1,36026 - 0,26690 - 0,21665$$
$$+0,02972 + 0,00810$$

ex quibus colligitur fore

$$J = -14,09600 \ldots l\text{-}J = 1,149096$$
$$K = -0,41676 \ldots l\text{-}K = 9,619888$$

Hinc

Hinc $J' = -2,65933$ . . $K' = -1,26020$

atque $\frac{2\kappa J + \mathfrak{J}}{nn} = +0,02057$. $l\frac{2\kappa J + \mathfrak{J}}{nn} = 8,313172$

$\frac{2\kappa K + \mathfrak{K}}{nn} = -0,01063$. $l\frac{2\kappa K + \mathfrak{K}}{nn} = 8,026598$

§. 117. Quaeramus iam valorem ipsius $\mathfrak{H}$, ex aequatione

$2\mathfrak{H} = -0,45434 - 0,39771 - 0,01652 + 0,62330$

erit $\mathfrak{H} = -0,12264$ . . . $l-\mathfrak{H} = 9,088632$

hincque reperitur: $H' = 2H + 0,08011$

vnde aequatio XI praebet:

$2,04688 + 0,24748 - 1,01591\,H + 0,27805 - 3,06194$
$+ 0,36275 + 0,00118 - 0,00623 + 0,02224 + 0,03552$
$- 0,62485 - 0,33247 - 0,83753 + 0,49710 =$
$- 4H - 0,16022 - 0,04851 - 0,53944 - 0,37802$
$+ 0,07384$

seu $2,98409\,H = -2,68053$

Ergo $H = -0,89829$ . . . $l-H = 9,953417$

$H' = -1,71647$; $\frac{2\kappa H + \mathfrak{H}}{nn} = -0,01102$

§. 118. Tandem superſunt litterae $\mathfrak{L}$ et $\mathfrak{M}$

$2(2\alpha - 1)\,\mathfrak{L} = -0,45434 - 0,05280 - 0,01652 - 1,31570$
$+ 0,00158 \qquad - 0,60396$
$- 0,00015$

$2(2\alpha + 1)\,\mathfrak{M} = + 0,00158 + 0,00368 + 0,01935$
$- 0,00015$

Hinc $\mathfrak{L} = +1,40715$ . . $l-\mathfrak{L} = 0,148340$

$\mathfrak{M} = +0,00426$ . . $l\mathfrak{M} = 7,629896$

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Deinde vero habebitur:

$L' = 2(2\alpha-1)\,L - 0{,}00213 + 0{,}17162 - 12{,}31170 - 0{,}02596 + 0{,}00013 - 0{,}26555$

$M' = 2(2\alpha+1)\,M - 0{,}00213 - 000254 - 0{,}00742 + 0{,}00013$

seu

$L' = 2(2\alpha-1)\,L - 12{,}43359$

$M' = 2(2\alpha+1)\,M - 0{,}01196$

§. 119. Nunc denique aggrediamur aequationes XIV et XV

XIV. $-2{,}83960 - 1{,}01591\,L - 0{,}88774 + 0{,}00702 + 0{,}36275 + 0{,}28733 - 0{,}06016 + 0{,}03675 + 7{,}60650 = -3{,}01144\,L + 21{,}57666 + 0{,}00255 - 0{,}14680 + 10{,}75272$

XV. $+0{,}00860 - 1{,}01591\,M + 0{,}01317 + 0{,}00702 - 0{,}00623 - 0{,}00494 - 0{,}00176 + 0{,}00336 + 0{,}01360 = -32{,}89415\,M + 0{,}06859 + 0{,}00255 + 0{,}00325$

ex quibus eruitur

$L = +13{,}86720 \cdots \quad lL = 1{,}141988$

$M = +0{,}00131 \cdots \quad lM = 7{,}117165$

hincque $L' = 11{,}3090$ et $M' = -0{,}00445$

$\frac{2nL+\mathfrak{L}}{nn} = 0{,}15125 \cdots \quad l\frac{2nL+\mathfrak{L}}{nn} = 9{,}179684$

$\frac{2nM+\mathfrak{M}}{nn} = 0{,}00004 \quad l\frac{2nM+\mathfrak{M}}{nn} = 5{,}592770$

Ex his valoribus nouae correctiones inueniri possent, sed differentiae prodirent tam exiguae, vt operae pretium non sit eas inuestigare.

§. 120.

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§. 120. His igitur valoribus inuentis, denotante iam a distantiam Lunae mediam a Terra, et eius distantia curtata $= x$, cum sit $x = \frac{(1-kk)a}{1-k\cos r}u$, erit;

	log. coefficient:
$u = 1 - 0{,}0074991 \cos 2\eta$	7,875009
$+ 0{,}0000532 \cos 4\eta$	5,725912
$+ 0{,}191557k \cos(2\eta - r)$	9,282297
$- 0{,}003293k \cos(2\eta + r)$	7,517525
$- 0{,}003321k \cos(4\eta - r)$	7,521296
$+ 0{,}000049k \cos(4\eta + r)$	5,692584
$- 0{,}00511kk \cos 2r$	7,708601
$- 0{,}08022kk \cos(2\eta - 2r)$	8,904280
$- 0{,}00237kk \cos(2\eta + 2r)$	7,375072
$+ 0{,}07892kk \cos(4\eta - 2r)$	8,897172
$+ 0{,}00001kk \cos(4\eta + 2r)$	4,872349

vbi quidem in duobus primis terminis simul eos, qui per kk erant affecti, sumus complexi, posito $k = 0{,}05445$. Etiamsi enim hic valor non omnino esset iustus, tamen inde in his terminis minimis nullus error nasci poterit.

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§. 121. Porro quoque hinc ex §. 116. valorem ipsius $\frac{d\Phi}{dr}$ determinabimus, quatenus a sola excentricitate orbitae lunaris pendet.

	log. coeff.
$\frac{d\Phi}{dr} = 1,009276$	0,004010
$+ 0,0195144 \cos 2\eta$	8,290355
$- 0,0000322 \cos 4\eta$	5,507856
$- 0,001231k \cos r$	7,090258
$- 0,366103k \cos(2\eta - r)$	9,563604
$+ 0,012832k \cos(2\eta + r)$	8,108292
$+ 0,002829k \cos(4\eta - r)$	7,451633
$- 0,000171k \cos(4\eta + r)$	6,232305
$+ 0,01182kk \cos 2r$	8,072618
$- 0,02057kk \cos(2\eta - 2r)$	8,313172
$+ 0,01063kk \cos(2\eta + 2r)$	8,026598
$- 0,09883kk \cos(4\eta - 2r)$	8,994889
$- 0,00004kk \cos(4\eta + 2r)$	5,592770

§. 122.

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§. 122. Cum nunc sit $\frac{d\hat{v}}{dr} = \frac{ds}{dr} = \frac{1+2ee}{n} + \frac{2}{n}k\cos r + \frac{3}{2n}kk\cos 2r$, erit

		log. coeff.
$\frac{d\eta}{dr} =$	$0,933838$	9,970272
$+$	$0,0195144\cos 2\eta$	8,290355
$-$	$0,0000322\cos 4\eta$	5,507856
$-$	$0,152101k\cos r$	9,182132
$-$	$0,366103k\cos(2\eta - r)$	9,563604
$+$	$0,012829k\cos(2\eta + r)$	8,108292
$+$	$0,002829k\cos(4\eta - r)$	7,451633
$-$	$0,000171k\cos(4\eta + r)$	6,232305
$-$	$0,10133kk\cos 2r$	9,005738
$-$	$0,02057kk\cos(2\eta - 2r)$	8,313172
$+$	$0,01063kk\cos(2\eta + 2r)$	8,026598
$-$	$0,09883kk\cos(4\eta - 2r)$	8,994889
$-$	$0,00004kk\cos(4\eta + 2r)$	5,592770

quae formulae ad motum Lunae horarium tam absolutum quam a sole adhiberi possunt, quemadmodum illa distantiam definiens diametro apparenti et parallaxi horizontali inuestigandae inseruit.

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§. 23. Quaeramus nunc valorem integralem pro longitudine Lunae Φ, quatenus a sola excentricitate orbitae lunaris pendet, ac ponamus,

$$\Phi = Or + \mathfrak{A}'\sin 2\eta + a'kk\sin 2\eta + \mathfrak{B}'\sin 4\eta + b'kk\sin 4\eta$$
$$+ \mathfrak{C}'k\sin r + \mathfrak{D}'k\sin(2\eta - r) + \mathfrak{F}'k\sin(4\eta - r)$$
$$+ \mathfrak{E}'k\sin(2\eta + r) + \mathfrak{G}'k\sin(4\eta + r)$$
$$+ \mathfrak{H}'kk\sin 2r + \mathfrak{J}'kk\sin(2\eta - 2r) + \mathfrak{L}'kk\sin(4\eta - 2r)$$
$$+ \mathfrak{K}'kk\sin(2\eta + 2r) + \mathfrak{M}'kk\sin(4\eta + 2r)$$

atque sequentes obtinebimus formulas:

$$+0,0188387 = 2a\mathfrak{A}' - \frac{\mathfrak{A}'(2n\mathfrak{B}+\mathfrak{B})}{nn} - \frac{2\mathfrak{B}'(2n\mathfrak{A}+\mathfrak{A})}{nn}$$

$$-0,0000370 = 4a\mathfrak{B}' - \frac{\mathfrak{A}'(2n\mathfrak{A}+\mathfrak{A})}{nn}$$

$$-0,001231 = \mathfrak{C}' - \frac{\mathfrak{A}'(2n\mathfrak{D}+\mathfrak{D})}{nn} + \frac{\mathfrak{A}'(2n\mathfrak{E}+\mathfrak{E})}{nn} - \frac{(\mathfrak{D}'+\mathfrak{E}')(2n\mathfrak{A}+\mathfrak{A})}{nn}$$

$$-0,366103 = (2a-1)\mathfrak{D}' + \frac{\mathfrak{A}'(2n+\mathfrak{C})}{nn}$$

$$+0,012832 = (2a+1)\mathfrak{E}' - \frac{\mathfrak{A}'(2n+\mathfrak{C})}{nn}$$

$$+0,002829 = (4a-1)\mathfrak{F}' - \frac{4\mathfrak{B}'}{n} - \frac{\mathfrak{A}'(2n\mathfrak{D}+\mathfrak{D})}{nn} - \frac{\mathfrak{D}'(2n\mathfrak{A}+\mathfrak{A})}{nn}$$

$$-0,000171 = (4a+1)\mathfrak{G}' - \frac{4\mathfrak{B}'}{n} - \frac{\mathfrak{A}'(2n\mathfrak{E}+\mathfrak{E})}{nn} - \frac{\mathfrak{E}'(2n\mathfrak{A}+\mathfrak{A})}{nn}$$

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$$+0,22790 = 2\alpha\alpha' - \frac{(\mathfrak{D}'+\mathfrak{E}')(2n+\mathfrak{C})}{nn} - \frac{2\mathfrak{F}'(2\kappa\mathrm{D}+\mathfrak{D})}{nn} - \frac{2\mathfrak{b}'(2\kappa\mathrm{A}+\mathfrak{A})}{nn}$$

$$+0,00163 = 4\alpha\mathfrak{b}' - \frac{\mathfrak{a}'(2\kappa\mathrm{A}+\mathfrak{A})}{nn} - \frac{\mathfrak{E}'(2\kappa\mathrm{D}+\mathfrak{D})}{nn} - \frac{2(\mathfrak{F}'+\mathfrak{G}')(2n+\mathfrak{C})}{nn}$$

$$+0,01182 = 2\mathfrak{H}' - \frac{\mathfrak{E}'(2\kappa\mathrm{D}+\mathfrak{D})}{nn} - \frac{\mathfrak{A}'(2\kappa\mathrm{J}+\mathfrak{J})}{nn} - \frac{(\mathfrak{J}'+\mathfrak{K}')(2\kappa\mathrm{A}+\mathfrak{A})}{nn} - \frac{\mathfrak{D}'(2\kappa\mathrm{E}+\mathfrak{E})}{nn}$$

$$-0,02057 = 2(\alpha-1)\mathfrak{J}' - \frac{3\mathfrak{A}'}{2n} - \frac{\mathfrak{D}'(2n+\mathfrak{C})}{nn} - \frac{\mathfrak{A}'(2\kappa\mathrm{H}+\mathfrak{H})}{nn} - \frac{2\mathfrak{L}'(2\kappa\mathrm{A}+\mathfrak{A})}{nn}$$

$$+0,01063 = 2(\alpha+1)\mathfrak{K}' - \frac{3\mathfrak{A}'}{2n} - \frac{\mathfrak{E}'(2n+\mathfrak{C})}{nn} - \frac{\mathfrak{A}'(2\kappa\mathrm{H}+\mathfrak{H})}{nn} - \frac{2\mathfrak{G}'(2\kappa\mathrm{D}+\mathfrak{D})}{nn}$$

$$-0,09883 = 2(2\alpha-1)\mathfrak{L}' - \frac{3\mathfrak{B}'}{n} + \frac{2\mathfrak{F}'(2n+\mathfrak{C})}{nn} + \frac{\mathfrak{A}'(2\kappa\mathrm{J}+\mathfrak{J})}{nn} - \frac{\mathfrak{J}'(2\kappa\mathrm{A}+\mathfrak{A})}{nn} - \frac{\mathfrak{D}'(2\kappa\mathrm{D}+\mathfrak{D})}{nn}$$

$$-0,00004 = 2(2\alpha+1)\mathfrak{M}' - \frac{3\mathfrak{B}'}{n} - \frac{2\mathfrak{G}'(2n+\mathfrak{C})}{nn}$$

§. 124.

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§. 124. Ex his eliciuntur valores sequentes:

$\mathfrak{A}' = +0{,}0100887$	$l\,\mathfrak{A}' = 8{,}003837$	$a' = 0{,}09140$	
$\mathfrak{B}' = -0{,}0000409$	$l - \mathfrak{B}' = 5{,}611723$	$l\,a' = 8{,}960934$	
$\mathfrak{C}' = +0{,}010146$	$l\,\mathfrak{C}' = 8{,}006295$	$b' = 0{,}00089$	
$\mathfrak{D}' = -0{,}420226$	$l - \mathfrak{D}' = 9{,}623483$	$l\,b' = 6{,}949340$	
$\mathfrak{E}' = +0{,}004992$	$l\,\mathfrak{E}' = 7{,}698261$	$\mathfrak{A}' + a'kk = 0{,}0103597$	
$\mathfrak{F}' = +0{,}005286$	$l\,\mathfrak{F}' = 7{,}723163$	$\mathfrak{B}' + b'kk = -0{,}0000382$	
$\mathfrak{G}' = -0{,}000086$	$l - \mathfrak{G}' = 5{,}935307$	$l(\mathfrak{A}' + a'kk) = 8{,}015347$	
$\mathfrak{H}' = +0{,}00420$	$l\,\mathfrak{H}' = 7{,}623250$	$l - (\mathfrak{B}' + b'kk) = 5{,}582063$	
$\mathfrak{J}' = +0{,}57328$	$l\,\mathfrak{J}' = 9{,}758367$		
$\mathfrak{K}' = +0{,}00318$	$l\,\mathfrak{K}' = 7{,}502427$		
$\mathfrak{L}' = -0{,}15083$	$l - \mathfrak{L}' = 9{,}178488$		
$\mathfrak{M}' = -0{,}00002$	$l - \mathfrak{M}' = 5{,}301030$		

§. 125. Pro longitudine ergo Lunae habemus hactenus hanc formulam

		log. coeff.
$\varphi = \text{Const.}$	$+ 1{,}0085272\ r$	0,003687
	$+ 0{,}0103597 \sin 2\eta$	8,015347
	$- 0{,}0000382 \sin 4\eta$	5,582063
	$+ 0{,}010146k \sin r$	8,006295
	$- 0{,}420226k \sin(2\eta - r)$	9,623483
	$+ 0{,}004992k \sin(2\eta + r)$	7,698261
	$+ 0{,}005286k \sin(4\eta - r)$	7,723163
	$- 0{,}000086k \sin(4\eta + r)$	5,935307
	$+ 0{,}00420kk \sin 2r$	7,623250
	$+ 0{,}57328kk \sin(2\eta - 2r)$	9,758367
	$+ 0{,}00318kk \sin(2\eta + 2r)$	7,402427
	$- 0{,}15083kk \sin(4\eta - 2r)$	9,178488
	$- 0{,}00002kk \sin(4\eta + 2r)$	5,301030

§. 126.

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§. 126. Quodsi iam ponamus $k = 0{,}05445$, et hos coefficientes ad minuta secunda cum partibus decimalibus reducamus, longitudo $\odot$ ita exprimetur vt sit:

			log. coeff.
$\odot =$ Const.	$+$	$1{,}0085272\ r$	
	$+$	$2136'',8 \sin 2\eta$	3,329772
	$-$	$7,8 \sin 4\eta$	0,895488
	$+$	$113,9 \sin r$	2,056718
	$-$	$4719,6 \sin(2\eta - r)$	3,673906
	$+$	$56,1 \sin(2\eta + r)$	1,748684
	$+$	$59,4 \sin(4\eta - r)$	1,773586
	$-$	$1,0 \sin(4\eta + r)$	9,985730
	$+$	$2,5 \sin 2r$	0,409671
	$+$	$350,6 \sin(2\eta - 2r)$	2,544788
	$+$	$1,5 \sin(2\eta + 2r)$	0,188848
	$-$	$92,2 \sin(4\eta - 2r)$	1,964909
	$-$	$0,0 \sin(4\eta + 2r)$	8,087451

Hisque formulis praecipuae inaequalitates, quibus motus Lunae perturbatur, continentur.

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CAPUT VIII.

DE MOTU APOGEI LUNAE.

§. 127.

His inuentis iam arduam illam de motu apogei Lunae quaestionem examinare, atque adeo decidere licebit. Quanquam enim in praecedentibus calculis vbique verum apogei motum, quem obseruationes ostendunt, introduxi, ita vt id ipsum, quod in controversia est, assumsisse videar; tamen quoniam in hunc ipsum finem terrae vim, qua luna vrgetur, indefinitam sum contemplatus, dum rationi distantiarum reciprocae duplicatae terminum indefinitum adiunxi, vnde littera μ in calculum est ingressa, iudicium de eo apogei motu, qui Theoriae Neutonianae esset consentaneus, non erit difficile. Quodsi enim valor litterae μ nihilo aequalis reperiatur, hinc concludendum erit Theoriam Neutoni cum phaenomenis perfecte consentire; sin autem pro littera μ notabilis prodeat valor, Theoria ista insufficiens erit censenda.

§. 128. Motus autem apogei, quoniam huius rei in calculo nusquam mentio est facta, in ea continetur proportione, quam motus lunae medius ad motum anomaliae tenere est positus. Cum enim remotis lunae inaequalitatibus, quae regulae Keplerianae aduersantur, longitudo lunae vera obtineatur, si eius anomalia vera r ad longitudinem apogei addatur: denotet π longitudinem apogei, eritque longitudo vera $\varphi = \pi + r$, vnde

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vnde fit $\varpi = \Phi - r$. Ex quo intelligitur, si $\Phi - r$ quantitatem designet constantem, apogeum in quiete relinqui, sin autem $\Phi - r$ valorem variabilem obtineat, tum apogeum quoque lunae motum esse habiturum.

§. 129. Cum autem terminos illos omnes, qui sinus angulorum implicant, ideoque inaequalitates periodicas continent, quibus apogei motus non afficitur, omittimus, per integrationem deducimur ad huiusmodi formulam $\Phi = \text{Const.} + \Theta r$, vnde propterea habetur longitudo apogei $\varpi = \text{Const.} + (\Theta - 1)\, r$. Hinc consequimur sequentes proportiones:

I. Vt 1 ad $\Theta - 1$, ita motus anomaliae lunae ad motum apogei.

II. Vt Θ ad 1, ita motus lunae medius ad motum anomaliae.

III. Vt Θ ad $\Theta - 1$, ita motus lunae medius ad motum apogei.

§. 130. Si obseruationes consulamus, valor litterae Θ reperitur $= 1{,}0085272$, quem etiam in calculo vbique adhibui; propterea quod propositum erat non tam in istum valorem a priori inquirere, quam ipsam potius Theoriam ita instituere, atque si opus fuerit, emendare, vt motus inde apogei experientiae consentaneus resultaret. Vicissim autem Theoria stabilita, siue Neutoniana siue alia, quae ex determinato pro μ substituto valore oriatur, facile erit valorem ipsius Θ a priori eruere, quem deinceps cum valore vero 1,0085272 conferre licebit. Vel inuento valore ipsius Θ, apogeum

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lunae interuallo menſis apogiſtici progredietur per ſpatium $(O-1)\,360°$, interuallo autem menſis periodici per ſpatium $(1-\frac{1}{O})\,360°$. Secundum obſeruationes autem apogeum promouetur

vno menſe apogiſtico per ſpatium $3°, 4', 11''$
vno menſe periodico per ſpatium $3, 2, 38$

§. 131. Ex calculo autem §. 107. expoſito valor litterae O ex elementis ante aſſumtis ita definitur, vt ſit $O+0,000649 = \varkappa+0,000285$ ſiue $O = \varkappa-0,000364$ Etſi enim haec exigua particula 0,000364 iam ex valore ipſius $\varkappa$ veritati conſentanee aſſumto eſt orta, tamen perſpicuum eſt, tenuem differentiam nullius hic momenti futuram fuiſſe. Verum littera $\varkappa$ per Theoriam ita erat aſſumta, vt eſſet

$$\varkappa = V\left(1+\frac{3+4\mu+\delta}{2nn}\right)$$

vbi quidem valor ipſius nn ex motu medio lunae ad motum ſolis relato habetur, ita vt ſit ſine reſpectu ad motum apogei habito, $nn = 175,71795$. Ergo pro Theoria Neutoniana eſt

$$\varkappa = V\left(1+\frac{3+\delta}{2nn}\right) \text{ et } O = V\left(1+\frac{3+\delta}{2nn}\right)-0,000364.$$

§. 132. Hic igitur patet totam hanc inueſtigationem ad inuentionem litterae δ reduci, cuius valor, vti ex ſuperiori calculo manifeſtum eſt, a pluribus litteris et coefficientibus terminorum, quos ante eruere oportebat, pendet, ita vt neglecta hac littera δ motus apogei nullo modo recte definiri queat. Initio quidem vbi hanc litteram

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teram in calculum induximus, quod factum est §. 44. haec res leuis momenti est visa; cum enim pro CC, quae erat constans per integrationem in calculum ingressa, valorem vero proximum inueniſſemus $1+\frac{3+4\mu}{2nn}$, quoniam facile erat praeuidere, reliquis adhibitis elementis ad motum lunae pertinentibus, hunc valorem aliquantum immutari posse, pro vero valore ipsius $\frac{CC}{1-kk}$ posuimus $1+\frac{3+4\mu+\delta}{2nn}$. Deinde autem valor ipsius δ potissimum pendet a valore litterae γ, qua vsi sumus ad verum valorem constantis $\frac{m}{nn}=1+\frac{2+3\mu+\gamma}{nn}$ obtinendum, cum proxime verus esset inuentus $=1+\frac{2+3\mu}{nn}$.

§. 133. Ab his ergo litteris γ et δ, quae initio nullius fere vsus esse videbantur, determinatio motus apogei potissimum pendet, quae cum ex pluribus atque adeo omnibus inaequalitatibus lunae ab excentricitate orbis determinari debeant, mirum sane non est, quod legitima motus apogei designatio, cum tantis implicata sit difficultatibus, tam dudum fuerit abscondita. Plerique enim, qui motum apogei ex sola Theoria concludere sunt anniſi, ad omnes has inaequalitates non respexerunt, atque calculum perinde adminiſtrauerunt, ac si hic litteras γ et δ neglexiſſemus. Ac si non defuere, qui sibi persuaserunt, motum apogei cum Theoria Neutoniana consentire, ii plerumque per errorem calculi seducti ad veritatem peruenisse sibi sunt visi. Quin etiam Ipse Neu-

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tonus Theoriae ſuae in motu apogei determinando parum tribuiſſe videtur.

§. 134. Hinc ex neglectu harum litterarum γ et δ, ſeu ex alia omiſſione eodem recidente, factum eſt, vt Theoria Neutoni obſeruationibus circa motum apogei lunae inſtitutis plane non ſatisfacere ſit putata; quae opinio etiam ita inualuit, vt perſpicaciſſimus quisque hanc Theoriam inſufficientem pronunciaret. Atque ſagaciſſimus Clairaltius huic opinioni vehementiſſime erat addictus, antequam publice in contrarias partes diſceſſerat. Eadem ſcilicet ratione ob neglectum minutarum illarum particularum erat deceptus, qua et ego fateri cogor, me per complures annos conſtanter eſſe opinatum, ex Theoria Neutoni pro motu apogei Lunae non vltra ſemiſſem prodire, ita vt error vltra ſemiſſem exſurgens committeretur.

§. 135. Fons itaque huius erroris, qui niſi ſumma circumſpectio adhibeatur, vix euitatur, in eo latet, quod in calculo debita illa conſtantium determinatio, pro qua equidem hic litteras γ et δ adhibui, negligatur. Quemadmodum per hanc omiſſionem dimidius tantum apogei motus eliciatur, oſtendiſſe iuuabit. Sit igitur $\delta = o$, atque littera illius O ſecundum Theoriam Neutonianam, qua eſt $\mu = o$, valor erit $O = \sqrt{(1 + \frac{3}{2nn})} - 0{,}000364$; qui euolutus fit: $O = 1{,}0042592 - 0{,}000364$. Quare etiamſi particula $0{,}000364$ vtpote ex profundiori indagine nata praetermittatur, tamen iſte valor pro $O = 1{,}0042592$, ſi cum vero per obſeruationes cognito $O = 1{,}0085272$ comparetur, exacte fere dimidium motum apogei praebet;

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bet; atque adeo haec tam accurata medietas non parum digna videtur.

§. 136. Jam videamus, quam prope valorem litterae δ adhibendo ad veritatem perducamur. Inuenimus autem (115) $\delta = 3{,}20892$, vnde prodit

$$V\left(1+\frac{3+\delta}{2nn}\right) = 1{,}0087947$$

qui valor iam maior est quam verus 1,0085272, sed recordandum est inde subtrahi debere 0,000364, sicque relinquetur $O = 1{,}0084307$, ex quo motus progressiuus apogei pro interuallo mensis apogistici prodibit $=$ 3° 2′ 9″ et pro interuallo mensis periodici $=$ 3° 0′ 37″, qui numeri duobus tantum minutis a vero deficiunt. Ad hunc defectum supplendum litterae μ tribui poterit valor conueniens ex formula $\mu = \frac{1}{2}(nn-1)nn - \frac{3}{4} - \frac{1}{4}\delta$, vnde reperitur $\mu = 0{,}03782$, qui valor tantillus est, vt nisi de motu apogei sit quaestio, semper pro nihilo haberi possit.

§. 137. Verum nullo modo affirmare possumus, valores illos pro γ et δ inuentos ita esse absolutos, vt nulla amplius correctione indigeant. Quin potius, si formulas supra exhibitas attentius perpendamus, tantum abest vt eas pro completis habere possimus, vt potius manifestum sit, omnes reliquas inaequalitates motus luae perinde ac eas quas iam definiuimus, terminos quoque in eas suppeditare. Qui etsi admodum erunt parvi, tamen omnino sufficere poterunt ad exiguum istud supplementum, quo adhuc a vero distamus, conficiendum.

Cum

Cum enim sola fere inaequalitas ab angulo $2\mathfrak{y} - \mathfrak{o}$ pendens motum apogei a dimidio tantopere auxisset, vt valor ipsius O ab 1,0042592 vsque ad 1,0084307 increvisset, nullum fere est dubium, quin leuis defectus huius numeri a vero valore 1,0085272 a reliquis inaequalitatibus proficiscatur.

§. 138. Hinc igitur concludere debemus, Theoriam Neutonianam cum motu apogei obseruato tam exacte conuenire, vt aberratio, si quidem vlla locum habeat, tam sit exigua, vt merito pro nihilo reputari possit: neque etiam calculi ope ob summam paruitatem eam certo definire licebit. Cum itaque hoc pacto Theoria Neutoniana a fortissima obiectione sit vindicata, gloria huius insignis inuenti cum industriae tum candori excellentissimi Clairaki debetur, qui primus egregium hunc Theoriae consensum cum veritate detexit et publice est professus: cui ea re eo maiores debemus gratias, quod sine eius studio summo, quod in hac inuestigatione consumsit, Theoria Neutoniana fortasse vix vnquam ab hac suspicione insufficientiae esset liberata. Atque nunc demum pleno lumine veritas istius Theoriae, cui vni Astronomiae Theoria vniuersa innititur, fulgere est censenda, cum antea non mediocribus tenebris fuisset inuoluta.

CAPUT

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CAPUT IX.

INUESTIGATIO INAEQUALITATUM LUNAE A SOLA EXCENTRICITATE ORBITAE SOLIS PENDENTIUM.

§. 139.

Quoniam in hac inueſtigatione excentricitas orbitae lunaris non in cenſum venit, inaequalitates quas ſcrutamur partim ab anomalia vera ſolis s partim ab angulo 2η pendebunt. Cum igitur ſit

$$\frac{ds}{dr} = \frac{d\theta}{dr} = \frac{1+2ee}{n} + \frac{2}{n}k\cos r - \frac{2}{n}e\cos s - \frac{2}{n}ek\cos(r-s) + \frac{3}{2n}kk\cos 2r + \frac{1}{2n}ee\cos 2s - \frac{2}{n}ek\cos(r+s)$$

hinc differentiale ds ad differentiale dr reducitur. Atque hoc quidem capite, quia ad excentricitatem Lunae non attendimus, erit

$$\frac{ds}{dr} = \frac{d\theta}{dr} = \frac{1+2ee}{n} - \frac{2}{n}e\cos s + \frac{1}{2n}ee\cos 2s$$

§. 140. Incipiamus ergo a formulis $\int R\,dr$ et v, quas omiſſis terminis ab angulo r pendentibus ponamus

$$\int R\,dr = \mathfrak{A}\cos 2\eta + \mathfrak{P}e\cos s + \mathfrak{Q}e\cos(2\eta - s) + \mathfrak{R}e\cos(2\eta + s) + \mathfrak{S}ee\cos 2s + \mathfrak{T}ee\cos(2\eta - 2s) + \mathfrak{V}ee\cos(2\eta + 2s)$$

$$v = A\cos 2\eta + Pe\cos s + Qe\cos(2\eta - s) + Re\cos(2\eta + s) + See\cos 2s + Tee\cos(2\eta - 2s) + Vee\cos(2\eta + 2s)$$

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vbi quidem pro $\mathfrak{A}$ et A valores supra inuentos completos accipi oportet, ita vt in iis termini akk et akk sint comprehensi; erit ergo

$\mathfrak{A} = -0,81033$ $\quad l-\mathfrak{A} = 9,908662$

$A = -1,31773$ $\quad l-A = 0,119826$

Valores autem hinc deriuati erunt:

$\frac{2\varkappa A+\mathfrak{A}}{nn} = -0,019744$, $l\frac{-(2\varkappa A+\mathfrak{A})}{nn} = 8,295442$

$A' = -2,47576$ $\quad l-A' = 0,393708$

$\mathfrak{A}' = +0,01036$ $\quad l\,\mathfrak{A}' = 8,015347$

Terminos autem angulum quadruplum 4η inuoluentes hic ob summam paruitatem omisi, quoniam in combinatione cum angulo s plane fierent imperceptibiles.

§. 141. Hinc iam primo colligitur:

$$\frac{d\Phi}{d\tau} = \varkappa - \frac{(2\varkappa A+\mathfrak{A})}{nn}\cos 2\eta - \frac{(2\varkappa P+\mathfrak{P})}{nn}e\cos s$$
$$- \frac{(2\varkappa Q+\mathfrak{Q})}{nn}e\cos(2\eta-s) - \frac{(2\varkappa R+\mathfrak{R})}{nn}e\cos(2\eta+s)$$
$$- \frac{(2\varkappa S+\mathfrak{S})}{nn}ee\cos 2s$$
$$- \frac{(2\varkappa T+\mathfrak{T})}{nn}ee\cos(2\eta-2s) - \frac{(2\varkappa V+\mathfrak{V})}{nn}ee\cos(2\eta+2s)$$

atque porro

$$\frac{d\eta}{d\tau} = a - \frac{(2\varkappa A+\mathfrak{A})}{nn}\cos 2\eta - \left(\frac{2\varkappa P+\mathfrak{P}}{nn} - \frac{2}{n}\right)e\cos s$$
$$- \frac{(2\varkappa Q+\mathfrak{Q})}{nn}e\cos(2\eta-s) + \frac{(2\varkappa R+\mathfrak{R})}{nn}e\cos(2\eta+s)$$
$$- \left(\frac{(2\varkappa S+\mathfrak{S})}{nn} - \frac{1}{2n}\right)ee\cos 2s$$

Deinde

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Deinde quia est proxime $kk = 9ee$, erit

$$R = \tfrac{3}{2}(1 + \tfrac{27}{2}ee)\sin 2\eta + \left(\frac{3Q}{2nn} - \frac{3R}{2nn}\right)e\sin s$$

$$- \left(\tfrac{9}{4} - \frac{3P}{2nn}\right)e\sin(2\eta - s) - \left(\tfrac{9}{4} - \frac{3P}{2nn}\right)e\sin(2\eta + s)$$

$$+ \left(\frac{3T}{2nn} - \frac{3V}{2nn}\right)ee\sin 2s$$

$$+ \left(\tfrac{9}{8} + \frac{3S}{2nn}\right)ee\sin(2\eta - 2s) + \left(\tfrac{9}{8} + \frac{3S}{2nn}\right)ee\sin(2\eta + 2s)$$

atque omissis terminis, quibus non est opus

$$\frac{ddv}{dr^2} = e\cos s\left\{\begin{array}{l} -\tfrac{3}{2} - 2\varkappa\mathfrak{P} - \mathfrak{C}P - \dfrac{9A}{4nn} + \dfrac{\mathfrak{A}\mathfrak{Q}}{nn} + \dfrac{\mathfrak{A}\mathfrak{R}}{nn} \\ + \dfrac{3AQ}{nn} + \dfrac{3AR}{nn} + \dfrac{3\mathfrak{A}(Q+R)}{nn} + \dfrac{3A(\mathfrak{Q}+\mathfrak{R})}{nn} \end{array}\right.$$

$$+ e\cos(2\eta - s)\left\{\begin{array}{l} -\tfrac{9}{4} - 2\varkappa\mathfrak{Q} - \mathfrak{C}Q - \dfrac{3A}{4nn} + \dfrac{3P}{4nn} \\ + \dfrac{\mathfrak{A}\mathfrak{P}}{nn} + \dfrac{3AP}{nn} + \dfrac{3\mathfrak{A}P}{nn} + \dfrac{3A\mathfrak{P}}{nn} \end{array}\right.$$

$$+ e\cos(2\eta + s)\left\{\begin{array}{l} -\tfrac{9}{4} - 2\varkappa\mathfrak{R} - \mathfrak{C}R - \dfrac{3A}{4nn} + \dfrac{3P}{4nn} \\ + \dfrac{\mathfrak{A}\mathfrak{P}}{nn} + \dfrac{3AP}{nn} + \dfrac{3\mathfrak{A}P}{nn} + \dfrac{3A\mathfrak{P}}{nn} \end{array}\right.$$

$$+ ee\cos 2s\left\{\begin{array}{l} +\tfrac{3}{4} - 2\varkappa\mathfrak{S} - \mathfrak{C}S - \dfrac{3P}{4nn} + \dfrac{\mathfrak{P}\mathfrak{P}}{2nn} \\ + \dfrac{3PP}{2nn} + \dfrac{3P\mathfrak{P}}{nn} \end{array}\right.$$

$$+ ee\cos(2\eta-2s)\left\{\begin{array}{l}+\frac{9}{8}-2\varkappa\mathfrak{T}-6T-\frac{9P}{8nn}+\frac{\mathfrak{A}\mathfrak{S}}{nn}\\ +\frac{3AS}{nn}+\frac{3\mathfrak{A}S}{nn}+\frac{3A\mathfrak{S}}{nn}\end{array}\right.$$

$$+ ee\cos(2\eta+2s)\left\{\begin{array}{l}+\frac{9}{8}-2\varkappa\mathfrak{V}-6V-\frac{9P}{8nn}+\frac{\mathfrak{A}\mathfrak{S}}{nn}\\ +\frac{3AS}{nn}+\frac{3\mathfrak{A}S}{nn}+\frac{3A\mathfrak{S}}{nn}\end{array}\right.$$

§. 142. Quodsi iam forma pro $\int R\,dr$ assumta differentietur, orietur:

$$R = -2\alpha\mathfrak{A}\sin 2\eta$$
$$+ e\sin s\left(-\frac{1}{n}\mathfrak{P}-\frac{(\mathfrak{Q}-\mathfrak{R})(2\varkappa A+\mathfrak{A})}{nn}\right)$$
$$+ e\sin(2\eta-s)\left(-\frac{2\mathfrak{A}}{n}\pm\frac{\mathfrak{A}(2\varkappa P+\mathfrak{P})}{nn}-(2\alpha-\frac{1}{n})\mathfrak{Q}\right)$$
$$+ e\sin(2\eta+s)\left(-\frac{2\mathfrak{A}}{n}+\frac{\mathfrak{A}(2\varkappa P+\mathfrak{P})}{nn}-(2\alpha+\frac{1}{n})\mathfrak{R}\right)$$
$$+ ee\sin 2s\left(\frac{1}{n}\mathfrak{P}-\frac{2}{n}\mathfrak{S}-\frac{(\mathfrak{T}-\mathfrak{V})(2\varkappa A+\mathfrak{A})}{nn}\right)$$
$$+ ee\sin(2\eta-2s)\left(\frac{1}{2n}\mathfrak{A}-\frac{3}{n}\mathfrak{Q}-(2\alpha-\frac{2}{n})\mathfrak{T}\right)$$
$$+ ee\sin(2\eta+2s)\left(\frac{1}{2n}\mathfrak{A}-\frac{1}{n}\mathfrak{R}-(2\alpha+\frac{2}{n})\mathfrak{V}\right)$$

§. 143. Comparatione ergo instituta habebitur

$$-\frac{1}{n}\mathfrak{P}-\frac{(\mathfrak{Q}-\mathfrak{R})(2\varkappa A+\mathfrak{A})}{nn}=\frac{3(Q-R)}{2nn}$$

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$$-\frac{2}{n}\mathfrak{A}+\frac{\mathfrak{A}(2nP+\mathfrak{P})}{nn}-(2\alpha-\frac{1}{n})\mathfrak{Q}=-\frac{3}{4}+\frac{3P}{2nn}$$

$$-\frac{2}{n}\mathfrak{A}+\frac{\mathfrak{A}(2nP+\mathfrak{P})}{nn}-(2\alpha+\frac{1}{n})\mathfrak{R}=-\frac{3}{4}+\frac{3P}{2nn}$$

$$\frac{1}{n}\mathfrak{P}-\frac{2}{n}\mathfrak{S}-\frac{(\mathfrak{T}-\mathfrak{V})(2nA+\mathfrak{A})}{nn}=\frac{3(T-V)}{2nn}$$

$$\frac{1}{2n}\mathfrak{A}-\frac{3}{n}\mathfrak{Q}-(2\alpha-\frac{2}{n})\mathfrak{T}=\frac{3}{8}+\frac{3S}{2nn}$$

$$\frac{1}{2n}\mathfrak{A}-\frac{1}{n}\mathfrak{R}-(2\alpha+\frac{2}{n})\mathfrak{V}=\frac{3}{8}+\frac{3S}{2nn}$$

vnde deinceps valores litterarum germanicarum $\mathfrak{P}, \mathfrak{Q}, \mathfrak{R}, \mathfrak{S}, \mathfrak{T}, \mathfrak{V}$ ſumus inueſtigaturi.

§. 144. Differentietur ſimili modo quantitas v, ac ponatur:

$$\frac{dv}{dr}=-A'\sin 2\eta-P'e\sin s-Q'e\sin(2\eta-s)-S'ee\sin 2s-T'ee\sin(2\eta-2s)$$
$$-R'e\sin(2\eta+s)\qquad -V'ee\sin(2\eta+2s)$$

eritque

$A'=2\alpha\mathfrak{A}$, cuius quidem valor iam ſupra habetur

$$P'=\frac{1}{n}P+\frac{(Q-R)(2nA+\mathfrak{A})}{nn}$$

$$Q'=(2\alpha-\frac{1}{n})Q+\frac{2}{n}A-\frac{A(2nP+\mathfrak{P})}{nn}$$

$$R'=(2\alpha+\frac{1}{n})R+\frac{2}{n}A-\frac{A(2nP+\mathfrak{P})}{nn}$$

$$S'=\frac{2}{n}S-\frac{1}{n}P+\frac{(T-V)(2nA+\mathfrak{A})}{nn}$$

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$$T' = (2\alpha - \frac{2}{n})\,T + \frac{3}{n}\,Q - \frac{1}{2n}\,A$$

$$V' = (2\alpha + \frac{2}{n})\,V + \frac{1}{n}\,R - \frac{1}{2n}\,A$$

vnde denuo differentiando ernitur.

$$\frac{ddv}{dt^2} = e\cos s\,(-\frac{1}{n}P' + \frac{(Q'+R')(2\kappa A+\mathfrak{A})}{nn}$$

$$e\cos(2\eta - s)\,(-(2\alpha - \frac{1}{n})\,Q' - \frac{2}{n}A' + \frac{A'(2\kappa P+\mathfrak{P})}{nn}$$

$$e\cos(2\eta + s)\,(-(2\alpha + \frac{1}{n})\,R' - \frac{2}{n}A' + \frac{A'(2\kappa P+\mathfrak{P})}{nn}$$

$$ee\cos 2s\,(-\frac{2}{n}S' + \frac{1}{n}P' + \frac{(T'+V')(2\kappa A+\mathfrak{A})}{nn}$$

$$ee\cos(2\eta - 2s)\,(-(2\alpha - \frac{2}{n})\,T' + \frac{1}{2n}A' - \frac{3}{n}Q'$$

$$ee\cos(2\eta + 2s)\,(-(2\alpha + \frac{2}{n})\,V' + \frac{1}{2n}A' - \frac{1}{n}R'$$

§. 145. Sequentes ergo aequationes resoluendae occurrent

$$-\tfrac{1}{2} - 2\kappa\mathfrak{P} - 6P - \frac{9A}{4nn} + \frac{(\mathfrak{A}+3A)(Q+R)}{nn} + \frac{(3\mathfrak{A}+3A)(Q+R)}{n^3} =$$

$$-\frac{1}{n}P' + \frac{(Q'+R')(2\kappa A+\mathfrak{A})}{nn}$$

$$-\tfrac{3}{4} - 2\kappa\Omega - 6Q - \frac{3A}{4nn} + \frac{3P}{4nn} + \frac{(\mathfrak{A}+3A)\mathfrak{P}}{nn} + \frac{(3\mathfrak{A}+3A)P}{n^3} =$$

$$-(2\alpha - \frac{1}{n})\,Q' + \frac{2}{n}A' + \frac{A'(2\kappa P+\mathfrak{P})}{nn}$$

$$-\tfrac{3}{4}-2\alpha\mathfrak{R}-6R-\frac{3A}{4nn}+\frac{3P}{4nn}+\frac{(\mathfrak{A}+3A)\mathfrak{P}}{nn}+\frac{(3\mathfrak{A}+3A)P}{nn}=$$
$$-\left(2\alpha+\frac{1}{n}\right)R'-\frac{2}{n}A'+\frac{A'(2\alpha P+\mathfrak{P})}{nn}$$

$$+\tfrac{3}{4}-2\alpha\mathfrak{S}-6S-\frac{3P}{4nn}+\frac{\mathfrak{P}\mathfrak{P}+6P\mathfrak{P}+3PP}{2nn}=$$
$$-\frac{2}{n}S'+\frac{1}{n}P'+\frac{(T'+V')(2\alpha A+\mathfrak{A})}{nn}$$

$$+\tfrac{3}{8}-2\alpha\mathfrak{T}-6T+\frac{9P}{8nn}+\frac{(\mathfrak{A}+3A)\mathfrak{S}}{nn}+\frac{(3\mathfrak{A}+3A)S}{nn}=$$
$$-\left(2\alpha-\frac{2}{n}\right)T'+\frac{1}{2n}A'+\frac{3}{n}Q'$$

$$+\tfrac{3}{8}-2\alpha\mathfrak{V}-6V-\frac{9P}{8nn}+\frac{(\mathfrak{A}+3A)\mathfrak{S}}{nn}+\frac{(3\mathfrak{A}+3A)S}{nn}=$$
$$-\left(2\alpha+\frac{2}{n}\right)V'+\frac{1}{2n}A'-\frac{1}{n}R'$$

Neglectis primo terminis minimis, qui adhuc sunt incogniti, reperitur

$\mathfrak{Q}=+1,3238$	$Q=+2,5714$	$Q'=1,10094$
$\mathfrak{R}=+1,2210$	$R=+2,0571$	$R'=3,79793$
$\mathfrak{P}=-0,0313$	$P=-1,4807$	$P'=-0,12185$

§. 146. Ex his autem accuratius ita definientur

$\mathfrak{Q}=+1,33859$	$Q=0,126649$
$\mathfrak{R}=+1,23468$	$R=0,091545$
$Q=+2,60087$	$Q=0,415119$
$R=+2,00590$	$R=0,302308$
$Q'=4,44801$	$R'=3,67581$

et

et $\mathfrak{P} = -0{,}04010 \ldots l-\mathfrak{P} = 8{,}603133$
$P = -1{,}46488 \ldots l-P = 0{,}165801$
$P' = -0{,}12222$

Deinde reperitur

$\mathfrak{S} = +0{,}03911 \ldots l\mathfrak{S} = 8{,}592230$
$S = +0{,}60882 \ldots lS = 9{,}784486$
$\mathfrak{T} = -0{,}85267 \ldots l-\mathfrak{T} = 9{,}930827$
$\mathfrak{V} = -0{,}62125 \ldots l-\mathfrak{V} = 9{,}793266$
$T = -2{,}60380 \ldots l-T = 0{,}465615$
$V = -1{,}02720 \ldots l-V = 0{,}011672$

Pro sequentibus vero calculis est

$\frac{2nP+\mathfrak{P}}{nn} = -0{,}017051 \ldots l-\frac{(2nP+\mathfrak{P})}{nn} = 8{,}231755$

$\frac{2nQ+\mathfrak{Q}}{nn} = +0{,}037487 \ldots l+\frac{2nQ+\mathfrak{Q}}{nn} = 8{,}573878$

$\frac{2nR+\mathfrak{R}}{nn} = +0{,}030062 \ldots l+\frac{2nR+\mathfrak{R}}{nn} = 8{,}478023$

§. 147. Hinc igitur pro distantia lunae a terra curtata $x = \frac{(1-kk)au}{1-k\cos v}$, pars quantitatis u ab excentricitate orbitae solaris tantum pendens erit

		log. coeff.
$u =$ Praeced.	$- 0{,}008336e\cos s$	7,920985
	$+ 0{,}014801e\cos(2\eta - s)$	8,170303
	$+ 0{,}011415e\cos(2\eta + s)$	8,057492
	$+ 0{,}00364ee\cos 2s$	7,539670
	$- 0{,}01482ee\cos(2\eta - 2s)$	8,170799
	$- 0{,}00584ee\cos(2\eta + 2s)$	7,766856

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Deinde vero erit

		log. coeff.
$\frac{d\Phi}{dv} =$ Praec.	$+0{,}017041\,e\cos s$	8,231755
	$-0{,}037487\,e\cos(2\eta - s)$	8,573878
	$-0{,}030062\,e\cos(2\eta + s)$	8,478023
	$-0{,}00722\,ee\cos 2s$	7,858166
	$+0{,}03470\,ee\cos(2\eta - 2s)$	8,540319
	$+0{,}01533\,ee\cos(2\eta + 2s)$	8,185614

§. 148. Ponatur nunc integrale

$$\Phi = \text{Praec.} + \mathfrak{A}'\sin 2\eta + \mathfrak{P}'e\sin s + \mathfrak{Q}'e\sin(2\eta - s) + \mathfrak{R}'e\sin(2\eta + s) + \mathfrak{S}'ee\sin 2s + \mathfrak{T}'ee\sin(2\eta - 2s) + \mathfrak{V}'ee\sin(2\eta + 2s)$$

erit differentiatione peracta:

$$+0{,}017051 = \frac{1}{n}\mathfrak{P}' - \frac{(\mathfrak{Q}' + \mathfrak{R}')(2\alpha A + \mathfrak{A})}{nn}$$

$$-0{,}037489 = \left(2\alpha - \frac{1}{n}\right)\mathfrak{Q}' + \frac{2}{n}\mathfrak{A}' - \frac{\mathfrak{A}'(2\alpha P + \mathfrak{P})}{nn}$$

$$-0{,}030062 = \left(2\alpha + \frac{1}{n}\right)\mathfrak{R}' + \frac{2}{n}\mathfrak{A}' - \frac{\mathfrak{A}'(2\alpha P + \mathfrak{P})}{nn}$$

$$-0{,}00722 = \frac{2}{n}\mathfrak{S}' - \frac{1}{n}\mathfrak{P}' - \frac{(\mathfrak{T}' + \mathfrak{V}')(2\alpha A + \mathfrak{A})}{nn}$$

$$+0{,}03470 = \left(2\alpha - \frac{2}{n}\right)\mathfrak{T}' - \frac{1}{2n}\mathfrak{A}' + \frac{3}{n}\mathfrak{Q}'$$

$$+0{,}01533 = \left(2\alpha + \frac{2}{n}\right)\mathfrak{V}' - \frac{1}{2n}\mathfrak{A}' + \frac{1}{n}\mathfrak{R}'$$

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fietque his valoribus determinatis

		log. coeff.
$\varphi =$ Praec.	$+ 0{,}236034\, e \sin s$	9,372974
	$- 0{,}021889\, e \sin(2\eta - s)$	8,340237
	$- 0{,}016368\, e \sin(2\eta + s)$	8,214002
	$+ 0{,}06615\, ee \sin 2s$	8,820508
	$+ 0{,}02332\, ee \sin(2\eta - 2s)$	8,367825
	$+ 0{,}00840\, ee \sin(2\eta + 2s)$	7,924429

§. 149. Reducamus has inaequalitates etiam ad minuta secunda, ponendo excentricitatem orbitae solaris $e = 0{,}01680$, atque habebimus

		log. coeff.
$\varphi =$ Praec.	$+ 817'',9 \sin s$	2,912708
	$- 75,8 \sin(2\eta - s)$	1,879971
	$- 56,7 \sin(2\eta + s)$	1,753736
	$+ 3,8 \sin 2s$	0,585551
	$+ 1,4 \sin(2\eta - 2s)$	0,132868
	$+ 0,5 \sin(2\eta + 2s)$	9,689472

Denotat hic s anomaliam veram solis; vnde patet eam Lunae inaequalitatem, quae sinui huius anomaliae est proportionalis, admodum esse notabilem, dum ad $13', 38''$ exsurgit. Tabulae autem Astronomicae, vbi haec inaequalitas aequatio solaris nominatur, eam multo minorem faciunt, cuius rei causam inuestigari adhuc conveniet.

§. 150.

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§. 150. Quodsi enim litteram $\mathfrak{P}'$ accuratius definire velimus, habebimus has formulas resoluendas:

$$-\frac{1}{n}\mathfrak{P}+\frac{\mathfrak{A}(2\varkappa Q+\mathfrak{Q})}{nn}-\frac{\mathfrak{A}(2\varkappa R+\mathfrak{R})}{nn}-\frac{(\mathfrak{Q}-\mathfrak{R})(2\varkappa A+\mathfrak{A})}{nn}=\frac{3(Q-R)}{nn}$$

$$P'=\frac{1}{n}P-\frac{A(2\varkappa Q+\mathfrak{Q})}{nn}+\frac{A(2\varkappa R+\mathfrak{R})}{nn}+\frac{(Q-R)(2\varkappa A+\mathfrak{A})}{nn}$$

$$-\tfrac{3}{2}-2\varkappa\mathfrak{P}-\mathfrak{C}P-\frac{9A}{4nn}+\frac{(\mathfrak{A}+3A)(\mathfrak{Q}+\mathfrak{R})}{nn}+\frac{(3\mathfrak{A}+3A)(Q+R)}{nn}=$$

$$-\frac{1}{n}P'+\frac{A'(2\varkappa Q+\mathfrak{Q})}{nn}+\frac{A'(2\varkappa R+\mathfrak{R})}{nn}+\frac{(Q'+R')(2\varkappa A+\mathfrak{A})}{nn}$$

vnde elicimus:

$$\mathfrak{P}=-0,1183\ ;\quad P=-1,1356\ ;\quad \frac{P}{nn}=-0,0064$$

atque $\frac{2\varkappa P+\mathfrak{P}}{nn}=-0,01376$, fierique iam oportet

$$+0,01376=\frac{1}{n}\mathfrak{P}'-\frac{\mathfrak{A}'(2\varkappa Q+\mathfrak{Q})}{nn}-\frac{\mathfrak{A}'(2\varkappa R+\mathfrak{R})}{nn}-\frac{(\mathfrak{Q}'+\mathfrak{R}')(2\varkappa A+\mathfrak{A})}{nn}$$

vnde oritur $\mathfrak{P}'=+0,201385$. Quare accuratius habemus

n = Praec.	—	0,006400 e cos s		7,806180
$\frac{d\Phi}{dr}$ = Praec.	+	0,013760 e cos s		8,138618
Φ = Praec.	+	0,201385 e sin s		9,304026
		seu		
Φ = Praec.	+	701″, 1 sin s		2,845780

Ergo aequatio sinui Anguli s proportionalis tantum est 11′, 41″.

CAPUT X.

INUESTIGATIO INAEQUALITATVM LUNAE AB VTRIUSQUE ORBITAE EXCENTRICITATE SIMUL PENDENTIUM.

§. 151.

Quoniam praeuidemus inaequalitates huius generis, quae altiores litterarum k et e poteſtates ſimul complectuntur, minimas eſſe futuras, alios terminos non ſcrutabimur, niſi qui producto ſimplici ek ſint affecti. Habebimus ergo

$$\frac{ds}{dv} = \frac{dδ}{dv} = \frac{1}{n} + \frac{2}{n}k\cos r - \frac{2}{n}e\cos s - \frac{2}{n}ek\cos(r-s) - \frac{2}{n}ek\cos(r+s)$$

Cum igitur ad hanc inueſtigationem opus non ſit illis terminis ex praecedentibus, qui vel per k^2 vel per e^2 erant affecti, quia litterae alphabethi deficere incipiunt, litteris S, T et ſequentibus denuo utemur; quare cauendum, ne iſtae litterae cum ante adhibitis confundantur.

§. 152. Aſſumtis ergo ex terminis iam ante definitis, iis qui in eos, quos iam inueſtigamus, vim exſerunt, ponamus

$$\int R\,dv = \mathfrak{A}\cos 2\eta + \mathfrak{C}k\cos r + \mathfrak{D}k\cos(2\eta-r) + \mathfrak{P}e\cos s + \mathfrak{Q}e\cos(2\eta-s) + \mathfrak{E}k\cos(2\eta+r) + \mathfrak{R}e\cos(2\eta+s) + \mathfrak{S}ek\cos(r-s) + \mathfrak{V}ek\cos(2\eta-r+s) + \mathfrak{Y}ek\cos(2\eta-r-s) + \mathfrak{T}ek\cos(r+s) + \mathfrak{X}ek\cos(2\eta+r+s) + \mathfrak{Z}ek\cos(2\eta+r+s)$$

$v =$

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$$v = A\cos 2\eta \ldots + Dk\cos(2\eta - r) + P e\cos s + Q e\cos(2\eta - s)$$
$$+ Ek\cos(2\eta + r) + Re\cos(2\eta + s)$$
$$+ Sek\cos(r - s) + Vek\cos(2\eta - r + s) + Yek\cos(2\eta - r - s)$$
$$+ Tek\cos(r + s) + Xek\cos(2\eta + r - s) + Zek\cos(2\eta + r + s)$$

eritque ex praecedentibus

$\mathfrak{A} = -0,81033$	;	$l - \mathfrak{A} = 9,908662$
$\mathfrak{C} = -0,64383$	;	$l - \mathfrak{C} = 9,808771$
$\mathfrak{D} = -3,59362$	;	$l - \mathfrak{D} = 0,555531$
$\mathfrak{E} = -1,08732$	;	$l - \mathfrak{E} = 0,036358$
$\mathfrak{P} = -0,11830$	;	$l - \mathfrak{P} = 9,072985$
$\mathfrak{Q} = +1,33859$	;	$l\ \mathfrak{Q} = 0,126649$
$\mathfrak{R} = +1,23468$	;	$l\ \mathfrak{R} = 0,091545$
$A = -1,31773$	;	$l - A = 0,119826$
. . .	.	. . .
$D = +33,6600$	;	$l\ D = 1,527113$
$E = -0,5785$	;	$l - E = 9,762341$
$P = -1,1356$	;	$l - P = 0,055225$
$Q = +2,60087$	;	$l - Q = 0,415119$
$R = +2,00590$	;	$l\ R = 0,302308$

§. 153. Reliqui vero valores hinc deriuati, quibus opus habemus, sunt:

$A' = -2,47576$	;	$l - A' = 0,393708$
$C' = -0,13847$	;	$l - C' = 9,141356$
$D' = 29,39153$	;	$l\ D' = 1,468222$
$E' = -1,47347$	;	$l - E' = 0,168341$
$P' = -0,0260$	;	$l - P' = 8,414973$
$Q' = 4,40924$	;	$l\ Q' = 0,644363$
$R' = 3,79793$	;	$l\ R' = 0,579548$

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$$\mathfrak{A}' = +0{,}01036 \;;\; l\,\mathfrak{A}' = 8{,}015347$$
$$\mathfrak{C}' = +0{,}01015 \;;\; l\,\mathfrak{C}' = 8{,}006295$$
$$\mathfrak{D}' = -0{,}42023 \;;\; l{-}\mathfrak{D}' = 9{,}623483$$
$$\mathfrak{E}' = +0{,}00499 \;;\; l{-}\mathfrak{E}' = 7{,}698261$$
$$\mathfrak{P}' = +0{,}20138 \;;\; l\,\mathfrak{P}' = 9{,}304016$$
$$\mathfrak{Q}' = -0{,}02189 \;;\; l{-}\mathfrak{Q}' = 8{,}340237$$
$$\mathfrak{R}' = -0{,}01637 \;;\; l{-}\mathfrak{R}' = 8{,}214002$$

Sit breuitatis gratia

$$a' = \frac{2\kappa A + \mathfrak{A}}{nn} = -0{,}019744 \;;\; l{-}\frac{(2\kappa A + \mathfrak{A})}{nn} = 8{,}295442$$
$$d' = \frac{2\kappa D + \mathfrak{D}}{nn} = +0{,}36611 \;;\; l\,\frac{2\kappa D + \mathfrak{D}}{nn} = 9{,}563604$$
$$e' = \frac{2\kappa E + \mathfrak{E}}{nn} = -0{,}01283 \;;\; l{-}\frac{(2\kappa E + \mathfrak{E})}{nn} = 8{,}108292$$
$$p' = \frac{2\kappa P + \mathfrak{P}}{nn} = -0{,}01376 \;;\; l{-}\frac{(2\kappa P + \mathfrak{P})}{nn} = 8{,}138618$$
$$q' = \frac{2\kappa Q + \mathfrak{Q}}{nn} = +0{,}03749 \;;\; l\,\frac{2\kappa Q + \mathfrak{Q}}{nn} = 8{,}573878$$
$$r' = \frac{2\kappa R + \mathfrak{R}}{nn} = +0{,}03006 \;;\; l\,\frac{2\kappa R + \mathfrak{R}}{nn} = 8{,}478023$$

§. 154. Si simili modo vlterius ponatur:

$$s' = \frac{2\kappa S + \mathfrak{S}}{nn}\;;\quad t' = \frac{2\kappa T + \mathfrak{T}}{nn}\;;\quad v' = \frac{2\kappa V + \mathfrak{V}}{nn}\;;$$
$$x' = \frac{2\kappa X + \mathfrak{X}}{nn}\;;\quad y' = \frac{2\kappa Y + \mathfrak{Y}}{nn}\;;\quad z' = \frac{2\kappa Z + \mathfrak{Z}}{nn}$$

habe-

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habebimus:

$$\frac{dQ}{dv} = \text{Praec.} - a' \operatorname{cof} 2\eta - \frac{\mathfrak{C}}{nn} k \operatorname{cof} r - d' k \operatorname{cof}(2\eta - r) - p' e \operatorname{cof} s - q' e \operatorname{cof}(2\eta - s)$$
$$- e' k \operatorname{cof}(2\eta + r) \quad - r' e \operatorname{cof}(2\eta + s)$$
$$- s' ek \operatorname{cof}(r - s) - v' ek \operatorname{cof}(2\eta - r + s) - y' ek \operatorname{cof}(2\eta - r - s)$$
$$- t' ek \operatorname{cof}(r + s) - x' ek \operatorname{cof}(2\eta + r - s) - z' ek \operatorname{cof}(2\eta + r + s)$$

atque posito $\frac{2}{n} + \frac{\mathfrak{C}}{nn} = c' = 0,147197$; $l\, c' = 9,167900$

$$\frac{d\eta}{dv} = a - a' \operatorname{cof} 2\eta - c' k \operatorname{cof} r - d' k \operatorname{cof}(2\eta - r) + (\tfrac{2}{n} - p') e \operatorname{cof} s - q' e \operatorname{cof}(2\eta - s)$$
$$- e' k \operatorname{cof}(2\eta + r) \quad - r' e \operatorname{cof} 2\eta + s)$$
$$+ (\tfrac{2}{n} - s') ek \operatorname{cof}(r - s) - v' ek \operatorname{cof}(2\eta - r + s) - y' ek \operatorname{cof}(2\eta - r - s)$$
$$+ (\tfrac{2}{n} - s') ek \operatorname{cof}(r + s) - x' ek \operatorname{cof}(2\eta + r - s) - z' ek \operatorname{cof}(2\eta + r + s)$$

vbi cum sit $\frac{2}{n} = 0,150876$, erit $\frac{2}{n} - p' = 0,16464$

§. 155. Nunc termini coefficiente ek affecti, qui in formis $R =$ et $\frac{ddv}{dr^2}$ insunt, colligantur: eritque

$$R = ek \operatorname{sin}(r-s)\left(+\frac{3}{2nn}V - \frac{3}{2nn}X - \frac{3Q}{nn} + \frac{3R}{nn} - \frac{9D}{4nn} + \frac{9E}{4nn}\right)$$
$$ek \operatorname{sin}(r+s)\left(+\frac{3}{2nn}Y - \frac{3}{2nn}Z - \frac{3R}{nn} + \frac{3Q}{nn} + \frac{9E}{4nn} - \frac{9D}{4nn}\right)$$
$$ek \operatorname{sin}(2\eta - r + s)\left(-\frac{g}{2} + \frac{3}{2nn}S + \frac{3P}{nn}\right)$$
$$ek \operatorname{sin}(2\eta + r - s)\left(-\frac{g}{2} + \frac{3}{2nn}S + \frac{3P}{nn}\right)$$

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$$ek \sin(2\eta - r - s)\left(-\tfrac{1}{2} + \frac{3}{2nn}T + \frac{3P}{nn}\right)$$

$$ek \sin(2\eta + r + s)\left(-\tfrac{1}{2} + \frac{3}{2nn}T + \frac{3P}{nn}\right)$$

et $\frac{ddv}{dr^2} =$

$$ek\cos(r-s)\left\{\begin{array}{l} -3-2n\mathfrak{S}-6S+\frac{1}{2}bP+\frac{3V}{4nn}+\frac{3X}{4nn}+\frac{3Q}{2nn}+\frac{3R}{2nn} \\ -\frac{9D}{8nn}-\frac{9E}{8nn}+\frac{3\mathfrak{V}}{nn}+\frac{3\mathfrak{Y}}{nn}+\frac{3\mathfrak{P}}{nn}+\frac{3\mathfrak{Q}}{nn}+\frac{3\mathfrak{R}}{nn} \\ +\frac{3AV}{nn}+\frac{3AX}{nn}+\frac{3DQ}{nn}+\frac{3ER}{nn}-\frac{3AQ}{2nn}-\frac{3AR}{2nn} \\ +\frac{3\mathfrak{A}V}{nn}+\frac{3A\mathfrak{V}}{nn}+\frac{3\mathfrak{A}X}{nn}+\frac{3A\mathfrak{Y}}{nn}+\frac{3\mathfrak{C}P}{nn} \\ +\frac{3\mathfrak{D}Q}{nn}+\frac{3D\mathfrak{Q}}{nn}+\frac{3\mathfrak{C}R}{nn}+\frac{3E\mathfrak{R}}{nn} \end{array}\right.$$

$$ek\cos(r+s)\left\{\begin{array}{l} -3-2n\mathfrak{T}-6T+\frac{1}{2}bP+\frac{3Y}{4nn}-\frac{3Z}{4nn}+\frac{3Q}{2nn}+\frac{3\mathfrak{N}}{2nn} \\ -\frac{9D}{8nn}-\frac{9E}{8nn}+\frac{3\mathfrak{Y}}{nn}+\frac{3\mathfrak{Z}}{nn}+\frac{3\mathfrak{P}}{nn}+\frac{3\mathfrak{R}}{nn}+\frac{3\mathfrak{Q}}{nn} \\ +\frac{3AY}{nn}+\frac{3AZ}{nn}+\frac{3DR}{nn}+\frac{3EQ}{nn}-\frac{3AQ}{2nn}-\frac{3AR}{2nn} \\ +\frac{3\mathfrak{A}Y}{nn}+\frac{3A\mathfrak{Y}}{nn}+\frac{3\mathfrak{A}Z}{nn}+\frac{3A\mathfrak{Z}}{nn}+\frac{3\mathfrak{C}P}{nn} \\ +\frac{3\mathfrak{C}Q}{nn}+\frac{3E\mathfrak{Q}}{nn}+\frac{3\mathfrak{D}R}{nn}+\frac{3D\mathfrak{R}}{nn} \end{array}\right.$$

ek

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$$ek\cos(2\eta-r+s)\begin{cases}-\frac{9}{2}-2\kappa\mathfrak{V}-6V+\frac{15}{2}bR+\frac{3S}{4nn}-\frac{3D}{4mn}+\frac{3P}{2mm}\\+\frac{\mathfrak{A}\mathfrak{S}}{nn}+\frac{\mathfrak{C}\mathfrak{R}}{nn}+\frac{\mathfrak{D}\mathfrak{P}}{mm}+\frac{3AS}{nn}+\frac{3DP}{nn}-\frac{3AP}{2mm}\\+\frac{3\mathfrak{A}S}{nn}+\frac{3A\mathfrak{S}}{nn}+\frac{3\mathfrak{C}R}{mn}+\frac{3\mathfrak{D}P}{nn}+\frac{3D\mathfrak{P}}{mm}\end{cases}$$

$$ek\cos(2\eta+r-s)\begin{cases}-\frac{9}{2}-2\kappa\mathfrak{X}-6X+\frac{15}{2}bQ+\frac{3S}{4nn}-\frac{3E}{4mn}+\frac{3P}{2mn}\\+\frac{\mathfrak{A}\mathfrak{S}}{nn}+\frac{\mathfrak{C}\mathfrak{Q}}{nn}+\frac{\mathfrak{E}\mathfrak{P}}{mn}+\frac{3AS}{nn}+\frac{3EP}{nn}-\frac{3AP}{2mm}\\+\frac{3\mathfrak{A}S}{nn}+\frac{3A\mathfrak{S}}{nn}+\frac{3\mathfrak{C}Q}{nn}+\frac{3\mathfrak{E}P}{mn}+\frac{3E\mathfrak{P}}{nn}\end{cases}$$

$$ek\cos(2\eta-r-s)\begin{cases}-\frac{9}{2}-2\kappa\mathfrak{Y}-6Y+\frac{15}{2}bQ+\frac{3T}{4nn}-\frac{3D}{4mn}+\frac{3P}{2mn}\\+\frac{\mathfrak{A}\mathfrak{C}}{nn}+\frac{\mathfrak{C}\mathfrak{Q}}{nn}+\frac{\mathfrak{D}\mathfrak{P}}{nn}+\frac{3AT}{nn}+\frac{3DP}{mn}-\frac{3AP}{2mm}\\+\frac{3\mathfrak{A}T}{nn}+\frac{3A\mathfrak{T}}{mn}+\frac{3\mathfrak{C}Q}{nn}+\frac{3\mathfrak{D}P}{nn}+\frac{3D\mathfrak{P}}{nn}\end{cases}$$

$$ek\cos(2\eta+r+s)\begin{cases}-\frac{9}{2}-2\kappa\mathfrak{Z}-6Z+\frac{15}{2}bR+\frac{3T}{4nn}-\frac{3E}{4mn}+\frac{3P}{2mm}\\+\frac{\mathfrak{A}\mathfrak{T}}{mn}+\frac{\mathfrak{C}\mathfrak{R}}{nn}+\frac{\mathfrak{E}\mathfrak{P}}{mn}+\frac{3AT}{nn}+\frac{3EP}{nn}-\frac{3AP}{2mm}\\+\frac{3\mathfrak{A}T}{mn}+\frac{3A\mathfrak{T}}{nn}+\frac{3\mathfrak{C}R}{mn}+\frac{3\mathfrak{E}P}{nn}+\frac{3E\mathfrak{P}}{nn}\end{cases}$$

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§. 156. Quaeramus ergo quoque ex formula assumta $\int R\,dr$ differentiale, quod erit

$$R =$$

$$ek\sin(r-s)(+\mathfrak{A}v'-\mathfrak{A}x'-\mathfrak{D}q'+\mathfrak{E}r'+\tfrac{1}{n}\mathfrak{P}+\mathfrak{Q}d'-\mathfrak{R}c'-(1-\tfrac{1}{n})\mathfrak{S}-\mathfrak{V}a'+\mathfrak{X}a')$$

$$ek\sin(r+s)(+\mathfrak{A}y'-\mathfrak{A}z'-\mathfrak{D}r'+\mathfrak{E}q'-\tfrac{1}{n}\mathfrak{P}-\mathfrak{Q}c'+\mathfrak{R}d'-(1+\tfrac{1}{n})\mathfrak{T}-\mathfrak{Y}a'+\mathfrak{Z}a')$$

$$ek\sin(2\eta-r+s)(-\mathfrak{A}(\tfrac{2}{n}-s')-\mathfrak{D}(\tfrac{2}{n}-p')+\mathfrak{R}c'-\tfrac{1}{n}\mathfrak{R}-(2\alpha-1+\tfrac{1}{n})\mathfrak{V})$$

$$ek\sin(2\eta+r-s)(-\mathfrak{A}(\tfrac{2}{n}-s')-\mathfrak{E}(\tfrac{2}{n}-p')+\mathfrak{Q}c'+\tfrac{1}{n}\mathfrak{Q}-(2\alpha+1-\tfrac{1}{n})\mathfrak{X})$$

$$ek\sin(2\eta\cdot r-s)(-\mathfrak{A}(\tfrac{2}{n}-t')-\mathfrak{D}(\tfrac{2}{n}-p')+\mathfrak{Q}c'+\tfrac{1}{n}\mathfrak{Q}-(2\alpha-1-\tfrac{1}{n})\mathfrak{Y})$$

$$ek\sin(2\eta+r+s)(-\mathfrak{A}(\tfrac{2}{n}-t')-\mathfrak{E}(\tfrac{2}{n}-p')+\mathfrak{R}c'-\tfrac{1}{n}\mathfrak{R}-(2\alpha+1+\tfrac{1}{n})\mathfrak{Z})$$

§. 157. Ponatur ex differentiatione formae v;

$$S' = (1-\tfrac{1}{n})S-A(v'-x')+Dq'-Er'-\tfrac{1}{n}P-Qd'+Rc'+(V-X)a'$$

$$T' = (1+\tfrac{1}{n})T-A(y'-z')+Dr'-Eq'+\tfrac{1}{n}P+Qc'-Rd'+(Y-Z)a'$$

$$V' = (2\alpha-1+\tfrac{1}{n})V+A(\tfrac{2}{n}-s')+D(\tfrac{2}{n}-p')-Rc'+\tfrac{1}{n}R$$

$$X' = (2\alpha+1-\tfrac{1}{n})X+A(\tfrac{2}{n}-s')+E(\tfrac{2}{n}-p')-Qc'-\tfrac{1}{n}Q$$

$$Y' = (2\alpha-1-\tfrac{1}{n})Y+A(\tfrac{2}{n}-t')+D(\tfrac{2}{n}-p')-Qc'-\tfrac{1}{n}Q$$

$$Z' = (2\alpha+1+\tfrac{1}{n})Z+A(\tfrac{2}{n}-t')+E(\tfrac{2}{n}-p')-Rc'+\tfrac{1}{n}R$$

vt

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vt habeatur

$$\frac{dv}{dr} = -A'\sin 2\eta - C'k\sin r - D'k\sin(2\eta - r) - E'k\sin(2\eta + r)$$
$$- P'e\sin s - Q'e\sin(2\eta - s) - R'e\sin(2\eta + s)$$
$$- S'ek\sin(r - s) - V'ek\sin(2\eta - r + s) - Y'ek\sin(2\eta - r - s)$$
$$- T'ek\sin(r + s) - X'ek\sin(2\eta + r - s) - Z'ek\sin(2\eta + r + s)$$

§. 158. Haec iam forma denuo differentiata dabit

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+ek\cos(r-s)(+A'(v'+x')+D'q'+E'p'-\tfrac{1}{n}P'+Q'd'+R'c'-(1-\tfrac{1}{n})S'+(V'+X')a'$$

$$+ek\cos(r+s)(+A'(y'+z')+D'r'+E'q'-\tfrac{1}{n}P'+Q'e'+R'd'-(1+\tfrac{1}{n})T'+(Y+Z')a'$$

$$+ek\sin(2\eta-r+s)(-A'(\tfrac{2}{n}-s')-D'(\tfrac{2}{n}-p')+R'c'-\tfrac{1}{n}R'-(2a-1+\tfrac{1}{n})V')$$

$$+ek\sin(2\eta+r-s)(-A'(\tfrac{2}{n}-s')-E'(\tfrac{2}{n}-p')+Q'c'+\tfrac{1}{n}Q'-(2a+1-\tfrac{1}{n})X')$$

$$+ek\sin(2\eta-r-s)(-A'(\tfrac{2}{n}-s')-D'(\tfrac{2}{n}-p')+Q'c'+\tfrac{1}{n}Q'-(2a-1-\tfrac{1}{n})Y')$$

$$+ek\sin(2\eta+r+s)(-A'(\tfrac{2}{n}-s')-E'(\tfrac{2}{n}-p')+R'c'-\tfrac{1}{n}R'-(2a+1+\tfrac{1}{n})Z')$$

§. 159. Priores autem expreſſiones, ſi litterarum cognitarum valores ſubſtituantur, ſequenti modo prodibunt.

$$R = ek\sin(r-s)(-0,44856 + 0,00854(V-X))$$
$$ek\sin(r+s)(-0,42826 + 0,00854(Y-Z))$$
$$ek\sin(2\eta-r+s)(-4,51938 + 9,00854\ S)$$
$$ek\sin(2\eta+r-s)(-4,51938 + 0,00854\ S)$$
$$ek\sin(2\eta-r-s)(-4,51939 + 0,00854\ T)$$
$$ek\sin(2\eta+r+s)(-4,51938 + 0,00854\ T)$$

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§. 160. Altera vero forma pro $\frac{ddv}{dr^2}$ fit

$$\frac{ddv}{dr^2} =$$

$$ekcs(r-s)(-2,83505-2k\mathfrak{S}-6S-0,02711(\mathfrak{V}+\mathfrak{X})-0,03207(V+X))$$
$$ekcs(r+s)(-3,21669-2k\mathfrak{T}-6T-2,02711(\mathfrak{Y}+\mathfrak{Z})-0,03207(Y+Z))$$
$$ek\,cs(2\eta-r+s)(-2,26927-2k\mathfrak{V}-6V-0,02711\,\mathfrak{S}-0,03207\,S)$$
$$ek\,cs(2\eta+r-s)(-0,53441-2k\mathfrak{X}-6X-0,02711\,\mathfrak{S}-0,03207\,S)$$
$$ek\,cs(2\eta-r-s)(-1,36456-2k\mathfrak{Y}-6Y-0,02711\,\mathfrak{T}-0,03207\,T)$$
$$ek\,cs(2\eta+r+s)(-1,43912-2k\mathfrak{Z}-6Z-0,02711\,\mathfrak{T}-0,03207\,T)$$

§. 161. Deinde ſimili modo alterae formulae per differentiationem erutae, ſubſtitutis valoribus cognitis ita ſe habebunt.

$$R =$$

$$ek\,\sin(r-s)\,(+0,59883+\mathfrak{A}(v'-x')-(1-\tfrac{1}{n})\mathfrak{S})+0,01974(\mathfrak{V}-\mathfrak{X})$$

$$ek\,\sin(r+s)\,(+0,54556+\mathfrak{A}(y'-z')-(1+\tfrac{1}{n})\mathfrak{T})+0,01974(\mathfrak{Y}-\mathfrak{Z})$$

$$ek\,\sin(2\eta-r+s)\,(+0,80251+\mathfrak{A}s'-(2\alpha-1+\tfrac{1}{n})\,\mathfrak{V})$$

$$ek\,\sin(2\eta+r-s)\,(+0,59936+\mathfrak{A}s'-(2\alpha+1-\tfrac{1}{n})\,\mathfrak{X})$$

$$ek\,\sin(2\eta-r-s)\,(+1,01199+\mathfrak{A}s'-(2\alpha-1-\tfrac{1}{n})\,\mathfrak{Y})$$

$$ek\,\sin(2\eta+r+s)\,(+0,38988+\mathfrak{A}s'-(\alpha+1+\tfrac{1}{n})\,\mathfrak{Z})$$

Porro

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Porro reperiemus sequentes valores

$$S' = (1-\tfrac{1}{n})\,S - A(v'-x') + 0,38693 - 0,01974\,(V-X)$$

$$T' = (1+\tfrac{1}{n})\,T - A(y'-z') + 0,18018 - 0,01974\,(Y-Z)$$

$$V' = (2a-1+\tfrac{1}{n})\,V - A s' + 5,19902$$

$$X' = (2a+1-\tfrac{1}{n})\,X - A s' - 0,87311$$

$$Y' = (2a-1-\tfrac{1}{n})\,Y - A s' + 4,76391$$

$$Z' = (2a+1+\tfrac{1}{n})\,Z - A s' - 0,43800$$

ac denique $\frac{ddv}{dr^2} =$ Praec.

$$+ ekcs(r-s)(-(1-\tfrac{1}{n})S' + A'(v'+x') + 2,62592 - 0,01974\,(V'+X'))$$

$$+ ekcs(r+s)(-(1+\tfrac{1}{n})T' + A'(y'+z') + 2,16417 - 0,01974\,(Y'+Z'))$$

$$+ ek\cos(2\eta-r+s)(-(2a-1+\tfrac{1}{n})\,V' + A's' - 4,19296)$$

$$+ ek\cos(2\eta+r-s)(-(2a+1-\tfrac{1}{n})\,X' + A's' + 1,59777)$$

$$+ ek\cos(2\eta-r-s)(-(2a-1-\tfrac{1}{n})\,Y' + A's' - 3,48384)$$

$$+ ek\cos(2\eta+r+s)(-(2a+1+\tfrac{1}{n})\,Z' + A's' + 0,88865)$$

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§. 162. Hinc ergo pro determinandis coefficientibus sequentes obtinemus aequationes

$$(1-\tfrac{1}{n})\mathfrak{S} = 1{,}04739 - 0{,}00854(V-X) + \mathfrak{A}(v'-x') + 0{,}01974(\mathfrak{V}-\mathfrak{X})$$

$$(1+\tfrac{1}{n})\mathfrak{T} = 0{,}97382 - 0{,}00858(Y-Z) + \mathfrak{A}(y'-z') + 0{,}01974(\mathfrak{Y}-\mathfrak{Z})$$

$$(2\alpha-1+\tfrac{1}{n})\,\mathfrak{V} = 5{,}32189 - 0{,}00854\,S + \mathfrak{A}\,s'$$

$$(2\alpha+1-\tfrac{1}{n})\,\mathfrak{X} = 5{,}11874 - 0{,}00854\,S + \mathfrak{A}\,s'$$

$$(2\alpha-1-\tfrac{1}{n})\,\mathfrak{Y} = 5{,}53137 - 0{,}00854\,T + \mathfrak{A}\,s'$$

$$(2\alpha+1+\tfrac{1}{n})\,\mathfrak{Z} = 4{,}90926 - 0{,}00854\,T + \mathfrak{A}\,s'$$

Deinde

$$+5{,}46097 = (1-\tfrac{1}{n})S' - 2n\mathfrak{S} - 6S - 0{,}02711(\mathfrak{V}+\mathfrak{X}) - 0{,}03207(V+X) - A'(v'+x') + 0{,}01974\,(V'+X')$$

$$+5{,}38086 = (1+\tfrac{1}{n})T' - 2n\mathfrak{T} - 6T - 0{,}02711(\mathfrak{Y}+\mathfrak{Z}) - 0{,}03207(Y+Z) - A'(y'+z') + 0{,}01974\,(Y'+Z')$$

$$-1{,}92371 = (2\alpha-1+\tfrac{1}{n})V' - 2n\mathfrak{V} - 6V - 0{,}02711\mathfrak{S} - 0{,}03207\,S - A's'$$

$$+2{,}13218 = (2\alpha+1-\tfrac{1}{n})X' - 2n\mathfrak{X} - 6X - 0{,}02711\,\mathfrak{S} - 0{,}03207\,S - A's'$$

$$-2{,}11928 = (2\alpha-1-\tfrac{1}{n})Y' - 2n\mathfrak{Y} - 6Y - 0{,}02711\,\mathfrak{T} - 0{,}03207\,T - A's'$$

$$+2{,}32777 = (2\alpha+1+\tfrac{1}{n})Z' - 2n\mathfrak{Z} - 6Z - 0{,}02711\,\mathfrak{T} - 0{,}03207\,T - A's'$$

§. 163.

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§. 163, Pro vlteriori calculo eſt

$1-\frac{1}{n}=0,924562 \ldots\ldots l(1-\frac{1}{n})=9,965935$

$1+\frac{1}{n}=1,075438 \ldots\ldots l(1+\frac{1}{n})=0,031570$

$2\alpha-1+\frac{1}{n}=0,942914 \ldots l(2\alpha-1+\frac{1}{n})=9,965935$

$2\alpha+1-\frac{1}{n}=2,792038 \ldots l(2\alpha+1-\frac{1}{n})=0,445915$

$2\alpha-1-\frac{1}{n}=0,792038 \ldots l(2\alpha-1-\frac{1}{n})=9,898747$

$2\alpha+1+\frac{1}{n}=2,942914 \ldots l(2\alpha+1+\frac{1}{n})=0,468775$

Hinc in aequationibus poſterioribus valores litterarum S′, T′, V′ ſubſtituantur, et ob $\mathfrak{C}=1,01591$, erit

$$0,16110\,S = -5,10323-2\alpha\mathfrak{S}-A'(v'+x')-0,02711(\mathfrak{V}+\mathfrak{X})+0,01974(V'+X')+1,21835(v'-x')-0,03207(V+X)-0,01825(V-X)$$

$$0,14059\,T = +5,18709+2\alpha\mathfrak{T}+A'(y'+z')+0,02711(\mathfrak{Y}+\mathfrak{Z})-0,01974(Y'+Z')-1,41710(y'-z')+0,03207(Y+Z)+0,02124(Y-Z)$$

$$0,12683\,V = +6,82591-2\alpha\mathfrak{V}+3,718\,s'-0,02711\,\mathfrak{S}-0,03207\,S$$

$$6,77934\,X = +4,56988+2\alpha\mathfrak{X}-6,1548\,s'+0,02711\,\mathfrak{S}+0,03207\,S$$

$$0,38858\,Y = +5,89253-2\alpha\mathfrak{Y}+3,5194\,s'-0,02711\,\mathfrak{T}-0,03207\,T$$

$$7,64474\,Z = +3,61676+2\alpha\mathfrak{Z}-6,3536\,s'+0,02711\,\mathfrak{T}+0,03207\,T$$

§. 164.

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§. 164. Commodiſſime hi coefficientes inueniri videntur, ſi primo $\mathfrak{V}$, $\mathfrak{X}$, $\mathfrak{Y}$, $\mathfrak{Z}$ et V, X, Y, Z proxime quaerantur, quod fiet terminos minimos negligendo:

$\mathfrak{V} = +5,7560 \ldots l\mathfrak{V} = 0,760125$
$\mathfrak{X} = +1,8334 \ldots l\mathfrak{X} = 0,263245$
$\mathfrak{Y} = +6,9837 \ldots l\mathfrak{Y} = 0,844088$
$\mathfrak{Z} = +1,6643 \ldots l\mathfrak{Z} = 0,221240$
$V = -37,7650 \ldots l{-}V = 1,577086$
$X = +1,2198 \ldots lX = 0,086293$
$Y = -21,1040 \ldots l{-}Y = 1,324360$
$Z = +0,9125 \ldots lZ = 9,960209$
$V' = -29,7170 \ldots v' = -0,398$
$X' = +2,5326 \ldots x' = +0,024$
$Y' = -11,9510 \ldots y' = -0,201$
$Z' = +2,2472 \ldots z' = +0,020$

§. 165. Hic autem valores pro V' et Y' tam fiunt magni, vt viciſſim poſt inuentas litteras S, T nimium valores modo erutos afficiant, vnde neceſſe erit reſolutionem harum aequationum ordinario modo inſtruere. Reperitur ergo

$\mathfrak{V} = 5,7560 - 0,0193\,S - 0,0050\,\mathfrak{S}$
$\mathfrak{X} = 1,8333 - 0,0064\,S - 0,0016\,\mathfrak{S}$
$\mathfrak{Y} = 6,9837 - 0,0225\,T - 0,0058\,\mathfrak{T}$
$\mathfrak{Z} = 1,6643 - 0,0061\,T - 0,0016\,\mathfrak{T}$
$2\varkappa\mathfrak{V} = 11,6155 - 0,0390\,S - 0,0100\,\mathfrak{S}$
$2\varkappa\mathfrak{X} = 3,6996 - 0,0129\,S - 0,0033\,\mathfrak{S}$
$2\varkappa\mathfrak{Y} = 14,0930 - 0,0455\,T - 0,0118\,\mathfrak{T}$
$2\varkappa\mathfrak{Z} = 3,3586 - 0,0122\,T - 0,0032\,\mathfrak{T}$

qui

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qui valores ſubſtituti dant:

$$V = -37,7647 + 0,3912\,S + 0,0031\,\mathfrak{S}$$
$$X = +1,2198 - 0,0076\,S - 0,0016\,\mathfrak{S}$$
$$Y = -21,1038 + 0,1385\,T + 0,0121\,\mathfrak{T}$$
$$Z = +0,9125 - 0,0069\,T - 0,0016\,\mathfrak{T}$$

$$2\varkappa V = -76,2085 + 0,7893\,S + 0,0064\,\mathfrak{S}$$
$$2\varkappa X = +2,4616 - 0,0153\,S - 0,0033\,\mathfrak{S}$$
$$2\varkappa Y = -42,5870 + 0,2794\,T + 0,0244\,\mathfrak{T}$$
$$2\varkappa Z = +1,8413 - 0,0140\,T - 0,0033\,\mathfrak{T}$$

§. 166. Hinc porro valores deriuati erunt

$$V' = -29,7167 + 0,3767\,S + 0,0094\,\mathfrak{S}$$
$$X' = +2,5326 - 0,0061\,S + 0,0029\,\mathfrak{S}$$
$$Y' = -11,9511 + 0,1248\,T + 0,0161\,\mathfrak{T}$$
$$Z' = +2,2472 - 0,0054\,T + 0,0028\,\mathfrak{T}$$

$$v' = -0,4009 + 0,0044\,S + 0,0001\,\mathfrak{S}$$
$$x' = +0,0244$$
$$y' = -0,2026 + 0,0015\,T + 0,0001\,\mathfrak{T}$$
$$z' = +0,0200$$

ac porro

$$\mathfrak{X} - \mathfrak{x} = 3,9227 - 0,0129\,S - 0,0034\,\mathfrak{S}$$
$$\mathfrak{Y} - \mathfrak{Z} = 5,3194 - 0,0164\,T - 0,0042\,\mathfrak{T}$$
$$V - X = -38,9845 + 0,3988\,S + 0,0047\,\mathfrak{S}$$
$$Y - Z = -22,0163 + 0,1454\,T + 0,0137\,\mathfrak{T}$$

$$\mathfrak{X} + \mathfrak{x} = 7,5893 - 0,0257\,S - 0,0066\,\mathfrak{S}$$
$$\mathfrak{Y} + \mathfrak{Z} = 8,6480 - 0,0286\,T - 0,0074\,\mathfrak{T}$$
$$V + X = -36,5449 + 0,3836\,S + 0,0015\,\mathfrak{S}$$
$$Y + Z = -20,1913 + 0,1316\,T + 0,0105\,\mathfrak{T}$$

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$$V' + X' = -27,1841 + 0,3706\,S + 0,0123\,\mathfrak{S}$$
$$Y' + Z' = -\ 9,7039 + 0,1194\,T + 0,0189\,\mathfrak{T}$$
$$v' - x' = -\ 0,4253 + 0,0044\,S + 0,0001\,\mathfrak{S}$$
$$y' - z' = -\ 0,2226 + 0,0015\,T + 0,0001\,\mathfrak{T}$$
$$v' + x' = -\ 0,3765 + 0,0044\,S + 0,0001\,\mathfrak{S}$$
$$y' + z' = -\ 0,1826 + 0,0015\,T + 0,0001\,\mathfrak{T}$$

§. 167. His valoribus ſubſtitutis reperitur

$$(1 - \tfrac{1}{n})\,\mathfrak{S} = +\ 1,8024 - 0,0070\,S$$
$$(1 + \tfrac{1}{n})\,\mathfrak{T} = +\ 1,4453 - 0,0027\,T$$

vnde concluditur

$$\mathfrak{S} = 1,9495 - 0,0075\,S \qquad 2n\,\mathfrak{S} = 3,9340 - 0,0160\,S$$
$$\mathfrak{T} = 1,3440 - 0,0025\,T \qquad 2n\,\mathfrak{T} = 2,7121 - 0,0053\,T$$
$$0,1401\,S = -9,2444 \qquad 0,1479\,T = +7,9767$$

§. 168. Nunc igitur habebimus

$$S = -66,6980 \ldots l - S = 1,824113$$
$$T = +53,9330 \ldots l\,T = 1,731855$$
$$\mathfrak{S} = +\ 2,4497 \ldots l\,\mathfrak{S} = 0,389113$$
$$\mathfrak{T} = +\ 1,2092 \ldots l\,\mathfrak{T} = 0,082498$$
$$s' = -0,75204 \ldots l - s' = 9,876241$$
$$t' = +0,62626 \ldots l\,t' = 9,796755$$
$$v' = -\ 0,6942 \ldots l - v' = 9,841484$$
$$x' = +\ 0,0244 \ldots l\,x' = 8,387390$$
$$y' = -\ 0,1216 \ldots l - y' = 9,084933$$
$$z' = +\ 0,0200 \ldots l\,z' = 9,301030$$

$\mathfrak{V} =$

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$\mathfrak{B} = +7{,}0311 \ldots l\mathfrak{B} = 0{,}847029$
$\mathfrak{X} = +2{,}2561 \ldots l\mathfrak{X} = 0{,}353358$
$\mathfrak{Y} = +5{,}7659 \ldots l\mathfrak{Y} = 0{,}760867$
$\mathfrak{Z} = +1{,}3339 \ldots l\mathfrak{Z} = 0{,}125123$
$V = -63{,}8498 \ldots l{-}V = 1{,}805169$
$X = +1{,}7451 \ldots lX = 0{,}241820$
$Y = -13{,}6222 \ldots l{-}Y = 1{,}134241$
$Z = +0{,}5356 \ldots lZ = 9{,}728840$

§. 169. His iam valoribus inuentis pro distantia lunae $x = \frac{(1-kk)\,au}{1-k\cos r}$ erit valoris ipsius u portio ab his terminis pendens;

		Log. coeff.
$u =$ Praec.	$-\ 0{,}3796\ ek\cos(r-s)$	9,579297
	$+\ 0{,}3069\ ek\cos(r+s)$	9,487093
	$-\ 0{,}3634\ ek\cos(2\eta-r+s)$	9,560344
	$+\ 0{,}0099\ ek\cos(2\eta+r-s)$	7,997004
	$-\ 0{,}0775\ ek\cos(2\eta-r-s)$	8,889425
	$+\ 0{,}0030\ ek\cos(2\eta+r+s)$	7,484024

et pro longitudine lunae

$\frac{d\Phi}{dr} =$ Praec. $+\ 0{,}7520\ ek\cos(r-s)$
$-\ 0{,}6263\ ek\cos(r+s)$
$+\ 0{,}6942\ ek\cos(2\eta-r+s)$
$-\ 0{,}0244\ ek\cos(2\eta+r-s)$
$+\ 0{,}1216\ ek\cos(2\eta-r-s)$
$-\ 0{,}0200\ ek\cos(2\eta+r+s)$

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cuius integrale si ponatur:

$$\varphi = \text{Praec.} + \mathfrak{S}' ek \sin(r-s) + \mathfrak{V}' ek \sin(2\eta - r + s) + \mathfrak{Y}' ek \sin(2\eta - r - s) + \mathfrak{T}' ek \sin(r+s) + \mathfrak{X}' ek \sin(2\eta + r - s) + \mathfrak{Z} ek \sin(2\eta + r + s)$$

erit

$$+0{,}7520 = (1 - \tfrac{1}{n})\mathfrak{S}' - \mathfrak{A}'(v' + x') - \mathfrak{D}' q' - \mathfrak{E}' r' + \tfrac{1}{n}\mathfrak{P}' - \mathfrak{Q}' d' - \mathfrak{R}' e' - (\mathfrak{V}' + \mathfrak{X}')a'$$

$$-0{,}6263 = (1 + \tfrac{1}{n})\mathfrak{T}' - \mathfrak{A}'(y' + z') - \mathfrak{D}' r' - \mathfrak{E}' q' + \tfrac{1}{n}\mathfrak{P}' - \mathfrak{Q}' e' - \mathfrak{R}' d' - (\mathfrak{Y}' + \mathfrak{Z}')a'$$

$$+0{,}6942 = (2a - 1 + \tfrac{1}{n})\mathfrak{V}' + \mathfrak{A}'(\tfrac{2}{n} - s') + \mathfrak{D}'(\tfrac{2}{n} - p') - \mathfrak{R}'(c' - \tfrac{1}{n})$$

$$-0{,}0244 = (2a + 1 - \tfrac{1}{n})\mathfrak{X}' + \mathfrak{A}'(\tfrac{2}{n} - s') + \mathfrak{E}'(\tfrac{2}{n} - p') - \mathfrak{Q}'(c' + \tfrac{1}{n})$$

$$+0{,}1216 = (2a - 1 - \tfrac{1}{n})\mathfrak{Y} + \mathfrak{A}'(\tfrac{2}{n} - s') + \mathfrak{D}'(\tfrac{2}{n} - p') - \mathfrak{Q}'(c' + \tfrac{1}{n})$$

$$-0{,}0200 = (2a + 1 + \tfrac{1}{n})\mathfrak{Z} + \mathfrak{A}'(\tfrac{2}{n} - s') + \mathfrak{E}'(\tfrac{2}{n} - p') - \mathfrak{R}'(c' - \tfrac{1}{n})$$

§. 170. Hinc autem reperitur

$$\mathfrak{S}' = +0{,}7467 \ldots l\,\mathfrak{S}' = 9{,}873165$$
$$\mathfrak{T}' = -0{,}6185 \ldots l - \mathfrak{T}' = 9{,}791317$$
$$\mathfrak{V}' = +0{,}8143 \ldots l\,\mathfrak{V}' = 9{,}910800$$
$$\mathfrak{X}' = -0{,}0142 \ldots l - \mathfrak{X}' = 8{,}150690$$
$$\mathfrak{Y}' = +0{,}2396 \ldots l\,\mathfrak{Y}' = 9{,}379550$$
$$\mathfrak{Z}' = -0{,}0061 \ldots l - \mathfrak{Z}' = 7{,}788910$$

§. 171.

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§. 171. Quatenus ergo longitudo Lunae ab excentricitate orbitae solis pendet, erit

		log. coeff.
$\varphi =$ Praec.	$+ 0{,}201385\, e \sin s$	9,304026
	$- 0{,}021889\, e \sin(2\eta - s)$	8,340237
	$- 0{,}016368\, e \sin(2\eta + s)$	8,214002
	$+ 0{,}06615\, ee \sin 2s$	8,820508
	$+ 0{,}02332\, ee \sin(2\eta - 2s)$	8,367825
	$+ 0{,}00840\, ee \sin(2\eta + 2s)$	7,924429
	$+ 0{,}7476\, e\,k \sin(r - s)$	9,873165
	$- 0{,}6185\, e\,k \sin(r + s)$	9,791317
	$+ 0{,}8143\, e\,k \sin(2\eta - r + s)$	9,910800
	$- 0{,}0142\, e\,k \sin(2\eta + r - s)$	8,150690
	$+ 0{,}2396\, e\,k \sin(2\eta - r - s)$	9,379550
	$- 0{,}0061\, e\,k \sin(2\eta + r + s)$	7,788910

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§. 172. Hae autem ſingulae inaequalitates ad numerum minutorum ſecundorum reductae dabunt:

		log. coeff.
$\Phi =$ Praec.	$+701'',1 \sin s$	2,845789
	$-75,8 \sin(2\eta - s)$	1,879971
	$-56,7 \sin(2\eta + s)$	1,753736
	$+3,8 \sin 2s$	0,585551
	$+1,4 \sin(2\eta - 2s)$	0,132862
	$+0,5 \sin(2\eta + 2s)$	9,689472
	$+140,9 \sin(r - s)$	2,148800
	$-116,7 \sin(r + s)$	2,067000
	$+153,7 \sin(2\eta - r + s)$	2,186530
	$-2,7 \sin(2\eta + r - s)$	0,426300
	$+45,2 \sin(2\eta - r - s)$	1,655200
	$+1,2 \sin(2\eta + r + s)$	0,064600

Hic ſcilicet et inaequalitates, quas in capite praecedente inuenimus, et iſtas in hoc capite erutas ſimul ſum complexus, vt coniunctim conſpectui exponerentur.

CAPUT

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CAPUT XI.

INUESTIGATIO INAEQUALITATUM LUNAE A PARALLAXI SOLIS PENDENTIUM.

§. 173.

Jam in formulis noſtris primariis ad eos quoque terminos progrediamur, qui littera v ſunt affecti, et quoniam eſt $1 : v$ vt diſtantia Solis media ad diſtantiam Lunae mediam a Terra, erit $1 : v$ vt parallaxis Lunae media ad parallaxin ſolis: ex quo inaequalitates Lunae, quae hinc oriuntur, a parallaxi ſolis pendere dicuntur. Quoniam vero valor ipſius v eſt valde paruus, quippe $\frac{1}{280}$ propemodum, alios terminos non contemplabimur, niſi qui per v ac per vk et ve ſunt multiplicati, propterea quod magis compoſiti fiant minimi.

§. 174. Ex terminis ergo iam inuentis hic retineamus eos, qui ſunt alicuius momenti, et cum iis nouos determinandos coniungamus; ſit ergo:

$$\int R\,dr = \mathfrak{A}\cos 2\eta + \mathfrak{C}k\cos r + \mathfrak{D}k\cos(2\eta - r) + \mathfrak{P}e\cos s + \mathfrak{Q}e\cos(2\eta - s)$$
$$+ \mathfrak{E}k\cos(2\eta + r) \qquad + \mathfrak{R}e\cos(2\eta + s)$$
$$+ \mathfrak{F}v\cos v + \mathfrak{H}vk\cos(\eta - r) + \mathfrak{K}ve\cos(\eta - s)$$
$$+ \mathfrak{G}v\cos 3\eta + \mathfrak{J}vk\cos(\eta + r) + \mathfrak{L}ve\cos(\eta + s)$$

$$v = A\cos 2\eta \ldots + Dk\cos(2\eta - r) + Pe\cos s + Qe\cos(2\eta - s)$$
$$+ Ek\cos(2\eta + r) \qquad + Re\cos(2\eta + s)$$
$$+ Fv\cos\eta + Hvk\cos(\eta - r) + Kve\cos(\eta - s)$$
$$+ Gv\cos 3\eta + Jvk\cos(\eta + r) + Lve\cos(\eta + s)$$

Non

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Non difficulter enim praeuidere licet, terminos, qui angulos r et s cum angulo 3η habeant coniunctos, fore tam exiguos, vt sine errore praetermitti queant.

§. 175. Quodsi iam retentis litterarum §. 153. inductarum valoribus, praeterea ponamus:

$$f' = \frac{2\varkappa F + \mathfrak{F}}{nn} \; ; \; g' = \frac{2\varkappa G + \mathfrak{G}}{nn} \; ; \; b' = \frac{2\varkappa H + \mathfrak{H}}{nn}$$

$$i' = \frac{2\varkappa J + \mathfrak{J}}{nn} \; ; \; k' = \frac{2\varkappa K + \mathfrak{K}}{nn} \; ; \; l' = \frac{2\varkappa L + \mathfrak{L}}{nn}$$

habebimus:

$$\frac{d\Phi}{dr} = \text{Praec.} - a'\cos 2\eta - \frac{\mathfrak{C}}{nn}k\cos r - d'k\cos(2\eta - r) - p'e\cos s - q'e\cos(2\eta - s)$$
$$- e'k\cos(2\eta + r) \quad - r'e\cos(2\eta + s)$$
$$- f'v\cos\eta - b'vk\cos(\eta - r) - k've\cos(\eta - s)$$
$$- g'v\cos 3\eta - i'vk\cos(\eta + r) - l've\cos(\eta + s)$$

atque

$$\frac{d\eta}{dr} = a - a'\cos 2\eta - c'k\cos r - d'k\cos(2\eta - r) + (\tfrac{2}{n} - p')e\cos s - q'e\cos(2\eta - s)$$
$$- e'k\cos(2\eta + r) \quad - r'e\cos(2\eta + s)$$
$$- f'v\cos\eta - b'vk\cos(\eta - r) - k've\cos(\eta - s)$$
$$- g'v\cos\eta - i'vk\cos(\eta + s) - l've\cos(\eta + s)$$

§. 176. Jam vero pro his terminis ab v pendentibus sequentes colligemus aequationes.

$$\mathfrak{R} = v\sin\eta\left(\tfrac{3}{8} + \frac{3F - 3G}{2nn}\right)$$
$$v\sin 3\eta\left(\tfrac{1}{8} + \frac{3F}{2nn}\right)$$

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$$+ k \sin(\eta - r)\left(\tfrac{15}{16} + \frac{3J}{2nn} + \frac{3F}{nn} - \frac{3G}{nn}\right)$$

$$+ k \sin(\eta + r)\left(\tfrac{15}{16} + \frac{3H}{2nn} - \frac{3G}{nn} + \frac{3F}{nn}\right)$$

$$+ e \sin(\eta - s)\left(-\tfrac{3}{4} + \frac{3L}{2nn} - \frac{9F}{4nn} + \frac{9G}{4nn}\right)$$

$$+ e \sin(\eta + s)\left(-\tfrac{3}{4} + \frac{3K}{2nn} + \frac{9G}{4nn} - \frac{9F}{4nn}\right)$$

$$\frac{ddv}{dt^2} =$$

$$+ \cos\eta \left\{\tfrac{9}{8} - 5F + \frac{3F}{4nn} + \frac{3G}{4nn} - 2\alpha\mathfrak{F} + \frac{\mathfrak{AF}}{nn} + \frac{\mathfrak{AG}}{nn}\right.$$

$$+ \cos 3\eta \left\{\tfrac{45}{16} - 6G + \frac{3F}{4nn} - 2\alpha\mathfrak{G} + \frac{\mathfrak{AF}}{nn}\right.$$

$$+ k \cos(\eta - r)\left\{\begin{array}{l}\tfrac{15}{16} - 6H + \tfrac{15}{16}F + \frac{3J}{4nn} + \frac{3F}{2nn} + \frac{3G}{2nn} - 2\alpha\mathfrak{H} \\ + \frac{\mathfrak{AJ}}{nn} + \frac{\mathfrak{CF}}{nn} + \frac{\mathfrak{DF}}{nn} + \frac{\mathfrak{EG}}{nn}\end{array}\right.$$

$$+ k \cos(\eta + r)\left\{\begin{array}{l}\tfrac{15}{16} - 6J + \tfrac{15}{16}F + \frac{3H}{4nn} + \frac{3G}{2nn} + \frac{3F}{2nn} - 2\alpha\mathfrak{J} \\ + \frac{\mathfrak{AH}}{nn} + \frac{\mathfrak{CF}}{nn} + \frac{\mathfrak{DG}}{nn} + \frac{\mathfrak{EF}}{nn}\end{array}\right.$$

$$\nu e \cos(\eta - s) \begin{cases} -\frac{9}{4} - 6K + \frac{3L}{4nn} - \frac{3F}{4nn} - \frac{9F}{8nn} - \frac{9G}{8nn} - 2\varkappa\mathfrak{K} \\ + \frac{3\mathfrak{L}}{nn} + \frac{3\mathfrak{P}}{nn} + \frac{3\mathfrak{Q}}{nn} + \frac{3\mathfrak{R}}{nn} \end{cases}$$

$$\nu e \cos(\eta + s) \begin{cases} -\frac{9}{4} - 6L + \frac{3K}{4nn} - \frac{3F}{4nn} - \frac{9F}{8nn} - \frac{9G}{8nn} - 2\varkappa\mathfrak{L} \\ + \frac{3\mathfrak{K}}{nn} + \frac{3\mathfrak{P}}{nn} + \frac{3\mathfrak{R}}{nn} + \frac{3\mathfrak{Q}}{nn} \end{cases}$$

sequentes partes seorsim exponamus:

$$\nu \cos\eta \begin{cases} +\frac{3\mathfrak{A}F}{nn} + \frac{3\mathfrak{A}G}{nn} + \frac{3A\mathfrak{F}}{nn} + \frac{3A\mathfrak{G}}{nn} + \frac{3AF}{nn} + \frac{3AG}{nn} \\ + \frac{9A}{8nn} + \frac{15A}{8nn} \end{cases}$$

$$\nu \cos 3\eta \begin{cases} +\frac{3\mathfrak{A}F}{nn} + \frac{3\mathfrak{A}\mathfrak{F}}{nn} + \frac{3AF}{nn} + \frac{9A}{8nn} \end{cases}$$

$$\gamma k \cos(\eta - r) \begin{cases} \frac{3\mathfrak{A}J}{nn} + \frac{3\mathfrak{C}F}{nn} + \frac{3\mathfrak{D}F}{nn} + \frac{3\mathfrak{C}G}{nn} + \frac{3A\mathfrak{J}}{nn} + \frac{3D\mathfrak{F}}{nn} \\ + \frac{3E\mathfrak{G}}{nn} + \frac{3AJ}{nn} + \frac{3DF}{nn} + \frac{3EG}{nn} - \frac{3AF}{nn} - \frac{3AG}{nn} \\ + \frac{9D}{8nn} + \frac{15E}{8nn} \end{cases}$$

$$\nu k \cos(\eta + r) \begin{cases} \frac{3\mathfrak{A}H}{nn} + \frac{3\mathfrak{C}F}{nn} + \frac{3\mathfrak{D}\mathfrak{G}}{nn} + \frac{3\mathfrak{C}F}{nn} + \frac{3A\mathfrak{H}}{nn} \\ + \frac{3D\mathfrak{G}}{nn} + \frac{3E\mathfrak{F}}{nn} + \frac{3AH}{nn} + \frac{3DG}{nn} + \frac{3EF}{nn} \\ - \frac{3AF}{2nn} - \frac{3AG}{2nn} + \frac{9E}{8nn} + \frac{15D}{8nn} \end{cases}$$

$\nu e \cos$

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$$\nu e \cos(\eta - s)\begin{cases} \frac{3\mathfrak{A}L}{nn} + \frac{3\mathfrak{P}F}{nn} + \frac{3\mathfrak{Q}F}{nn} + \frac{3\mathfrak{R}G}{nn} + \frac{3A\mathfrak{L}}{nn} \\ + \frac{3P\mathfrak{F}}{nn} + \frac{3Q\mathfrak{F}}{nn} + \frac{3R\mathfrak{G}}{nn} + \frac{3AL}{nn} + \frac{3PF}{nn} + \frac{3QF}{nn} \\ + \frac{3RG}{nn} + \frac{9P}{8nn} + \frac{9Q}{8nn} + \frac{15R}{8nn} \end{cases}$$

$$\nu e \cos(\eta + s)\begin{cases} \frac{3\mathfrak{A}K}{nn} + \frac{3\mathfrak{P}F}{nn} + \frac{3\mathfrak{Q}G}{nn} + \frac{3\mathfrak{R}F}{nn} + \frac{3A\mathfrak{K}}{nn} + \frac{3PF}{nn} \\ + \frac{3Q\mathfrak{G}}{nn} + \frac{3R\mathfrak{F}}{nn} + \frac{3AK}{nn} + \frac{3PF}{nn} + \frac{3QF}{nn} + \frac{3QG}{nn} \\ + \frac{9P}{8nn} + \frac{9R}{8nn} + \frac{15Q}{8nn} \end{cases}$$

§. 177. Verum differentiando nanciſcemur

R

$$\nu \sin \eta \quad \{\mathfrak{A}f' - \mathfrak{A}g' - \alpha\mathfrak{F} - \tfrac{1}{2}\mathfrak{F}\alpha' + \tfrac{3}{2}\mathfrak{G}\alpha'$$

$$\nu \sin 3\eta \quad \{\mathfrak{A}f' + \tfrac{1}{2}\mathfrak{F}\alpha' - 3\alpha\mathfrak{G}$$

$$\nu k \sin(\eta - r)\{\mathfrak{A}\iota' + \mathfrak{D}f' - \mathfrak{E}g' + \tfrac{1}{2}\mathfrak{F}c' - \tfrac{1}{2}\mathfrak{F}d' + \tfrac{3}{2}\mathfrak{G}e' - (\alpha-1)\mathfrak{H} - \tfrac{1}{2}\mathfrak{I}\alpha'$$

$$\nu k \sin(\eta + r)\{\mathfrak{A}h' - \mathfrak{D}g' + \mathfrak{E}f' + \tfrac{1}{2}\mathfrak{F}c' - \tfrac{1}{2}\mathfrak{F}e' + \tfrac{3}{2}\mathfrak{G}d' - \tfrac{1}{2}\mathfrak{H}\alpha' - (\alpha+1)\mathfrak{I}$$

$$\nu e \sin(\eta - s)\begin{cases} + \mathfrak{A}\iota' + \mathfrak{Q}f' - \mathfrak{R}g' - \tfrac{1}{2}\left(\frac{2}{n} - p'\right)\mathfrak{F} - \tfrac{1}{2}\mathfrak{F}q' + \tfrac{3}{2}\mathfrak{G}r' \\ \qquad - \left(\alpha - \frac{1}{n}\right)\mathfrak{K} - \tfrac{1}{2}\mathfrak{L}\alpha' \end{cases}$$

$$\nu e \sin(\eta + s)\begin{cases} + \mathfrak{A}k' - \mathfrak{Q}g' + \mathfrak{R}f' - \tfrac{1}{2}\left(\frac{2}{n} - p'\right)\mathfrak{F} - \tfrac{1}{2}\mathfrak{F}r' + \tfrac{3}{2}\mathfrak{G}q' \\ \qquad - \left(\alpha + \frac{1}{n}\right)\mathfrak{L} - \tfrac{1}{2}\mathfrak{K}\alpha' \end{cases}$$

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Deinde posito

$F' = \alpha F - A(f' - g') + \frac{1}{2} F a' - \frac{3}{2} G a'$

$G' = 3\alpha G - A f' - \frac{1}{2} F a'$

$H' = (\alpha - 1) H - A i' - D f' + E g' - \frac{1}{2} F c' + \frac{1}{2} F d' - \frac{3}{2} G e' + \frac{1}{2} J a'$

$J' = (\alpha + 1) J - A b' + D g' - E f' - \frac{1}{2} F c' + \frac{1}{2} F e' - \frac{3}{2} G d' + \frac{1}{2} H a'$

$K' = (\alpha - 1) K - A l' - Q f' + R g' + \frac{1}{2} F(\frac{2}{n} - p') + \frac{1}{2} F q' - \frac{3}{2} G r' + \frac{1}{2} L a'$

$L' = (\alpha + 1) L - A k' + Q g' - R f' + \frac{1}{2} F(\frac{2}{n} - p') + \frac{1}{2} F r' - \frac{3}{2} G q' + \frac{1}{2} K a'$

erit

$$\frac{dv}{dr} = -A' \sin 2\eta - D' k \sin(2\eta - r) - P' e \sin s - Q' e \sin(2\eta - s)$$
$$- E' k \sin(2\eta + r) - R' e \sin(2\eta + s)$$
$$- F' v \sin \eta - H' v k \sin(\eta - r) - K' v e \sin(\eta - s)$$
$$- G' v \sin 3\eta - J' v k \sin(\eta + r) - L' v e \sin(\eta + s)$$

§. 178. Hinc iam denuo differentiando consequemur: $\frac{ddv}{dr^2} =$ Praec.

$$+ v \cos \eta \left[+ A' f' + A' g' - \alpha F' + \frac{1}{2} F' a' + \frac{3}{2} G' a' \right.$$
$$+ v \cos 3\eta \left[+ A' f' + \frac{1}{2} F' a' - 3\alpha G' \right.$$
$$+ vk \cos(\eta - r) \left\{ \begin{array}{r} + A' i' + D' f' + E' g' + \frac{1}{2} F' c' + \frac{1}{2} F' d' + \frac{3}{2} G' e' \\ - (\alpha - 1) H' + \frac{1}{2} J' a' \end{array} \right.$$
$$+ vk \cos(\eta + r) \left\{ \begin{array}{r} + A' b' + E' f' + D' g' + \frac{1}{2} F' c' + \frac{1}{2} F' e' + \frac{3}{2} G' d' \\ + \frac{1}{2} H' a' - (\alpha + 1) J' \end{array} \right.$$
$$+ ve \cos(\eta - s) \left\{ \begin{array}{r} + A' l' + Q' f' + R' g' - \frac{1}{2}(\frac{2}{n} - p') F' + \frac{1}{2} F' g' + \frac{3}{2} G' r' \\ - (\alpha - \frac{1}{n}) K' + \frac{1}{2} L' a' \end{array} \right.$$

+

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$$+\nu\varepsilon\cos(\eta+\vartheta)\left\{\begin{array}{r}+A'k'+R'f'+Q'g'-\frac{1}{2}\left(\frac{2}{n}-p'\right)F'+\frac{3}{2}F'\gamma'+\frac{3}{2}G'\varphi'\\ +\frac{1}{2}K'\alpha'-\left(\alpha+\frac{1}{n}\right)L'\end{array}\right.$$

qui valores cum antecedentibus comparari debent, vt inde valores coefficientium eliciantur.

§. 179. Sumamus primo duos valores ab initio positos, quoniam hi a sequentibus non pendent, atque habebimus,

$$0=\alpha\mathfrak{F}-\mathfrak{A}f'+\mathfrak{A}g'+\tfrac{1}{2}\mathfrak{F}\alpha'-\tfrac{1}{2}\mathfrak{G}\alpha'+\tfrac{3}{8}+\frac{3F-3G}{2nn}$$

$$0=3\alpha\mathfrak{G}-\mathfrak{A}f'-\tfrac{1}{2}\mathfrak{F}\alpha'+\tfrac{1}{8}+\frac{3F}{2nn}$$

$$0=\alpha F'-A'f'-A'g'-\tfrac{1}{2}F'\alpha'-\tfrac{1}{2}G'\alpha'+\tfrac{2}{3}-6F-2\varkappa\mathfrak{F}$$
$$+\frac{3(F+G)}{4nn}+\frac{(\mathfrak{A}+3A)}{nn}(\mathfrak{F}+\mathfrak{G})+\frac{(3A+3A)}{nn}(F+G)+\frac{3A}{nn}$$

$$0=3\alpha G'-A'f'-\tfrac{1}{2}F'\alpha'+\tfrac{1}{8}-6G-2\varkappa G$$
$$+\frac{3F}{4nn}+\frac{(\mathfrak{A}+3A)}{nn}\mathfrak{F}+\frac{(3\mathfrak{A}+3A)}{nn}F+\frac{9A}{8nn}$$

et $F'=\alpha F-A(f'-g')+\tfrac{1}{2}F\alpha'-\tfrac{1}{2}G\alpha'$

$G'=3\alpha G-Af'-\tfrac{1}{2}F\alpha'$

at est $f'=\frac{2\varkappa F+\mathfrak{F}}{nn}$ et $g'=\frac{2\varkappa G+\mathfrak{G}}{nn}$

Hic igitur primum litterarum, quae sunt cognitae, valores in numeris substituantur, eritque

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$F' = 0,93905\,F + 0,00753\,\mathfrak{F} + 0,01443\,\mathfrak{G} - 0,00753\,\mathfrak{G}$
$G' = 2,80121\,G + 0,02505\,F + 0,00753\,\mathfrak{F}$
$0 = 0,92847\,\mathfrak{F} + 0,01776\,F - 0,01776\,G - 0,02501\,\mathfrak{G} + 0,37500$
$0 = 2,80121\,\mathfrak{G} + 0,01447\,\mathfrak{F} + 0,01776\,F + 1,87500$

hincque

$\mathfrak{F} = -0,01930\,F + 0,01913\,G - 0,42145$
$\mathfrak{G} = -0,00623\,F - 0,00010\,G - 0,66717$

§. 180. Inde porro colligemus

$F' = 0,93895\,F + 0,01457\,G + 0,00185$
$G' = 0,02491\,F + 2,80135\,G - 0,00317$
$f' = 0,01138\,F + 0,00011\,G - 0,00240$
$g' = -0,00004\,F + 0,01148\,G - 0,00380$

quibus valoribus substitutis peruenimus ad has aequationes:

$0,09337\,F = +0,05421\,G + 1,96867$
$6,83135\,G = -0,08830\,F - 3,20944$

vnde fit:

$F = 20,65700 \ldots l\,F = 1,315067$
$G = -0,73681 \ldots l\text{-}G = 9,867356$
$\mathfrak{F} = -0,83418 \ldots l\text{-}\mathfrak{F} = 9,921260$
$\mathfrak{G} = -0,79580 \ldots l\text{-}\mathfrak{G} = 9,900804$
$F' = +19,38712 \ldots l\,F' = 1,287512$
$G' = -1,55266 \ldots l\text{-}G' = 0,191075$
$f' = +0,23259 \ldots l\,f' = 9,366501$
$g' = -0,01309 \ldots l\text{-}g' = 8,116940$

§. 181.

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§. 181. His valoribus, qui ad inaequalitates absolutas pertinent, expeditis, progrediamur ad eos, qui ab excentricitate orbitae lunaris pendent, ac his aequationibus continentur:

$$(\alpha-1)\mathfrak{H} - \mathfrak{A}i' + \tfrac{1}{2}\mathfrak{J}a' - \mathfrak{D}f' + \mathfrak{E}g' - \tfrac{1}{2}\mathfrak{F}(c'-d') - \tfrac{3}{2}\mathfrak{G}e'$$
$$+ \tfrac{15}{16} + \frac{3J}{2nn} + \frac{3(F-G)}{nn} = 0$$

$$(\alpha+1)\mathfrak{J} - \mathfrak{A}b' + \tfrac{1}{2}\mathfrak{H}a' + \mathfrak{D}g' - \mathfrak{E}f' - \tfrac{1}{2}\mathfrak{F}(c'-e') - \tfrac{3}{2}\mathfrak{G}d'$$
$$+ \tfrac{15}{16} + \frac{3H}{2nn} + \frac{3(F-G)}{nn} = 0$$

$$H' = (\alpha-1)H - Ai' + \tfrac{1}{2}Ja' - Df' + Eg' - \tfrac{1}{2}F(c'-d') - \tfrac{3}{2}Ge'$$

$$J' = (\alpha+1)J - Ab' + \tfrac{1}{2}Ha' + Dg' - Ef' - \tfrac{1}{2}F(c'-e') - \tfrac{3}{2}Gd'$$

$$(\alpha-1)H' - A'i' - \tfrac{1}{2}J'a' - D'f' - E'g' - \tfrac{1}{2}F'(c'+d') - \tfrac{3}{2}G'e'$$
$$+ \tfrac{15}{16} - 6H - 2\alpha\mathfrak{H} + \tfrac{1}{2}bF + \frac{3J}{4nn} + \frac{3(F+G)}{2nn} + \frac{\mathfrak{C}\mathfrak{F}}{nn} + \frac{3\mathfrak{C}F}{nn}$$
$$+ \frac{(\mathfrak{A}+3A)}{nn}\mathfrak{J} + \frac{(3\mathfrak{A}+3A)}{nn}J + \frac{(\mathfrak{D}+3D)}{nn}\mathfrak{F} + \frac{(3\mathfrak{D}+3D)}{nn}F$$
$$+ \frac{(\mathfrak{E}+3E)}{nn}\mathfrak{G} + \frac{(3\mathfrak{E}+3E)}{nn}G - \frac{3A(F+G)}{2nn} + \frac{9D+15E}{8nn} = 0$$

$$(\alpha+1)J' - A'b' - \tfrac{1}{2}H'a' - E'f' - D'g' - \tfrac{1}{2}F'(c'+e') - \tfrac{3}{2}G'd'$$
$$+ \tfrac{15}{16} - 6J - 2\alpha J + \tfrac{1}{2}bF + \frac{3H}{4nn} + \frac{3(F+G)}{2nn} + \frac{\mathfrak{C}\mathfrak{F}}{nn} + \frac{3\mathfrak{C}F}{nn}$$
$$+ \frac{(\mathfrak{A}+3A)}{nn}\mathfrak{H} + \frac{(3\mathfrak{A}+3A)}{nn}H + \frac{(\mathfrak{D}+3D)}{nn}\mathfrak{G} + \frac{(3\mathfrak{D}+3D)}{nn}G$$
$$+ \frac{(\mathfrak{E}+3E)}{nn}\mathfrak{F} + \frac{(3\mathfrak{E}+3E)}{nn}F - \frac{3A(F+G)}{2nn} + \frac{9E+15D}{8nn} = 0$$

§. 182.

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§. 182. In his aequationibus ſubſtituantur valores iam cogniti, atque obtinebimus,

$$-0{,}06626\,\mathfrak{H}+0{,}81033\,i'-0{,}00987\,\mathfrak{J}+0{,}00854\,J\pm 2{,}04620=0$$
$$1{,}83374\,\mathfrak{J}+0{,}81033\,b'-0{,}00987\,\mathfrak{H}+0{,}00854\,H+2{,}10646=0$$
$$H'=-0{,}06626\,H+1{,}31773\,i'-0{,}00937\,J-5{,}57458$$
$$J'=\;1{,}83374\,J+1{,}31773\,b'-0{,}00987\,H-1{,}55427$$

$$\left.\begin{array}{rrr} -0{,}06626\,H' & +2{,}47576\,i' & +0{,}00987\,J'-11{,}86106 \\ -1{,}01591\,H & +0{,}00427\,J & -0{,}02711\,\mathfrak{J}+2{,}81250 \\ -2{,}01798\,\mathfrak{H} & -0{,}03634\,J & +31{,}31044 \\ & & +10{,}60834 \end{array}\right\}=0$$

$$\left.\begin{array}{rrr} 1{,}83374\,J' & +2{,}47576\,b' & +0{,}00987\,H'+0{,}27762 \\ -1{,}01591\,J & +0{,}00427\,H & -0{,}02711\,\mathfrak{H}+2{,}81250 \\ -2{,}01798\,\mathfrak{J} & -0{,}03634\,H & +31{,}81380 \\ & & -0{,}81380 \end{array}\right\}=0$$

§. 183. Subſtituamus primo loco b' et i' valores, atque noſtrae aequationes reducentur ad formas ſequentes,

$$-0{,}06626\,\mathfrak{H}+0{,}01785\,J-0{,}00526\,\mathfrak{J}+2{,}04620=0$$
$$+1{,}83374\,\mathfrak{J}+0{,}01785\,H-0{,}00526\,\mathfrak{H}+2{,}10646=0$$
$$H'=-0{,}06626\,H+0{,}00526\,J+0{,}00750\,\mathfrak{J}-5{,}57458$$
$$J'=+1{,}83374\,J+0{,}00526\,H+0{,}00750\,\mathfrak{H}-1{,}55427$$

qui valores in ſequentibus ſubſtituti dant

$$-1{,}01147\,H-2{,}01791\,\mathfrak{H}+0{,}01417\,J-0{,}01352\,\mathfrak{J}+33{,}22429=0$$
$$+2{,}34674\,J-2{,}01791\,\mathfrak{J}+0{,}00535\,H+0{,}00073\,\mathfrak{H}+30{,}68146=0$$

vnde elicimus:

$$\mathfrak{H}=\pm 0{,}00077\,H+0{,}26935\,J+30{,}96780$$
$$\mathfrak{J}=-0{,}00974\,H+0{,}00077\,J-1{,}05989$$

§. 184.

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§. 184. Hi autem valores in posterioribus aequationibus substituti producent

$$1,01290\,H + 0,52937\,J + 29,25137 = 0$$
$$2,34539\,J + 0,02500\,H + 32,84282 = 0$$

hincque tandem concluditur:

$$H = -21,68110 \ldots 1-H = 1,336081$$
$$J = -13,77206 \ldots 1-J = 1,138999$$
$$\mathfrak{H} = +27,24155 \ldots 1-\mathfrak{H} = 1,435232$$
$$\mathfrak{J} = -1,25933 \ldots 1-\mathfrak{J} = 0,100140$$
$$b' = -0,09396 \ldots 1-b' = 8,972943$$
$$i' = -0,16533 \ldots 1-i' = 9,218352$$

§. 185. Nunc pro excentricitate orbitae solaris hae restant aequationes,

$$(\ddot{a}-\tfrac{1}{n})\mathfrak{K} - \mathfrak{A}l'' + \tfrac{1}{2}\mathfrak{L}a' - \mathfrak{Q}f' + \mathfrak{R}g' + \tfrac{1}{2}(\tfrac{2}{n}-p')\mathfrak{F} + \tfrac{1}{2}\mathfrak{F}q' - \tfrac{3}{2}\mathfrak{G}r' - \tfrac{3}{4} + \frac{3\mathfrak{L}}{2nn} - \frac{9F}{4nn} + \frac{9G}{4nn} = 0$$

$$(a+\tfrac{1}{n})\mathfrak{L} - \mathfrak{A}k' + \tfrac{1}{2}\mathfrak{K}a' + \mathfrak{Q}g' - \mathfrak{R}f' + \tfrac{1}{2}(\tfrac{2}{n}-p')\mathfrak{F} + \tfrac{1}{2}\mathfrak{F}r' - \tfrac{3}{2}\mathfrak{G}q' - \tfrac{3}{4} + \frac{3K}{2nn} - \frac{9F}{4nn} + \frac{9G}{4nn} = 0$$

$$K' = (a-\tfrac{1}{n})K - Al'' + \tfrac{1}{2}La' - Qf' + Rg' + \tfrac{1}{2}(\tfrac{2}{n}-p')F + \tfrac{1}{2}Fq' - \tfrac{3}{2}Gr'$$

$$L' = (a+\tfrac{1}{n})L - Ak' + \tfrac{1}{2}Ka' + Qg' - Rf' + \tfrac{1}{2}(\tfrac{2}{n}-p')F + \tfrac{1}{2}Fr' - \tfrac{3}{2}Gq'$$

$$(a-\tfrac{1}{n})K' - A'l'' - \tfrac{1}{2}La' - Q'f' - R'g' + \tfrac{1}{2}(\tfrac{2}{n}-p')F' - \tfrac{1}{2}F'q' - \tfrac{3}{2}G'r' - \tfrac{3}{4} - 6K - 2n\mathfrak{K} - \frac{15F}{8nn} - \frac{9G}{8nn} + \frac{3L}{4nn} + \frac{(\mathfrak{A}+3A)}{nn}\mathfrak{L} + \frac{(3\mathfrak{A}+3A)L}{nn} +$$

$$+\frac{(\mathfrak{F}+3F)}{nn}(\mathfrak{P}+\mathfrak{Q})+\frac{(3\mathfrak{F}+3F)}{nn}(P+Q)+\frac{(\mathfrak{G}+3G)}{nn}\mathfrak{R}+\frac{(3\mathfrak{G}+3G)}{nn}R$$
$$+\frac{9P}{8nn}+\frac{9Q}{8nn}+\frac{15R}{8nn}=0$$

$$(\alpha+\frac{1}{n})L'-A'k'-\tfrac{1}{2}K'd'-R'f'-Q'g'+\tfrac{1}{2}(\frac{2}{n}-p')F'-\tfrac{1}{2}F'r-\tfrac{3}{2}G'q'$$
$$-\tfrac{9}{4}-6L-2\alpha\mathfrak{L}+\frac{3K}{4nn}-\frac{15F}{8nn}-\frac{9G}{8nn}+\frac{(\mathfrak{A}+3A)}{nn}\mathfrak{K}+\frac{(3\mathfrak{A}+3A)}{nn}K$$
$$+\frac{(\mathfrak{F}+3F)}{nn}(\mathfrak{P}+\mathfrak{R})+\frac{(3\mathfrak{F}+3F)}{nn}(P+R)+\frac{(\mathfrak{G}+3G)}{nn}\mathfrak{Q}+\frac{(3\mathfrak{G}+3G)}{nn}Q$$
$$+\frac{9P}{8nn}+\frac{9R}{8nn}+\frac{15Q}{8nn}=0$$

§. 186. Hic autem obſeruo, hanc determinationem maxime eſſe lubricam, cum coefficiens litterae L, quem poſtremo eſt habitura, admodum fiat paruus; vnde is a terminis, quos omiſimus, non mediocrem mutationem perpeti poſſet. Hanc ob cauſam conſultum iudico, in calculum quoque terminos $3\eta-s$ et $3\eta+s$ introducere, quia praeuideo ab iis coefficientes terminorum, quos quaerimus, non leuiter affici. Sequenti ergo modo calculum redintegro.

§. 187. In hunc finem quoque rationem habeamus angulorum $3\eta-s$ et $3\eta+s$, ſitque

$$\int R\,dr = \mathfrak{A}\cos 2\eta + \mathfrak{P}e\cos s + \mathfrak{Q}e\cos(2\eta-s)$$
$$+\mathfrak{R}e\cos(2\eta+s)$$
$$+\mathfrak{F}v\cos\eta+\mathfrak{K}ve\cos(\eta-s)+\mathfrak{M}ve\cos(3\eta-s)$$
$$+\mathfrak{G}v\cos\eta+\mathfrak{L}ve\cos(\eta+s)+\mathfrak{N}ve\cos(3\eta+s)$$
$$v=$$

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$$\begin{aligned} v = {} & A\cos 2\eta + Pe\cos s + Qe\cos(2\eta - s) \\ & \qquad + Re\cos(2\eta + s) \\ & + Fv\cos\eta + Kve\cos(\eta - s) + Mve\cos(3\eta - s) \\ & + Gv\cos 3\eta + Lve\cos(\eta + s) + Nve\cos(3\eta + s) \end{aligned}$$

§. 188. Quodsi iam ponamus:

$$\frac{2\varkappa K + \mathfrak{K}}{n n} = k' \; ; \quad \frac{2\varkappa L + \mathfrak{L}}{n n} = l'$$

$$\frac{2\varkappa M + \mathfrak{M}}{n n} = m' \quad \frac{2\varkappa N + \mathfrak{N}}{n n} = n'$$

erit

$$\begin{aligned} \frac{dv}{dt} = {} & \text{Praec.} - a'\cos 2\eta - p'e\cos s - q'e\cos(2\eta - s) \\ & \qquad - r'e\cos(2\eta + s) \\ & - f'v\cos\eta - k've\cos(\eta - s) - m've\cos(3\eta - s) \\ & - g'v\cos 3\eta - l've\cos(\eta + s) - n've\cos(3\eta + s) \end{aligned}$$

atque ob $\frac{ds}{dt} = \frac{1}{n} - \frac{2}{n}e\cos s$ erit

$$\begin{aligned} \frac{d\eta}{ds} = {} & \varkappa - a'\cos 2\eta + \left(\frac{2}{n} - p'\right)e\cos s - q'e\cos(2\eta - s) \\ & \qquad - r'e\cos(2\eta + s) \\ & - f'v\cos\eta - k've\cos(\eta - s) - m've\cos(3\eta - s) \\ & - g'v\cos 3\eta - l've\cos(\eta + s) - n've\cos(3\eta + s) \end{aligned}$$

§. 189. Formulas nunc assumtas differentiemus, solosque terminos, quibus opus habemus, in calculo exprimamus ac reperiemus:

$$\begin{aligned} R = {} & \text{Praec.} \\ & + ve\sin(\eta - s)\Big(+\mathfrak{A}l' - \mathfrak{A}m' + \mathfrak{C}f' - \mathfrak{K}g' - \tfrac{1}{2}\mathfrak{B}\left(\tfrac{2}{n} - p'\right) - \tfrac{1}{2}\mathfrak{B}q' + \tfrac{3}{2}\mathfrak{G}r' - \mathfrak{K}\left(a - \tfrac{1}{n}\right) \\ & \qquad + \tfrac{1}{2}\mathfrak{L}a' + \tfrac{1}{2}\mathfrak{M}a' \\ & + \end{aligned}$$

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$$+ve\sin(\eta+s)(+\mathfrak{A}k'-\mathfrak{A}n'+\mathfrak{R}f'-\mathfrak{Q}g'-\tfrac{1}{2}\mathfrak{F}(\tfrac{2}{n}-p')-\tfrac{1}{2}\mathfrak{F}r'+\tfrac{3}{2}\mathfrak{G}q'-\mathfrak{L}(\alpha+\tfrac{1}{n})$$
$$-\tfrac{3}{2}\mathfrak{K}a'+\tfrac{3}{2}\mathfrak{N}a'$$
$$+ve\sin(3\eta-s)(+\mathfrak{A}k'+\mathfrak{Q}f'+\tfrac{1}{2}\mathfrak{F}q'-\tfrac{3}{2}\mathfrak{G}(\tfrac{2}{n}-p')+\tfrac{1}{2}\mathfrak{K}a'-\mathfrak{M}(3\alpha-\tfrac{1}{n})$$
$$+ve\sin(3\eta-s)(+\mathfrak{A}l'+\mathfrak{R}f'+\tfrac{1}{2}\mathfrak{F}r'-\tfrac{3}{2}\mathfrak{G}(\tfrac{2}{n}-p')+\tfrac{1}{2}\mathfrak{L}a'-\mathfrak{N}(3\alpha+\tfrac{1}{n})$$

Ac si breuitatis gratia ponamus:

$$K' = (\alpha-\tfrac{1}{n})K-A(l'-m')-Qf'+Rg'+\tfrac{3}{2}F(\tfrac{2}{n}-p')+\tfrac{1}{2}Fq'-\tfrac{3}{2}Gr'$$
$$+\tfrac{1}{2}La'-\tfrac{3}{2}Ma'$$
$$L' = (\alpha+\tfrac{1}{n})L-A(k'-n')-Rf'+Qg'+\tfrac{1}{2}F(\tfrac{2}{n}-p')+\tfrac{3}{2}Fr'-\tfrac{3}{2}Gq'$$
$$+\tfrac{1}{2}Ka'-\tfrac{3}{2}Na'$$
$$M' = (3\alpha-\tfrac{1}{n})M-Ak'-Qf'-\tfrac{1}{2}Fq'+\tfrac{3}{2}G(\tfrac{2}{n}-p')-\tfrac{1}{2}Ka'$$
$$N' = (3\alpha+\tfrac{1}{n})N-Al'-Rf'-\tfrac{1}{2}Fr'+\tfrac{3}{2}G(\tfrac{2}{n}-p')-\tfrac{1}{2}La'$$

erit

$$\frac{dv}{dr} = -A'\sin 2\eta - P'e\sin s - Q'e\sin(2\eta-s)$$
$$-R'e\sin(2\eta+s)$$
$$-F'v\sin\eta \quad -K've\sin(\eta-s) \quad -M've\sin(3\eta-s)$$
$$-G'v\sin 3\eta \quad -L've\sin(\eta+s) \quad -N've\sin(3\eta+s)$$

§. 190. Hinc iam denuo differentiando nanciscemur

$$\frac{ddv}{dr^2} = \text{Praec.}$$
$$+ve\cos(\eta-s)\left\{\begin{array}{l} A'l'+A'm'+Q'f'+R'g'-\tfrac{1}{2}F'(\tfrac{2}{n}-p')+\tfrac{1}{2}F'q' \\ +\tfrac{3}{2}G'r'+\tfrac{1}{2}L'a'+\tfrac{3}{2}M'a'-(\alpha-\tfrac{1}{n})K' \end{array}\right.$$
$$+$$

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$$+ve\cos(\eta+s)\left\{\begin{array}{l} A'k'+A'm'+R'f'+Q'g'-\frac{1}{2}F'(\frac{2}{n}-p')+\frac{1}{2}F'p' \\ +\frac{1}{2}G'q'+\frac{1}{2}K'a'+\frac{1}{2}N'a'-(\alpha+\frac{1}{n})L' \end{array}\right.$$

$$+ve\cos(3\eta-s)\left\{\begin{array}{l} A'k'+Q'f'+\frac{1}{2}F'q'-\frac{1}{2}G'(\frac{2}{n}-p')+\frac{1}{2}K'a' \\ -(3\alpha-\frac{1}{n})M' \end{array}\right.$$

$$+ve\cos(3\eta+s)\left\{\begin{array}{l} A'k'+R'f'+\frac{1}{2}F'q'-\frac{1}{2}G'(\frac{2}{n}-p')+\frac{1}{2}L'a' \\ -(3\alpha+\frac{1}{n})N' \end{array}\right.$$

§. 191. Quodsi autem valores iam inuenti substituantur, habebitur

$$R = \text{Praec.}$$

$$+ve\sin(\eta-s)\left\{\begin{array}{l} -0,85830\mathfrak{K}+0,00526\mathfrak{L}-0,02500\mathfrak{M}+0,37592 \\ -0,00931L+0,00931M \end{array}\right.$$

$$+ve\sin(\eta+s)\left\{\begin{array}{l} -1,00918\mathfrak{L}+0,00526\mathfrak{K}-0,02500\mathfrak{N}+0,34115 \\ -0,00931K+0,00931N \end{array}\right.$$

$$+ve\sin(3\eta-s)\left\{\begin{array}{l} -2,72578\mathfrak{M}-0,01448\mathfrak{K}+0,49223 \\ -0,00931K \end{array}\right.$$

$$+ve\sin(3\eta+s)\left\{\begin{array}{l} -2,87666\mathfrak{N}-0,01448\mathfrak{L}+0,47116 \\ -0,00931L \end{array}\right.$$

$$K' = 0{,}85830\,K + 0{,}00527\,L + 0{,}01447\,M + 1{,}48970 + 0{,}00750\,\mathfrak{L} - 0{,}00750\,\mathfrak{M}$$
$$L' = 1{,}00916\,L + 0{,}00527\,K + 0{,}01447\,N + 1{,}55183 + 0{,}00750\,\mathfrak{K} - 0{,}00750\,\mathfrak{N}$$
$$M' = 2{,}72528\,M + 0{,}02501\,K - 1{,}17408 + 0{,}00750\,\mathfrak{K}$$
$$N' = 2{,}87666\,N + 0{,}02501\,L - 0{,}95901 + 0{,}00750\,\mathfrak{L}$$

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+ vecs(\eta - s)\begin{cases} -0{,}85830\,K' - 0{,}02843\,L - 0{,}02843\,M - 0{,}32674 \\ -0{,}00987\,L' - 0{,}01409\,\mathfrak{L} - 0{,}01409\,\mathfrak{M} \\ -0{,}02961\,M' \end{cases}$$

$$+ vecs(\eta + s)\begin{cases} -1{,}00918\,L' - 0{,}02843\,K - 0{,}02843\,N - 0{,}56620 \\ -0{,}00987\,K' - 0{,}01409\,\mathfrak{K} - 0{,}01409\,\mathfrak{N} \\ -0{,}02981\,N' \end{cases}$$

$$+ vecs(3\eta - s)\begin{cases} -2{,}72578\,M' - 0{,}02843\,K + 0{,}74683 \\ -0{,}00987\,K' - 0{,}01409\,\mathfrak{K} \end{cases}$$

$$+ vecs(3\eta + s)\begin{cases} -2{,}87661\,N' - 0{,}02853\,L + 0{,}67486 \\ -0{,}00987\,L' - 0{,}01409\,\mathfrak{L} \end{cases}$$

§. 192. Valores autem litterarum communate notatarum hic substituti dabunt

$$\frac{ddv}{dr^2} =$$

$$vecs(\eta - s)\begin{cases} -0{,}73747\,K + 0{,}04264\,L - 0{,}12156\,M - 0{,}00014\,N \\ -0{,}00029\,\mathfrak{K} - 0{,}02053\,\mathfrak{L} - 0{,}00765\,\mathfrak{M} + 0{,}00008\,\mathfrak{N} \\ -1{,}58590 \end{cases}$$

$$\text{vecs}(\eta+s)\begin{cases}-1{,}01923\mathfrak{L}-0{,}04222\mathfrak{K}-0{,}12821\mathfrak{N}-0{,}00014\mathfrak{M}\\-0{,}00029\mathfrak{L}-0{,}02166\mathfrak{K}-0{,}00642\mathfrak{N}+0{,}00008\mathfrak{M}\\\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad-2{,}11858\end{cases}$$

$$\text{vecs}(3\eta-s)\begin{cases}-7{,}43058\mathfrak{M}-0{,}09507\mathfrak{K}-0{,}00005\mathfrak{L}+3{,}93257\\+0{,}00008\mathfrak{N}-0{,}03453\mathfrak{K}-0{,}00007\mathfrak{L}\end{cases}$$

$$\text{vecs}(3\eta+s)\begin{cases}-8{,}27529\mathfrak{N}-0{,}11338\mathfrak{L}-0{,}00005\mathfrak{K}+3{,}41830\\+0{,}00008\mathfrak{N}-0{,}03566\mathfrak{L}-0{,}00007\mathfrak{K}\end{cases}$$

§. 193. His expreſſionibus ita euolutis atque ad calculum numericum praeparatis, quaeramus easdem expreſſiones ex formulis ſupra traditis pro R et $\frac{ddv}{dt^2}$, quae continentur in §. 52 et 54. Inde autem omittendis terminis, quos iam tractauimus, conſequemur.

$$R=\text{Pr.}+v\,\text{ſin}(\eta-s)\left(-\tfrac{3}{4}+\frac{3L}{2nn}-\frac{3M}{2nn}-\frac{9F}{4nn}+\frac{9G}{4nn}\right)$$

$$+v\,\text{ſin}(\eta+s)\left(-\tfrac{3}{4}+\frac{3K}{2nn}-\frac{3N}{2nn}+\frac{9G}{4nn}-\frac{9F}{4nn}\right)$$

$$+v\,\text{ſin}(3\eta-s)\left(-\tfrac{15}{4}+\frac{3K}{2nn}-\frac{9F}{4nn}\right)$$

$$+v\,\text{ſin}(3\eta+s)\left(-\tfrac{15}{4}+\frac{3L}{2nn}-\frac{9F}{4nn}\right)$$

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$$\frac{ddv}{dv^2} = \text{Praec.}$$

$$+ve\cos(\eta - s)\begin{cases} -\frac{9}{4} + \frac{9P}{8nn} + \frac{9Q}{8nn} + \frac{15R}{8nn} - 6K + \frac{3L}{4nn} + \frac{3M}{4nn} \\ -\frac{3F}{4nn} - \frac{9F}{8nn} - \frac{9G}{8nn} - 2n\mathfrak{K} + \frac{(\mathfrak{A}+3A)}{nn}(\mathfrak{L}+\mathfrak{M}) \\ +\frac{(3\mathfrak{A}+3A)}{nn}(L+M) + \frac{(\mathfrak{P}+3P)}{nn}\mathfrak{P} + \frac{(3\mathfrak{P}+3P)}{nn}F \\ +\frac{(\mathfrak{Q}+3Q)}{nn}\mathfrak{P} + \frac{(3\mathfrak{Q}+3Q)}{nn}F + \frac{(\mathfrak{R}+3R)}{nn}\mathfrak{G} \\ +\frac{(3\mathfrak{R}+3R)}{nn}G \end{cases}$$

$$+ve\cos(\eta + s)\begin{cases} -\frac{9}{4} + \frac{9P}{8nn} + \frac{9R}{8nn} + \frac{15Q}{8nn} - 6L + \frac{3K}{4nn} + \frac{3N}{4nn} \\ -\frac{3F}{4nn} - \frac{9F}{8nn} - \frac{9G}{8nn} - 2n\mathfrak{L} + \frac{(\mathfrak{A}+3A)}{nn}(\mathfrak{K}+\mathfrak{N}) \\ +\frac{(3\mathfrak{A}+3A)}{nn}(K+N) + \frac{(\mathfrak{P}+3P)}{nn}\mathfrak{F} + \frac{(3\mathfrak{P}+3P)}{nn}F \\ +\frac{(\mathfrak{Q}+3Q)}{nn}\mathfrak{G} + \frac{(3\mathfrak{Q}+3Q)}{nn}G + \frac{(\mathfrak{R}+3R)}{nn}\mathfrak{F} \\ +\frac{(3\mathfrak{R}+3R)}{nn}F \end{cases}$$

$$+ve\cos(3\eta - s)\begin{cases} -\frac{15}{4} + \frac{15P}{8nn} + \frac{9Q}{8nn} - 6M + \frac{3K}{4nn} - \frac{3G}{4nn} \\ -\frac{9F}{8nn} - 2n\mathfrak{M} + \frac{(\mathfrak{A}+3A)}{nn}\mathfrak{K} + \frac{(3\mathfrak{A}+3A)}{nn}K + \frac{(\mathfrak{P}+3P)}{nn}\mathfrak{G} \\ +\frac{(3\mathfrak{P}+3P)}{nn}G + \frac{(\mathfrak{Q}+3Q)}{nn}\mathfrak{F} + \frac{(3\mathfrak{Q}+3Q)}{nn}F \end{cases}$$

$+ve$

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$$+ve\cos(3\eta+s)\begin{cases} -\frac{15}{4}+\frac{15P}{8nn}+\frac{9R}{8nn}-6N+\frac{3L}{4nn}-\frac{3G}{4nn}-\frac{9F}{8nn}-2n\mathfrak{M} \\ +\frac{(\mathfrak{A}+3A)}{nn}\mathfrak{L}+\frac{(3\mathfrak{A}+3A)}{nn}L+\frac{(\mathfrak{P}+3P)}{nn}\mathfrak{G}+\frac{(3\mathfrak{P}+3P)}{nn}G \\ +\frac{(\mathfrak{R}+3R)}{nn}\mathfrak{F}+\frac{(3\mathfrak{R}+3R)}{nn}F \end{cases}$$

§. 194. Introducantur hic quoque valores iam cogniti, ac prodibit

$$\begin{aligned} R = Pr. &+ve\sin(\eta-s)[-1,02394+0,00854L-0,00854M] \\ &+ve\sin(\eta+s)[-1,02394+0,00854K-0,00854N] \\ &+ve\sin(3\eta-s)[-4,01450+0,00854K] \\ &+ve\sin(3\eta+s)[-4,01450+0,00854L] \end{aligned}$$

$$\frac{ddv}{dt^2} = \text{Praec.}$$

$$+vec\int(\eta-s)\begin{cases}-1,01591K-0,03207L-0,03207M-1,168671 \\ -2,01798\mathfrak{K}-0,02711\mathfrak{L}-0,02711\mathfrak{M}\end{cases}$$

$$+vec\int(\eta+s)\begin{cases}-1,01591L-0,03207K-0,03207N-1,93904 \\ -2,01798\mathfrak{L}-0,02711\mathfrak{K}-0,02711\mathfrak{N}\end{cases}$$

$$+vec\int(3\eta-s)\begin{cases}-1,01591M-0,03207K-2,49545 \\ -2,01798\mathfrak{M}-0,02711\mathfrak{K}\end{cases}$$

$$+vec\int(\eta 3+s)\begin{cases}-1,01591N-0,03207L-2,78450 \\ -2,01798\mathfrak{N}-0,02711\mathfrak{L}\end{cases}$$

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§. 195. Hinc ergo octo sequentes aequationes resultabunt

I. $0{,}85830\,\mathfrak{K} = +0{,}00526\,\mathfrak{L} - 0{,}02500\,\mathfrak{M} + 1{,}39986 - 0{,}01785\,L + 0{,}01785\,M$

II. $1{,}00918\,\mathfrak{L} = +0{,}00526\,\mathfrak{K} - 0{,}02500\,\mathfrak{N} + 1{,}36509 - 0{,}01785\,K + 0{,}01785\,N$

III. $2{,}72578\,\mathfrak{M} = -0{,}01448\,\mathfrak{K} + 4{,}50673 - 0{,}01785\,K$

IV. $2{,}87666\,\mathfrak{N} = -0{,}01448\,\mathfrak{L} + 4{,}48566 - 0{,}01785\,L$

V. $+0{,}27844K - 0{,}01057L - 0{,}08949M - 0{,}00014N + 0{,}10081 = 0$
$+2{,}01769\mathfrak{K} + 0{,}00658\,\mathfrak{L} - 0{,}01946\mathfrak{M} + 0{,}00008\mathfrak{N}$

VI. $-0{,}00332L - 0{,}01015K - 0{,}09614N - 0{,}00014M - 0{,}17954 = 0$
$+2{,}01769\,\mathfrak{L} + 0{,}00545\,\mathfrak{K} + 0{,}02069\mathfrak{N} + 0{,}00008\mathfrak{M}$

VII. $-6{,}41487\,M - 0{,}06300\,K - 0{,}00005\,L + 6{,}42802 = 0$
$+2{,}01806\mathfrak{M} - 0{,}00742\,\mathfrak{K} - 0{,}00007\,\mathfrak{L}$

VIII. $-7{,}25938\,N - 0{,}08131\,L - 0{,}00005\,K + 6{,}20280 = 0$
$+2{,}01806\,\mathfrak{N} - 0{,}00855\,\mathfrak{L} - 0{,}00007\,\mathfrak{K}$

§. 196. Ex aequationibus III et IV statim eliciuntur hi valores

$$\mathfrak{M} = -0{,}00531\,\mathfrak{K} - 0{,}00655\,K + 1{,}65336$$
$$\mathfrak{N} = -0{,}00503\,\mathfrak{L} - 0{,}00621\,L + 1{,}55933$$

qui

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qui in I et II ſubſtituti praebent:

$0{,}85817\mathfrak{K} = 1{,}35853 + 0{,}00526\mathfrak{L} - 0{,}01785(L-M) + 0{,}00016K$

$1{,}00905\mathfrak{L} = 1{,}32611 + 0{,}00526\mathfrak{K} - 0{,}01785(K-N) + 0{,}00015L$

vnde obtinetur:

$\mathfrak{K} = +1{,}59116 - 0{,}02080(L-M) + 0{,}00008K + 0{,}00010N$

$\mathfrak{L} = +1{,}32251 - 0{,}01769(K-N) + 0{,}00005L + 0{,}00011M$

$\mathfrak{M} = +1{,}64491 - 0{,}00655K + 0{,}00010(L-M)$

$\mathfrak{N} = +1{,}55268 - 0{,}00621L + 0{,}00009(K-N)$

§. 197. His valoribus ſubſtitutis caeterae aequationes abibunt in formas ſequentes:

$0{,}27862K - 0{,}05252L - 0{,}04754M + 0{,}00017N + 3{,}35313 = 0$

$-0{,}00346L - 0{,}04584K - 0{,}06045N + 0{,}00020M + 2{,}53002 = 0$

$-6{,}41502M - 9{,}07622K + 0{,}00930L + 9{,}73587 = 0$

$-7{,}25971N - 0{,}09384L + 0{,}00028K + 9{,}32499 = 0$

ex quarum binis poſtremis ſtatim obtinetur:

$M = -0{,}01188\,K + 0{,}00005\,L + 1{,}51765$

$N = -0{,}01293\,L + 0{,}00004\,K + 1{,}28448$

vnde colligitur:

$+0{,}27918\,K - 0{,}05252\,L + 3{,}28120 = 0$

$-0{,}00268\,L - 0{,}04584\,K + 2{,}45267 = 0$

ac denique

$K = +40{,}44710 \ldots 1/K = 1{,}606887$

$L = +368{,}40200 \ldots 1/L = 2{,}566322$

$M = +1{,}05555 \ldots 1/M = 0{,}023478$

$N = -3{,}47730 \ldots 1/N = 0{,}541242$

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§. 198. Litterarum germanicarum valores hinc erunt:

$\mathfrak{K} = -5{,}04677 \ldots 1-\mathfrak{K} = 0{,}703013$
$\mathfrak{L} = +0{,}56302 \ldots l\,\mathfrak{L} = 9{,}750524$
$\mathfrak{M} = +1{,}41672 \ldots l\,\mathfrak{M} = 0{,}151283$
$\mathfrak{N} = -0{,}73119 \ldots l\,\mathfrak{N} = 9{,}864030$

ac litterarum hinc deriuatarum:

$k' = +0{,}43578 \ldots l\,k' = 9{,}639267$
$l' = +4{,}23401 \ldots l\,l' = 0{,}626752$
$m' = +0{,}02018 \ldots l\,m' = 8{,}304921$
$n' = -0{,}04409 \ldots 1-n' = 8{,}644340$

§. 199. Nunc igitur intelligimus inaequalitates ab angulis $3\eta - s$ et $3\eta + s$ pendentes tam esse paruas, vt sine vllo errore reiici queant, etiamsi valores K et L aliquantum immutauerint. Distantia ergo lunae curtata a terra $x = \frac{(1-kk)\,au}{1-k\cos r}$ ita ab his inaequalitatibus parallacticis pendebit, vt sit

		Log. coeff.
$u =$ Praec. $+ 0{,}11756\ v \cos\eta$		9,070249
$- 0{,}00419\ v \cos 3\eta$		7,622540
$- 0{,}1234\ vk \cos(\eta - r)$		9,091265
$- 0{,}0784\ vk \cos(\eta + r)$		8,894183
$+ 0{,}2302\ ve \cos(\eta - s)$		9,362071
$+ 2{,}0965\ ve \cos(r + s)$		0,321506

Motus

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Motus autem momentaneus ita hinc afficietur, vt sit

$$\frac{d\Phi}{dr} = \text{Praec.} \quad \begin{array}{lll|l} -\ 0{,}23259 & \nu & \cos\eta & 9{,}366591 \\ +\ 0{,}01309 & \nu & \cos 3\eta & 8{,}116940 \\ +\ 0{,}0939 & \nu k & \cos(\eta - r) & 8{,}972943 \\ +\ 0{,}1653 & \nu k & \cos(\eta + r) & 9{,}218352 \\ -\ 0{,}4358 & \nu e & \cos(\eta - s) & 9{,}639267 \\ -\ 4{,}2340 & \nu e & \cos(\eta + s) & 0{,}626752 \end{array}$$

§. 200. Quodsi iam ipsam longitudinem lunae, quatenus ab his inaequalitatibus parallacticis pendet, ponamus:

$$\begin{aligned} \Phi = \text{Praec.} &+ \mathfrak{F}'\nu \sin\eta + \mathfrak{H}'\nu k \sin(\eta - r) + \mathfrak{K}'\nu e \sin(\eta - s) \\ &+ \mathfrak{G}'\nu \sin 3\eta + \mathfrak{J}'\nu k \sin(\eta + r) + \mathfrak{L}'\nu e \sin(\eta + s) \end{aligned}$$

sequentes obtinebimus aequationes pro horum coefficientium determinatione:

$$-0{,}23259 = \alpha\mathfrak{F}' - \mathfrak{A}'f' - \mathfrak{A}'g' - \tfrac{1}{2}\mathfrak{F}'a' - \tfrac{3}{2}\mathfrak{G}'a'$$

$$+0{,}01303 = 3\alpha\mathfrak{G}' - \mathfrak{A}'f' - \tfrac{1}{2}\mathfrak{F}'a'$$

$$\begin{aligned} +0{,}0939 = (\alpha - 1)\mathfrak{H}' - \mathfrak{A}'i' - \tfrac{1}{2}\mathfrak{J}'a' - \mathfrak{D}'f' - \mathfrak{E}'g' \\ - \tfrac{1}{2}\mathfrak{F}'c' - \tfrac{1}{2}\mathfrak{F}'d' - \tfrac{3}{2}\mathfrak{G}'e' \end{aligned}$$

$$\begin{aligned} +0{,}1653 = (\alpha + 1)\mathfrak{J}' - \mathfrak{A}'h' - \tfrac{1}{2}\mathfrak{H}'a' - \mathfrak{E}'f' - \mathfrak{D}'g' \\ - \tfrac{1}{2}\mathfrak{F}'c' - \tfrac{1}{2}\mathfrak{F}'e' - \tfrac{3}{2}\mathfrak{G}'d' \end{aligned}$$

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$$-0{,}4358 = (a - \tfrac{1}{n})\mathfrak{K}' - \mathfrak{A}'\mathfrak{l}' - \tfrac{1}{2}\mathfrak{L}'\mathfrak{m}' - \mathfrak{Q}'\mathfrak{f}' - \mathfrak{R}'\mathfrak{g}' + \tfrac{1}{2}\mathfrak{F}'(\tfrac{2}{n} - p') - \tfrac{1}{2}\mathfrak{F}'\mathfrak{q}' - \tfrac{1}{2}\mathfrak{G}'\mathfrak{r}'$$

$$-4{,}2340 = (a + \tfrac{1}{n})\mathfrak{L}' - \mathfrak{A}'\mathfrak{k}' - \tfrac{1}{2}\mathfrak{K}'\mathfrak{m}' - \mathfrak{R}'\mathfrak{f}' - \mathfrak{Q}'\mathfrak{g}' + \tfrac{1}{2}\mathfrak{F}'(\tfrac{2}{n} - p') - \tfrac{1}{2}\mathfrak{F}'\mathfrak{r}' - \tfrac{1}{2}\mathfrak{G}'\mathfrak{q}'$$

§. 201. Valoribus autem iam cognitis hic ſubſtitutis, aequationes iſtae in ſequentes abibunt formas;

$$-0{,}23031 = +0{,}94361\,\mathfrak{F}' + 0{,}02961\,\mathfrak{G}'$$

$$+0{,}01550 = +2{,}80122\,\mathfrak{G}' + 0{,}00987\,\mathfrak{F}'$$

$$-0{,}0056 = -0{,}06626\mathfrak{H}' + 0{,}00987\mathfrak{J}' - 0{,}25665\mathfrak{F}' + 0{,}01926\,\mathfrak{G}'$$

$$+0{,}1710 = +1{,}93374\mathfrak{J}' + 0{,}00987\mathfrak{H}' - 0{,}06717\,\mathfrak{F}' - 0{,}54918\,\mathfrak{G}'$$

$$-0{,}3968 = +0{,}85830\,\mathfrak{K}' + 0{,}00987\,\mathfrak{L}' + 0{,}06357\,\mathfrak{F}' - 0{,}04509\,\mathfrak{G}'$$

$$-4{,}2330 = +1{,}00918\,\mathfrak{L}' + 0{,}00987\,\mathfrak{K}' + 0{,}06729\,\mathfrak{F}' - 0{,}05625\,\mathfrak{G}'$$

vnde colligitur fore

$$\mathfrak{F}' = -0{,}24427 \ldots l\text{-}\mathfrak{F}' = 9{,}387868$$

$$\mathfrak{G}' = +0{,}00639 \ldots l\,\mathfrak{G}' = 7{,}805991$$

$$\mathfrak{H}' = +1{,}1959 \ldots l\,\mathfrak{H}' = 0{,}077694$$

$$\mathfrak{J}' = +0{,}0757 \ldots l\,\mathfrak{J}' = 8{,}879096$$

$$\mathfrak{K}' = -0{,}3959 \ldots l\text{-}\mathfrak{K}' = 9{,}597508$$

$$\mathfrak{L}' = -4{,}1738 \ldots l\text{-}\mathfrak{L}' = 0{,}620530$$

§. 201.

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§. 202. Hinc ergo habebimus sequentes partes pro longitudine Lunae, quas simul ope valorum proxime cognitorum pro v, k, e ad minuta secunda reducamus:

			Log coeff.	Val. coeff. in min sec.
Φ=Praec.	$-0,24427$	$v \sin \eta$	9,387868	$-175''$
	$+0,00639$	$v \sin 3\eta$	7,805991	$+ 4''$
	$+1,1959$	$vk \sin (\eta - r)$	0,077694	$+ 59''$
	$+0,0757$	$vk \sin (\eta + r)$	8,879096	$+ 4''$
	$-0,3959$	$ve \sin (\eta - s)$	9,597508	$- 5''$
	$-4,1738$	$ve \sin (\eta + s)$	0,620530	$- 49''$

Sicque omnes iam adepti sumus motus lunae inaequalitates, quae quidem ab inclinatione eius orbitae ad eclipticam non pendent. Interim tamen non diffiteor, dari aliquas insuper inaequalitates, quae alicuius forte sint momenti, quas in hac inuestigatione praeterimus, cuiusmodi sunt eae, quae ab angulis $2\eta - 3r$ et $2\eta - 2r + s$ pendent, quae ad plura minuta secunda assurgere posse videntur. Verum earum determinatio tam est taediosa, vt malim eam obseruationibus relinquere.

§. 203. Quae ergo hactenus inuenimus, in vnum colligamus ac primo pro distantia lunae a terra curuata $s = \frac{(1-kk)\,aa}{1-k\cos r}$ erit

$\Phi =$

		Log. coeff.	coeff. integri
$u = 1$	$-0{,}0074991 \cos 2\eta$	7,875009	$-0{,}007499$
	$+0{,}0000532 \cos 4\eta$	5,725912	$+0{,}000053$
	$+0{,}191557 k \cos(2\eta - r)$	9,282297	$+0{,}010430$
	$-0{,}003293 k \cos(2\eta + r)$	7,517525	$-0{,}000179$
	$-0{,}003321 k \cos(4\eta - r)$	7,521296	$-0{,}000181$
	$+0{,}000049 k \cos(4\eta + r)$	5,692584	$+0{,}000005$
	$-0{,}00511 kk \cos 2r$	7,708601	$-0{,}000015$
	$-0{,}08022 kk \cos(2\eta - 2r)$	8,904280	$-0{,}000238$
	$-0{,}00237 kk \cos(2\eta + 2r)$	7,375072	$-0{,}000007$
	$+0{,}07892 kk \cos(4\eta - 2r)$	8,897172	$+0{,}000234$
	$+0{,}00001 kk \cos(4\eta + 2r)$	4,872349	$+0{,}000000$
	$-0{,}006400 e \cos s$	7,806180	$-0{,}000206$
	$+0{,}014801 e \cos(2\eta - s)$	8,170303	$+0{,}000249$
	$+0{,}011415 e \cos(2\eta + s)$	8,057492	$+0{,}000192$
	$+0{,}00364 ee \cos 2s$	7,539670	$+0{,}000001$
	$-0{,}01482 ee \cos(2\eta - 2s)$	8,170799	$-0{,}000004$
	$-0{,}00584 ee \cos(2\eta + 2s)$	7,766856	$-0{,}000001$
	$-0{,}37957 ek \cos(r - s)$	9,579297	$-0{,}000347$
	$+0{,}30693 ek \cos(r + s)$	9,487039	$+0{,}000281$
	$-0{,}36337 ek \cos(2\eta - r + s)$	9,560344	$-0{,}000332$
	$+0{,}00993 ek \cos(2\eta + r - s)$	7,997004	$+0{,}000009$
	$-0{,}07752 ek \cos(2\eta - r - s)$	8,889425	$-0{,}000071$
	$+0{,}00305 ek \cos(2\eta + r + s)$	7,484024	$+0{,}000003$
	$+0{,}11756 \nu \cos \eta$	9,070249	$+0{,}000408$
	$-0{,}00419 \nu \cos 3\eta$	7,622540	$-0{,}000015$
	$-0{,}1234 \nu k \cos(\eta - r)$	9,091265	$-0{,}000024$
	$-0{,}0784 \nu k \cos(\eta + r)$	8,894183	$-0{,}000015$
	$+0{,}2302 \nu e \cos(\eta - s)$	9,362071	$+0{,}000013$
	$+2{,}0965 \nu e \cos(\eta + s)$	0,321506	$+0{,}000122$

Hic ad latus adiunxi valores coefficientium integrorum in numeris absolutis expressos, ponendo $k = 0{,}05445$, $e = 0{,}01680$ et $\nu = \frac{1}{188}$; quos proinde, si hi valores aliter per obseruationes determinentur, facile erit emendare.

§. 204.

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§. 204. Pro motu autem lunae momentaneo, ex quo eius motus horarius definiri poterit, habebimus:

$\frac{d\Phi}{dr} =$	Log. coeff.	Coeff. integri
$1,009176$	0,003967	+ 1,009176
$+ 0,019514 \cos 2\eta$	8,290355	+ 0,019514
$- 0,0000322 \cos 4\eta$	5,507856	— 0,000032
$- 0,001231 k \cos r$	7,090258	— 0,000067
$- 0,366103 k \cos(2\eta - r)$	9,563604	— 0,019934
$+ 0,012832 k \cos(2\eta + r)$	8,108292	+ 0,000699
$+ 0,002829 k \cos(4\eta - r)$	7,451633	+ 0,000154
$- 0,000171 k \cos(4\eta + r)$	6,232305	— 0,000009
$+ 0,01182 kk \cos 2r$	8,072618	+ 0,000035
$- 0,02057 kk \cos(2\eta - 2r)$	8,313172	— 0,000061
$+ 0,01063 kk \cos(2\eta + 2r)$	8,026598	+ 0,000032
$- 0,09883 kk \cos(4\eta - 2r)$	8,994889	— 0,000293
$- 0,00004 kk \cos(4\eta + 2r)$	5,592770	— 0,000000
$+ 0,013760 e \cos s$	8,138618	+ 0,000231
$- 0,037487 e \cos(2\eta - s)$	8,573878	— 0,000630
$- 0,030062 e \cos(2\eta + s)$	8,478023	— 0,000505
$- 0,00722 ee \cos 2s$	7,858166	— 0,000002
$+ 0,03470 ee \cos(2\eta - 2s)$	8,540319	+ 0,000010
$+ 0,01533 ee \cos(2\eta + 2s)$	8,185614	+ 0,000005
$+ 0,75204 ek \cos(r - s)$	9,876241	+ 0,000688
$- 0,62626 ek \cos(r + s)$	9,796755	— 0,000573
$+ 0,69420 ek \cos(2\eta - r + s)$	9,841484	+ 0,000635
$- 0,02440 ek \cos(2\eta + r - s)$	8,387390	— 0,000022
$+ 0,12160 ek \cos(2\eta - r - s)$	9,084933	+ 0,000111
$- 0,02000 ek \cos(2\eta + r + s)$	9,301030	— 0,000018
$- 0,23259 v \cos\eta$	9,366591	— 0,000808
$+ 0,01309 v \cos 3\eta$	8,116940	+ 0,000045
$+ 0,0939 vk \cos(\eta - r)$	8,972943	+ 0,000018
$+ 0,1653 vk \cos(\eta + r)$	9,218352	+ 0,000031
$- 0,4358 ve \cos(\eta - s)$	9,639267	— 0,000025
$- 4,2340 ve \cos(\eta + s)$	9,626753	— 0,000247

§. 205. Si iam longitudo Lunae per solam excentricitatem secundum regulas Keplerianas determinata ponatur $= \zeta$, ita vt posita eius anomalia vera $= r$, futurum sit $\zeta = C + 1{,}0085272\, r$, erit longitudo vera per hactenus inuentas inaequalitates.

	Log. coeff.	Val. coeff. in min. sec.
$\varphi = \zeta + 0{,}0103597 \sin 2\eta$	8,015347	$+2137''$
$- 0{,}0000382 \sin 4\eta$	5,582063	$- 8$
$+ 0{,}010146k \sin r$	8,006295	$+ 114$
$- 0{,}420226k \sin (2\eta - r)$	9,623483	$- 4720$
$+ 0{,}004992k \sin (2\eta + r)$	7,698261	$+ 56$
$+ 0{,}005286k \sin (4\eta - r)$	7,723163	$+ 59$
$- 0{,}000086k \sin (4\eta + r)$	5,935307	$- 1$
$+ 0{,}00420kk \sin 2r$	7,623250	$+ 2\frac{1}{2}$
$+ 0{,}57328kk \sin (2\eta - 2r)$	9,758367	$+ 351$
$+ 0{,}00318kk \sin (2\eta + 2r)$	7,402427	$+ 1\frac{1}{2}$
$- 0{,}15083kk \sin (4\eta - 2r)$	9,178488	$- 92$
$- 0{,}00002kk \sin (4\eta + 2r)$	5,301030	$- 0$
$+ 0{,}201385e \sin s$	9,304026	$+ 701$
$- 0{,}021889e \sin (2\eta - s)$	8,340237	$- 76$
$- 0{,}016368e \sin (2\eta + s)$	8,214002	$- 57$
$+ 0{,}06615ee \sin 2s$	8,820508	$+ 4$
$+ 0{,}02332ee \sin (2\eta - 2s)$	8,367825	$+ 1$
$+ 0{,}00840ee \sin (2\eta + 2s)$	7,924429	$+ \frac{1}{2}$
$+ 0{,}74760ek \sin (r - s)$	9,873165	$+ 141$
$- 0{,}61850ek \sin (r + s)$	9,791317	$- 118$
$+ 0{,}81430ek \sin (2\eta - r + s)$	9,910800	$+ 154$
$- 0{,}01420ek \sin (2\eta + r - s)$	8,150690	$- 3$
$+ 0{,}23960ek \sin (2\eta - r - s)$	9,379550	$+ 45$
$- 0{,}00610ek \sin (2\eta + r + s)$	7,788910	$- 1$
$- 0{,}24427v \sin \eta$	9,387868	$- 175$
$+ 0{,}00639v \sin 3\eta$	7,805991	$+ 4$
$+ 1{,}1959vk \sin (\eta - r)$	0,077694	$+ 59$
$+ 0{,}0757vk \sin (\eta + r)$	8,879096	$+ 4$
$- 0{,}3959ve \sin (\eta - s)$	9,597508	$- 5$
$- 4{,}1738ve \sin (\eta + s)$	0,620530	$- 49$

CAPUT

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CAPUT XII.

INUESTIGATIO INAEQUALITATUM MOTUM LINEAE NODORUM AFFICIENTIUM.

§. 206.

Antequam reliquas motus Lunae inaequalitates, quae ab inclinatione eius orbitae ad eclipticam pendent, definire liceat, cum variationes, quae in motu lineae nodorum Lunae, tum eas, quae in ipsa inclinatione eius orbitae ad eclipticam deprehenduntur, inuestigari oportet. Residua enim pars aequationis nostrae principalis, qua omnes motus Lunae inaequalitates continentur, litteras π et ϱ implicat, quarum illa longitudinem nodi ascendentis, haec vero ϱ inclinationem ad eclipticam designat. Nisi igitur vtriusque huius quantitatis incrementa vel decrementa ad differentiale dr reduxerimus, residuas motus Lunae inaequalitates determinare non poterimus.

207. Aequatio autem supra (55) pro motu lineae nodorum tradita, cum sit $\frac{2nv+\int R\,dr}{nn} = \alpha + \frac{1+2ee}{nn} - \frac{d\Phi}{dr}$

ideoque $\frac{2nv+\int R\,dr}{nn} = \alpha' \cos 2\eta - (\frac{2}{n} - c')k\cos r + d'k\cos(2\eta - r)$
$+ e'k\cos(2\eta + r)$
$+ \varrho' e\cos s + q'e\cos(2\eta - s)$
$+ r'e\cos(2\eta + s)$

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si ponamus breuitatis gratia $\frac{3(1+2kk+\frac{2}{3}ee)}{nn} = i$, vt sit $i = 0,0168918$, induet formam sequentem:

$$\frac{d\pi}{dr} = -i\left(n + \alpha' \cos 2\eta - (\tfrac{2}{n} - c')k\cos r + \delta' k\cos(2\eta - r) + \text{etc.} + \varepsilon' k\cos(2\eta + r)\right)$$
$$\left(1 + 4k\cos r + 5kk\cos 2r - 6ek\cos(r-s) - 3e\cos s + \tfrac{3}{2}ee\cos 2s - 6ek\cos(r+s)\right)\left(\tfrac{3}{4} + \tfrac{1}{4}\cos 2\eta - \tfrac{3}{4}\cos(2\varphi - 2\pi) - \tfrac{3}{4}\cos(2\theta - 2\pi)\right)$$
$$-\frac{nnv}{4}\left(\tfrac{11}{4}\sin\eta + \tfrac{5}{4}\sin 3\eta - \tfrac{3}{2}\sin(\varphi+\theta-2\pi) - \tfrac{5}{4}\cos(3\varphi - \theta - 2\pi) - \tfrac{5}{4}\cos(3\theta - \varphi - 2\pi)\right)$$

vbi quidem plurimi termini tam fiunt parui, vt facile negligi queant.

§. 208. Productum autem ex duobus prioribus factoribus, quoniam id in formula pro inclinatione recurrit, seorsim exhibeamus: fiet id autem reiectis terminis, qui prae reliquis admodum sunt parui vt sequitur:

$$\begin{array}{l|l|l}
-ni + 2i(\frac{2}{n} - c')kk - \frac{3}{2}ip'ee & = & -0,017043 \\
\cos 2\eta \quad (-ia' - 2id'kk & & +0,000161\cos 2\eta \\
k\cos r \quad (i(\frac{2}{n} - c') - 4ni & & -0,068110\, k\cos r \\
k\cos(2\eta - r) \quad (-id' - 2ia' & & -0,005791\, k\cos(2\eta - r) \\
k\cos(2\eta + r) \quad (-ie' - 2ia' & & +0,000828\, k\cos(2\eta + r) \\
kk\cos 2r \quad (-5ni + 2i(\frac{2}{n} - c') & & -0,085091\, kk\cos 2r \\
e\cos s \quad (-ip' + 3ni & & +0,051362\, e\cos s \\
e\cos(2\eta - s) \quad (iq' + \frac{3}{2}ia' & & -0,000929\, e\cos(2\eta - s) \\
e\cos(2\eta + s) \quad (-ir' + \frac{3}{2}ia' & & -0,000804\, e\cos(2\eta + s) \\
ee\cos 2s \quad (-\frac{3}{2}ni + \frac{3}{2}ip' & & -0,025914\, ee\cos 2s
\end{array}$$

§. 209.

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§. 209. His valoribus substitutis prodibit

$$\frac{d\pi}{dr} = -0{,}004261 + 0{,}000020$$

$\cos 2\eta$ $(+0{,}000040 - 0{,}004261$

$k \cos r$ $(-0{,}017043 - 0{,}000620$

$k \cos(2\eta - r)$ $(-0{,}001448 - 0{,}008514) - 0{,}010636 kk \cos(2\eta - 2r)$

$k \cos(2\eta + r)$ $(+0{,}000207 - 0{,}008514) - 0{,}010636 kk \cos(2\eta + 2r)$

$kk \cos 2r$ $(-0{,}021273$

$e \cos s$ $(+0{,}012840 - 0{,}000217$

$e \cos(2\eta - s)$ $(-0{,}000232 + 0{,}006420) - 0{,}003239 ee \cos(2\eta - 2s)$

$e \cos(2\eta + s)$ $(-0{,}000201 + 0{,}006420) - 0{,}003239 ee \cos(2\eta + 2s)$

$ee \cos 2s$ $(-0{,}006479)$

$\cos 2(\Phi - \pi)$ $(+0{,}003261 - 0{,}000020) - 0{,}000041 \cos\eta$

$+ 0{,}000019 \cos(3\Phi - \theta - 2\pi)$

$\cos 2(\theta - \pi)$ $(+0{,}004261 - 0{,}000020) - 0{,}000019 \cos 3\eta$

$+ 0{,}000019 \cos(3\theta - \Phi - 2\pi)$

$k \cos(2\Phi - 2\pi - r)$ $(+0{,}008514 + 0{,}000724)$

$+ 0{,}000022 \cos(\Phi + \theta - 2\pi)$

$k \cos(2\Phi - 2\pi + r)$ $(+0{,}008514 - 0{,}000103)$

$+ 0{,}010636 kk \cos 2(\Phi - r - \pi)$

$k \cos(2\theta - 2\pi - r)$ $(+0{,}008514 - 0{,}000103)$

$k \cos(2\theta - 2\pi + r)$ $(+0{,}008514 + 0{,}000724)$

$e \cos(2\Phi - 2\pi - s)$ $(-0{,}006420 + 0{,}000116)$

$e \cos(2\Phi - 2\pi + s)$ $(-0{,}006420 + 0{,}000100)$

$e \cos(2\theta - 2\pi - s)$ $(-0{,}006420 + 0{,}000100)$

$+ 0{,}003239 ee \cos 2(\theta - s - \pi)$

$e \cos(2\theta - 2\pi + s)$ $(-0{,}006420 + 0{,}000116)$

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§. 210. Habebimus ergo

$$\frac{d\pi}{dr} = -0{,}004241 - 0{,}000041\cos 2\eta$$
$$-0{,}004221\cos 2\eta \quad -0{,}000019\cos 3\eta$$
$$-0{,}017663\,k\cos r \quad +0{,}000020\cos 4\eta$$
$$-0{,}009962\,k\cos(2\eta - r) \quad +0{,}004241\cos(2\varphi - 2\pi)$$
$$-0{,}008307\,k\cos(2\eta + r) \quad +0{,}004241\cos(2\theta - 2\pi)$$
$$-0{,}010636\,kk\cos(2\eta - 2r) \quad +0{,}000022\cos(\varphi + \theta - 2\pi)$$
$$-0{,}010636\,kk\cos(2\eta + 2r) \quad +0{,}000019\cos(3\varphi - \theta - 2\pi)$$
$$-0{,}021273\,kk\cos r \quad +0{,}000019\cos(3\theta - \varphi - 2\pi)$$
$$+0{,}012623\,e\cos s$$
$$+0{,}006188\,e\cos(2\eta - s)$$
$$+0{,}006219\,e\cos(2\eta + s)$$
$$-0{,}006479\,ee\cos 2s$$
$$-0{,}003239\,ee\cos(2\eta - 2s)$$
$$-0{,}003239\,ee\cos(2\eta + 2s)$$
$$+0{,}009238\,k\cos(2\varphi - 2\pi - r) \quad -0{,}006304\,e\cos(2\varphi - 2\pi - s)$$
$$+0{,}008411\,k\cos(2\varphi - 2\pi + r) \quad -0{,}006320\,e\cos(2\varphi - 2\pi + s)$$
$$+0{,}008411\,k\cos(2\theta - 2\pi - r) \quad -0{,}006320\,e\cos(2\theta - 2\pi - s)$$
$$+0{,}009238\,k\cos(2\theta - 2\pi + r) \quad -0{,}006304\,e\cos(2\theta - 2\pi + s)$$
$$+0{,}010636\,kk\cos(2\varphi - 2\pi - 2r) \quad +0{,}003239\,ee\cos(2\theta - 2\pi - 2s)$$

§. 211. Quanquam plurimi horum terminorum tam ſunt parui, vt in ſe ſpectati tuto reiici poſſent, tamen quidam per integrationem ad magnitudinem ſatis notabilem excreſcere poſſunt. Huius autem indolis ſunt illi termini, qui eiusmodi complectuntur angulos, quorum

rum differentialia ad dr admodum paruam tenent rationem, cuiusmodi sunt anguli s, $2s$, $2\theta-2\pi$, $2\theta-2\pi-s$, $2\theta-2\pi+s$, $2\Phi-2\pi-2r$ et $2\theta-2\pi-2s$; quorum natura differentialium ex sequentibus formulis colligi potest:

$$\frac{d\eta}{dr} = \alpha - \alpha' \cos 2\eta - c'k \cos r - d'k \cos(2\eta - r) - e'k \cos(2\eta + r)$$
$$+ \left(\frac{2}{n} - p'\right) e \cos s - q'e \cos(2\eta - s) - r'e \cos(2\eta + s)$$

$$\frac{d\Phi}{dr} = \alpha + \frac{1+2ee}{n} - \alpha' \cos 2\eta + \left(\frac{2}{n} - c'\right) k \cos r$$
$$- d'k \cos(2\eta - r) - p'e \cos s - q'e \cos(2\eta - s)$$
$$- e'k \cos(2\eta + r) \qquad - r'e \cos(2\eta + s)$$

$$\frac{ds}{dr} = \frac{d\theta}{dr} = \frac{1+2ee}{n} + \frac{2}{n} k \cos r - \frac{2}{n} e \cos s$$

§. 212. Quaeramus primo inaequalitates motus nodorum, quae neque ab excentricitate orbitae lunaris neque solaris pendent, sitque:

$$\pi = \text{Const.} - Or + \mathfrak{A} \sin 2\eta + \mathfrak{B} \sin(2\Phi - 2\pi) + \mathfrak{C} \sin(2\theta - 2\pi)$$

reiectis reliquis terminis, quos praeuidemus fore minimos, ac differentiando obtinebimus:

$$\frac{d\pi}{dr} = -O - \mathfrak{A}\alpha' - 0{,}004241\,\mathfrak{B} - 0{,}004241\,\mathfrak{C}$$
$$+ \cos 2\eta \,(2\alpha\mathfrak{A} - 0{,}004241\,\mathfrak{B} - 0{,}004241\,\mathfrak{C}$$
$$+ \cos(2\Phi - 2\pi)\,(2(\alpha + \tfrac{1}{n})\mathfrak{B} + 0{,}008482\,\mathfrak{B} + 0{,}004221\,\mathfrak{C}$$
$$+ \cos(2\theta - 2\pi)\,(-\mathfrak{B}\alpha' + 0{,}004221\,\mathfrak{B} + \frac{2\mathfrak{C}}{n} + 0{,}008482\,\mathfrak{C}$$

vnde

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vnde oritur:

$$0 - 0{,}019744\,\mathfrak{A} + 0{,}004241\,\mathfrak{B} + 0{,}004241\,\mathfrak{C} = 0{,}004241$$
$$1{,}867476\,\mathfrak{A} - 0{,}004241\,\mathfrak{B} - 0{,}004241\,\mathfrak{C} = -0{,}004221$$
$$2{,}026834\,\mathfrak{B} + 0{,}004221\,\mathfrak{C} = 0{,}004241$$
$$0{,}023965\,\mathfrak{B} + 0{,}159358\,\mathfrak{C} = 0{,}004241$$

§. 213. Valores hinc igitur prodibunt sequentes:

$$\mathfrak{O} = +0{,}004078 \ldots l\,\mathfrak{O} = 7{,}610447$$
$$\mathfrak{A} = -0{,}002196 \ldots l\,-\mathfrak{A} = 7{,}341634$$
$$\mathfrak{B} = +0{,}002037 \ldots l\,\mathfrak{B} = 7{,}308991$$
$$\mathfrak{C} = +0{,}026307 \ldots l\,\mathfrak{C} = 8{,}420081$$

Vbi primum obseruo valorem ipsius O iam proxime accedere ad motum medium lunae nodorum, vti per obseruationes constat; inde enim esse deberet $\mathfrak{O} = 0{,}004053$; facile autem intelligitur, hunc exiguum defectum per reliquas inaequalitates suppleri posse. Quocirca hinc erit

$\pi =$ Const.	Valores in min. sec.
$-0{,}004078\,r$	
$-0{,}002196 \sin 2\eta$	$-\ 453''$
$+0{,}002037 \sin(2\phi - 2\pi)$	$+\ 420$
$+0{,}026307 \sin(2\theta - 2\pi)$	$+\ 5426$

quae inaequalitates mirifice conueniunt cum obseruationibus. His addi potest terminus:

$$+0{,}000336 \sin(4\theta - 4\pi)$$

cuius in minutis secundis valor est $+\ 69''$, qui terminus cum postremo illo facile coniungi potest.

§. 214. Quaeramus iam seorsim inaequalitates, quae ab excentricitate orbitae lunaris pendent, sitque

$\pi =$

$$\pi = \text{Const.} - Or + \mathfrak{A}\sin 2\eta + \mathfrak{B}\sin(2\varphi - 2\pi) + \mathfrak{C}\sin(2\vartheta - 2\pi)$$
$$+ \mathfrak{D}k\sin(2\eta - r) + \mathfrak{E}k\sin(2\eta + r) + \mathfrak{F}k\sin r$$
$$+ \mathfrak{G}kk\sin(2\eta - 2r) + \mathfrak{H}kk\sin 2r + \mathfrak{J}k\sin(2\varphi - 2\pi - r)$$
$$+ \mathfrak{K}k\sin(2\varphi - 2\pi + r) + \mathfrak{L}k\sin(2\vartheta - 2\pi - r) + \mathfrak{M}k\sin(2\vartheta - 2\pi + r)$$
$$+ \mathfrak{N}kk\sin(2\varphi - 2\pi - 2r)$$

eritque differentiando

$$\frac{d\pi}{dr} = \text{Pr.} + k\cos(2\eta - r)(-\mathfrak{A}c' - 0{,}009238\,\mathfrak{B} - 0{,}009238\,\mathfrak{C} + (2a-1)\mathfrak{D}$$
$$+ k\cos(2\eta + r)(-\mathfrak{A}c' - 0{,}008411\,\mathfrak{B} - 0{,}008411\,\mathfrak{C} + (2a+1)\mathfrak{E}$$
$$+ k\cos r(-\mathfrak{A}d' - \mathfrak{A}c' - 0{,}017649\,\mathfrak{B} - 0{,}017649\,\mathfrak{C} + \mathfrak{F} - \mathfrak{D}c' - \mathfrak{E}c'$$
$$+ kk\cos 2r\ (-0{,}010636\,\mathfrak{B} + 2\mathfrak{H} - \mathfrak{D}c' - \mathfrak{E}d' - \mathfrak{G}c'$$
$$+ kk\cos(2\eta - 2r)(-0{,}010636\,\mathfrak{C} + 2(a-1)\mathfrak{G} - \mathfrak{D}c'$$

$$+ k\cos(2\vartheta - 2\pi - r)\left\{\begin{array}{l} -\mathfrak{B}e' + 0{,}008307\,\mathfrak{B} + \frac{2}{n}\mathfrak{C} + 0{,}017663\,\mathfrak{C} \\ \qquad + (\frac{2}{n} - 1)\,\mathfrak{L} + 0{,}008482\,\mathfrak{L} \end{array}\right.$$

$$+ k\cos(2\vartheta - 2\pi + r)\left\{\begin{array}{l} -\mathfrak{B}d' + 0{,}009962\,\mathfrak{B} + \frac{2}{n}\mathfrak{C} + 0{,}017663\,\mathfrak{C} \\ \qquad + (\frac{2}{n} + 1)\,\mathfrak{M} + 0{,}008482\,\mathfrak{M} \end{array}\right.$$

$$+ k\cos(2\varphi - 2\pi - r)\left\{\begin{array}{l} + 0{,}017663\,\mathfrak{B} + 0{,}009962\,\mathfrak{C} + 2(a + \frac{1}{n})\mathfrak{J} \\ \qquad - \mathfrak{J} + 0{,}008482\,\mathfrak{J} \end{array}\right.$$

$$+ k\cos(2\varphi - 2\pi + r)\left\{\begin{array}{l} + 0{,}017663\,\mathfrak{B} + 0{,}008307\,\mathfrak{C} + 2(a + \frac{1}{n})\mathfrak{K} \\ \qquad + \mathfrak{K} + 0{,}008482\,\mathfrak{K} \end{array}\right.$$

$$+ kk\cos(2\varphi - 2\pi - 2r)\left\{\begin{array}{l} + 0{,}021273\,\mathfrak{B} + 0{,}010636\,\mathfrak{C} + 2(a + \frac{1}{n})\mathfrak{N} \\ \qquad - 2\mathfrak{N} + 0{,}008482\,\mathfrak{N} \end{array}\right.$$

§. 215. Superfluum foret maiorem curam in his differentialibus adhibere, quia vero proxime tantum rem determinare sufficit; erit ergo:

$$0,867476\,\mathfrak{D} = -0,009962$$
$$2,867476\,\mathfrak{E} = -0,008307$$

Hincque

	et in min sec.
$\mathfrak{D} = -0,011480$	$\mathfrak{D}k = -129''$
$\mathfrak{E} = -0,002900$	$\mathfrak{E}k = -33''$

quae inaequalitates in loco nodi vix alicuius sunt momenti, vnde eas exactius determinare non est opus.

§. 216. Calculo autem euoluto erit

$\pi =$ Pr. $-$	$0,011480k$	$\sin(2\eta - r)$	$8,059940$	$-129''$
$-$	$0,002900k$	$\sin(2\eta + r)$	$7,462400$	-33
$-$	$0,017663k$	$\sin r$	$8,247064$	-198
$+$	$0,090497kk$	$\sin(2\eta - 2r)$	$8,956634$	$+55$
$-$	$0,011978kk$	$\sin 2r$	$8,078384$	-7
$+$	$0,008707k$	$\sin(2\Phi - 2\pi - r)$	$7,939851$	$+98$
$+$	$0,002701k$	$\sin(2\Phi - 2\pi + r)$	$7,431516$	$+30$
$-$	$0,004680k$	$\sin(2\theta - 2\pi - r)$	$7,670224$	-53
$+$	$0,004685k$	$\sin(2\theta - 2\pi + r)$	$7,670680$	$+53$
$+$	$0,384848kk$	$\sin(2\Phi - 2\pi - 2r)$	$9,585289$	$+235$

§. 217. Simili modo inuestigemus inaequalitates motus nodorum, quae pendent ab excentricitate orbitae solaris sitque:

$\pi =$

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$$\pi = \text{Const.} - 0r + \mathfrak{A}\sin 2\eta + \mathfrak{B}\sin(2\Phi - 2\pi) + \mathfrak{C}\sin(2\theta - 2\pi)$$
$$+ \mathfrak{D}e\sin s + \mathfrak{E}e\sin(2\eta - s) + \mathfrak{F}e\sin(2\eta + s)$$
$$+ \mathfrak{G}ee\sin 2s + \mathfrak{H}e\sin(2\Phi - 2\pi - s) + \mathfrak{K}e\sin(2\theta - 2\pi - s)$$
$$+ \mathfrak{J}e\sin(2\Phi - 2\pi + s) + \mathfrak{L}e\sin(2\theta - 2\pi + s)$$
$$+ \mathfrak{M}ee\sin(2\theta - 2\pi - 2s)$$

vnde differentiando pro terminis quaesitis erit:

$$\frac{d\pi}{dr} = \text{Praec.}$$

$$+ e\cos s \left\{ \begin{array}{l} -\mathfrak{A}q' - \mathfrak{A}r' + 0{,}006304\mathfrak{B} + 0{,}006320\mathfrak{C} + \frac{1}{\pi}\mathfrak{D} - \mathfrak{E}a' - \mathfrak{F}a' \\ + 0{,}006320\mathfrak{B} + 0{,}006304\mathfrak{C} \end{array} \right.$$

$$+ e\cos(2\eta - s)\left\{ \mathfrak{A}\left(\frac{2}{\pi} - p'\right) + 0{,}006304\mathfrak{B} + 0{,}006304\mathfrak{C} + \left(2\alpha - \frac{1}{\pi}\right)\mathfrak{E} \right.$$

$$+ e\cos(2\eta + s)\left\{ \mathfrak{A}\left(\frac{2}{\pi} - p'\right) + 0{,}006320\mathfrak{B} + 0{,}006320\mathfrak{C} + \left(2\alpha + \frac{1}{\pi}\right)\mathfrak{F} \right.$$

$$+ ee\cos 2s \left\{ \begin{array}{l} -0{,}003239\,\mathfrak{C} - \frac{1}{\pi}\mathfrak{D} - \mathfrak{E}r' - \mathfrak{F}q' + \frac{2}{\pi}\mathfrak{G} \end{array} \right.$$

$$+ e\cos(2\Phi - 2\pi - s) \left\{ \begin{array}{l} -\mathfrak{B}p' - 0{,}012623\,\mathfrak{B} - 0{,}006188\,\mathfrak{C} \\ + \left(2\alpha \pm \frac{1}{\pi} + 0{,}098482\right)\mathfrak{H} \end{array} \right.$$

$$+ e\cos(2\Phi - 2\pi + s) \left\{ \begin{array}{l} -\mathfrak{B}p' - 0{,}012623\,\mathfrak{B} - 0{,}006219\,\mathfrak{C} \\ + \left(2\alpha + \frac{3}{\pi} + 0{,}098482\right)\mathfrak{J} \end{array} \right.$$

$$+ e\cos(2\theta - 2\pi - s)\begin{cases} - \mathfrak{B}r' - 0{,}006219\mathfrak{B} - \frac{2}{n}\mathfrak{C} - 0{,}012623\mathfrak{C} \\ + (\frac{1}{n} + 0{,}008482)\mathfrak{K} \end{cases}$$

$$+ e\cos(2\theta - 2\pi + s)\begin{cases} - \mathfrak{B}q' - 0{,}006188\mathfrak{B} - \frac{2}{n}\mathfrak{C} - 0{,}012623\mathfrak{C} \\ + (\frac{3}{n} + 0{,}008482)\mathfrak{L} \end{cases}$$

$$+ ee\cos(2\theta - 2\pi - 2s)\begin{cases} + 0{,}003239\mathfrak{B} + \frac{1}{2n}\mathfrak{C} + 0{,}006479\mathfrak{C} \\ + 0{,}008482\mathfrak{M} - \frac{1}{n}\mathfrak{K} - 0{,}012623\mathfrak{K} \end{cases}$$

§. 218. Hinc reperiuntur sequentes valores

$\mathfrak{D} = 0{,}159070$	$l\,\mathfrak{D} = 9{,}201585$	;	$\mathfrak{D}e = 551''$
$\mathfrak{C} = 0{,}003562$	$l\,\mathfrak{C} = 7{,}551680$	;	$\mathfrak{C}e = 12\frac{1}{2}$
$\mathfrak{F} = 0{,}003301$	$l\,\mathfrak{F} = 7{,}518677$	;	$\mathfrak{F}e = 11\frac{1}{2}$
$\mathfrak{G} = 0{,}031650$	$l\,\mathfrak{G} = 8{,}587191$	;	$\mathfrak{G}ee = 2''$
$\mathfrak{H} = -0{,}003153$	$l\,\mathfrak{H} = 7{,}498692$	;	$\mathfrak{H}e = -11''$
$\mathfrak{I} = -0{,}002932$	$l\,\mathfrak{I} = 7{,}467118$	;	$\mathfrak{I}e = -10''$
$\mathfrak{K} = -0{,}025750$	$l\,\mathfrak{K} = 8{,}410784$	;	$\mathfrak{K}e = -90$
$\mathfrak{L} = -0{,}009076$	$l\,\mathfrak{L} = 7{,}957885$	;	$\mathfrak{L}e = -32$

At valor ipsius $\mathfrak{M}$ tam sit paruus, vt merito pro nihilo haberi possit.

§. 219.

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§. 219. Colligamus ergo has inaequalitates in vnam ſummam, atque obtinebimus longitudinem veram nodi aſcendentis

		Valor. in minut. ſec.
π = Conſt.	$-0{,}004053\ r$	
	$-0{,}002196 \sin 2\eta$	$-\ 453''$
	$+0{,}002037 \sin(2\Phi-2\pi)$	$+\ 420$
	$+0{,}026307 \sin(2\theta-2\pi)$	$+\ 5426$
	$+0{,}000370 \sin(4\theta-4\pi)$	$+\ 75$
	$-0{,}01766k \sin r$	$-\ 198$
	$-0{,}01148k \sin(2\eta-r)$	$-\ 129$
	$-0{,}00290k \sin(2\eta+r)$	$-\ 33$
	$+0{,}0905kk \sin(2\eta-2r)$	$+\ 55$
	$-0{,}0120kk \sin 2r$	$-\ 7$
	$+0{,}00871k \sin(2\Phi-2\pi-r)$	$+\ 98$
	$+0{,}00270k \sin(2\Phi-2\pi+r)$	$+\ 30$
	$-0{,}00468k \sin(2\theta-2\pi-r)$	$-\ 53$
	$+0{,}00468k \sin(2\theta-2\pi+r)$	$+\ 53$
	$+0{,}3848kk \sin(2\Phi-2\pi-2r)$	$+\ 235$
	$+0{,}15907e \sin s$	$+\ 551$
	$-0{,}02575e \sin(2\theta-2\pi-s)$	$-\ 90$
	$-0{,}00907e \sin(2\theta-2\pi+s)$	$-\ 32$

omiſſis ſcilicet iis inaequalitatibus, quae non ſupra $30''$ exſurgunt.

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CAPUT XIII.

INUESTIGATIO INCLINATIONIS ORBITAE LUNARIS AD ECLIPTICAM.

§. 220.

Pro inclinatione orbitae lunaris ad eclipticam inuenienda, forma §. 208. euoluta multiplicari debet per $-\frac{1}{4}\sin 2\eta + \frac{1}{4}\sin 2(\Phi-\pi) + \frac{1}{4}\sin 2(\theta-\pi)$, ac productum erit $= \frac{d.l\tang\varrho}{dr}$: Hinc ergo habebitur:

$$\frac{d.l\tang\varrho}{dr} = +0{,}004261\sin 2\eta - 0{,}000020\sin 4\eta$$
$$+0{,}008514k\sin(2\eta-r)$$
$$+0{,}008514k\sin(2\eta+r)$$
$$+0{,}000827k\sin r$$
$$-0{,}004261\sin(2\Phi-2\pi) \text{ adiice}$$
$$-0{,}004261\sin(2\theta-2\pi) \text{ ad coeff.}$$
$$-0{,}008514k\sin(2\Phi-2\pi-r) - 0{,}000724$$
$$-0{,}008514k\sin(2\Phi-2\pi+r) + 0{,}000103$$
$$-0{,}008514k\sin(2\theta-2\pi-r) + 0{,}000103$$
$$-0{,}008514k\sin(2\theta-2\pi+r) - 0{,}000724$$
$$-0{,}010636kk\sin(2\Phi-2\pi-2r)$$
$$+0{,}006420e\sin(2\theta-2\pi-s) - 0{,}000100$$
$$+0{,}006420e\sin(2\theta-2\pi+s) - 0{,}000116$$

§. 221.

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§. 221. Quaeramus primo terminos, qui a neutra excentricitate pendent, ſitque

$$l\frac{\text{tang}\,\varrho}{\text{tang}\,\varepsilon} = \mathfrak{A}\cos 2\eta + a\cos 4\eta + \mathfrak{B}\cos(2\Phi - 2\pi) + \mathfrak{C}\cos(2\theta - 2\pi) + c\cos(4\theta - 4\pi)$$

eritque differentiando:

$$\frac{d\,l\text{tang}\varrho}{dr} = \sin 2\,[-2a\mathfrak{A} + 0{,}004241\,\mathfrak{B} - 0{,}004241\,\mathfrak{C}$$
$$\sin 4\eta\,[-4aa + \mathfrak{A}a'$$
$$\sin(2\Phi - 2\pi)\left\{-2(a+\frac{1}{n})\mathfrak{B} - 0{,}008482\mathfrak{B} - 0{,}004227\mathfrak{C}\right.$$
$$\sin(2\theta - 2\pi)\left\{-\frac{2}{n}\mathfrak{C} + \mathfrak{B}a' - 0{,}004221\mathfrak{B} - 0{,}008482\mathfrak{C}\right.$$
$$\sin(4\theta - 4\pi)\left\{+ 0{,}004241\,\mathfrak{C} - \frac{4}{n}c\right.$$

§. 222. Ex his iam reperitur:

$\mathfrak{A} = -0{,}002630 \ldots l\text{-}\mathfrak{A} = 7{,}419914$
$\mathfrak{B} = +0{,}002037 \ldots l\,\mathfrak{B} = 7{,}308991$
$\mathfrak{C} = +0{,}026307 \ldots l\,\mathfrak{C} = 8{,}420081$
$a = +0{,}000019 \ldots l\,a = 5{,}278753$
$c = +0{,}000370 \ldots l\,c = 6{,}567931$

ita vt hinc ſit:

$$l\frac{\text{tang}\,\varrho}{\text{tang}\,\varepsilon} = -0{,}002630\cos 2\eta + 0{,}000019\cos 4\eta + 0{,}002037\cos(2\Phi - 2\pi) + 0{,}026307\cos(2\theta - 2\pi) + 0{,}000370\cos(4\theta - 4\pi)$$

§. 223.

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§. 223. Quaeramus iam seorsim terminos ab excentricitate Lunae pendentes: sitque

$$l\frac{\text{tang}\varrho}{\text{tang}\varepsilon} = \mathfrak{A}\cos 2\eta + \mathfrak{B}\cos(2\Phi-2\pi) + \mathfrak{C}\cos(2\vartheta-2\pi)$$
$$+\mathfrak{D}k\cos(2\eta-r) + \mathfrak{E}k\cos(2\eta+r) + \mathfrak{F}k\cos r$$
$$+\mathfrak{G}k\cos(2\Phi-2\pi-r) + \mathfrak{J}k\cos(2\vartheta-2\pi-r)$$
$$+\mathfrak{H}k\cos(2\Phi-2\pi+r) + \mathfrak{K}k\cos(2\vartheta-2\pi+r)$$
$$+\mathfrak{L}k^2\cos(2\Phi-2\pi-2r)$$

vnde differentialibus sumendis habebitur: $\frac{d.l\text{tang}\varrho}{dr} =$

$$k\sin(2\eta-r)\left\{\begin{array}{l}+\mathfrak{A}c' + 0{,}09238\mathfrak{B} - 0{,}009238\mathfrak{C} + 0{,}004241\mathfrak{G}\\ +(2\alpha-1)\mathfrak{D} - 0{,}004241\mathfrak{K}\end{array}\right.$$

$$k\sin(2\eta+r)\left\{\begin{array}{l}+\mathfrak{A}c' + 0{,}008411\mathfrak{B} - 0{,}008411\mathfrak{C} + (2\alpha+1)\mathfrak{E}\\ +0{,}004241\mathfrak{H} - 0{,}004241\mathfrak{J}\end{array}\right.$$

$$k\sin r\left\{\begin{array}{l}+\mathfrak{A}d' - \mathfrak{A}c' + 0{,}000827\mathfrak{B} - 0{,}000827\mathfrak{C} - \mathfrak{F} - 0{,}004241\mathfrak{G}\\ +0{,}004241\mathfrak{H} - \mathfrak{D}d' + \mathfrak{E}d' - 0{,}004241\mathfrak{J} + 0{,}004241\mathfrak{K}\end{array}\right.$$

$$k\sin(2\Phi-2\pi-r)\left\{\begin{array}{l}-0{,}017663\mathfrak{B} - 0{,}009962\mathfrak{C} - 2(\alpha+\frac{1}{n})\mathfrak{G} + \mathfrak{G}\\ -0{,}008482\mathfrak{G} - 0{,}004221\mathfrak{J}\end{array}\right.$$

$$k\sin(2\Phi-2\pi+r)\left\{\begin{array}{l}-0{,}017663\mathfrak{B} - 0{,}008307\mathfrak{C} - 2(\alpha+\frac{1}{n})\mathfrak{H} - \mathfrak{H}\\ -0{,}008482\mathfrak{H} - 0{,}004221\mathfrak{K}\end{array}\right.$$

$$k\sin(2\vartheta-2\pi-r)\left\{\begin{array}{l}+\mathfrak{B}e' - 0{,}008307\mathfrak{B} - \frac{2}{n}\mathfrak{C} - 0{,}017663\mathfrak{C}\\ +\mathfrak{G}e' - 0{,}004221\mathfrak{G} - \frac{2}{n}\mathfrak{J} + \mathfrak{J} - 0{,}008482\mathfrak{J}\end{array}\right.$$

k sin

$$k\sin(2\theta-2\pi+r)\begin{cases}+\mathfrak{B}d'-0{,}009962\,\mathfrak{B}-\frac{2}{n}\mathfrak{C}-0{,}017663\,\mathfrak{C}\\+\mathfrak{H}a'-0{,}004221\,\mathfrak{H}-\frac{2}{n}\mathfrak{K}-\mathfrak{K}-0{,}008482\,\mathfrak{K}\end{cases}$$

$$kk\sin(2\phi-2\pi-2r)\begin{cases}-0{,}021273\,\mathfrak{B}-0{,}010636\,\mathfrak{C}-2(\alpha+\frac{1}{n})\,\mathfrak{L}+2\,\mathfrak{L}\\-0{,}008482\,\mathfrak{L}\end{cases}$$

hincque reperitur:

$\mathfrak{D}=$	$0{,}010487$	$l\,\mathfrak{D}=8{,}020638$
$\mathfrak{E}=$	$0{,}003166$	$l\,\mathfrak{E}=7{,}500439$
$\mathfrak{F}=$	$-0{,}001600$	$l\,\mathfrak{F}=7{,}204120$
$\mathfrak{G}=$	$+0{,}008719$	$l\,\mathfrak{G}=7{,}940484$
$\mathfrak{H}=$	$+0{,}002699$	$l\,\mathfrak{H}=7{,}431136$
$\mathfrak{J}=$	$-0{,}004460$	$l\,\mathfrak{J}=7{,}649305$
$\mathfrak{K}=$	$+0{,}004717$	$l\,\mathfrak{K}=7{,}623628$
$\mathfrak{L}=$	$+0{,}384890$	$l\,\mathfrak{L}=9{,}585335$

§. 224. Nunc denique pro inaequalitatibus ab excentricitate orbitae solaris pendentibus ponatur.

$$l\,\frac{\tang\varrho}{\tang\varepsilon}=\mathfrak{A}\cos 2\eta+\mathfrak{B}\cos(2\phi-2\pi)+\mathfrak{M}e\cos(2\theta-2\pi-s)$$
$$+\mathfrak{C}\cos(2\theta-2\pi)+\mathfrak{N}e\cos(2\theta-2\pi+s)$$

ac differentiando prodibit: $\frac{d.\,l\tang\varrho}{dr}=$

$$e\sin(2\theta-2\pi-s)\begin{cases}+\mathfrak{B}r'+0{,}006219\,\mathfrak{B}+\frac{2}{n}\mathfrak{C}+0{,}012623\,\mathfrak{C}\\-\frac{1}{n}\mathfrak{M}-0{,}008482\,\mathfrak{M}\end{cases}$$

$$e\sin(2\theta-2\pi+s)\left\{\begin{array}{l}+\mathfrak{B}q'+0{,}06188\,\mathfrak{B}+\frac{2}{n}\mathfrak{C}+0{,}012623\,\mathfrak{C}\\ \quad-\frac{3}{n}\mathfrak{N}-0{,}008482\,\mathfrak{N}\end{array}\right.$$

vnde reperitur

$$\mathfrak{M}=-0{,}024034 \quad . \quad . \quad l-\mathfrak{M}=8{,}380835$$

$$\mathfrak{N}=-0{,}008519 \quad . \quad . \quad l-\mathfrak{N}=7{,}930332$$

§. 225. Si ergo ε denotet inclinationem mediam orbitae lunaris ad eclipticam, et ϱ inclinationem veram, erit

			log. coeff.
$l\,\frac{\tang\varrho}{\tang\varepsilon}=$	$-0{,}002630$	$\cos 2\eta$	7,419915
	$+0{,}000019$	$\cos 4\eta$	5,278753
	$+0{,}002037$	$\cos(2\Phi-2\pi)$	7,308991
	$+0{,}026307$	$\cos(2\theta-2\pi)$	8,420081
	$+0{,}000370$	$\cos(4\theta-4\pi)$	6,567931
	$+0{,}01049k$	$\cos(2\eta-r)$	8,020638
	$+0{,}00317k$	$\cos(2\eta+r)$	7,500439
	$-0{,}00160k$	$\cos r$	7,204120
	$+0{,}00872k$	$\cos(2\Phi-2\pi-r)$	7,940484
	$+0{,}00270k$	$\cos(2\Phi-2\pi+r)$	7,431136
	$-0{,}00446k$	$\cos(2\theta-2\pi-r)$	7,649305
	$+0{,}00472k$	$\cos(2\theta-2\pi+r)$	7,673628
	$+0{,}3849kk$	$\cos(2\Phi-2\pi-2r)$	9,585335
	$-0{,}02403e$	$\cos(2\theta-2\pi-s)$	8,380835
	$-0{,}00852e$	$\cos(2\theta-2\pi+s)$	7,930332

§. 226.

§. 226. Quodsi iam ponatur $\frac{\text{tang}\,\varphi}{\text{tang}\,\varepsilon} = S$, erit ad numeros ipsos procedendo $\frac{\text{tang}\,\varphi}{\text{tang}\,\varepsilon} = 1 + S + \frac{1}{2}SS$ Hinc igitur negligendo terminos minimos, consequemur:

$$\begin{aligned}
\frac{\text{tang}\,\varphi}{\text{tang}\,\varepsilon} = 1 &- 0{,}002604\ \cos 2\theta \\
&+ 0{,}000020\ \cos 4\theta \\
&+ 0{,}026307\ \cos(2\theta - 2\pi) \\
&+ 0{,}000490\ \cos(4\theta - 4\pi) \\
&- 0{,}001504k\ \cos r \\
&+ 0{,}010426k\ \cos(2\pi - r) \\
&+ 0{,}000317k\ \cos(2\pi + r) \\
&+ 0{,}000885k\ \cos(2\theta - 2\pi - r) \\
&+ 0{,}000274k\ \cos(2\theta - 2\pi + r) \\
&- 0{,}000448k\ \cos(2\theta - 2\pi - \varepsilon) \\
&+ 0{,}000270k\ \cos(3\theta - 2\pi + \varepsilon) \\
&+ 0{,}000810kk\ \cos(2\theta - 2\pi - 2\varepsilon) \\
&- 0{,}02403e\ \cos(2\theta - 2\pi - \varepsilon) \\
&- 0{,}00852e\ \cos(4\theta - 2\pi - \varepsilon)
\end{aligned}$$

§. 227. Cum in aequatione noſtra principali, quae motum Lunae continet, inſit terminus $\frac{\tang \varrho^2}{\tang \varepsilon^2}$, huiusquoque valorem euolui conueniet: erit ergo

$$\frac{\tang \varrho^2}{\tang \varepsilon^2} = 1$$
$$- 0{,}005156 \cos 2\eta + 0{,}000040 \cos 4\eta$$
$$+ 0{,}003938 \cos(2\Phi - 2\pi)$$
$$+ 0{,}652614 \cos(2\theta - 2\pi) + 0{,}001320 \cos(4\theta - 4\pi)$$
$$- 0{,}00290k \cos r$$
$$+ 0{,}02098k \cos(2\eta - r)$$
$$+ 0{,}00634k \cos(2\eta + r)$$
$$+ 0{,}01796k \cos(2\Phi - 2\pi - r)$$
$$+ 0{,}00556k \cos(2\Phi - 2\pi + r)$$
$$- 0{,}00896k \cos(2\theta - 2\pi - r)$$
$$+ 0{,}00940k \cos(2\theta - 2\pi + r)$$
$$+ 0{,}7698kk \cos(2\Phi - 2\pi - 2r)$$
$$+ 0{,}04806e \cos(2\theta - 2\pi - s)$$
$$+ 0{,}01704e \cos(2\theta - 2\pi + s)$$

Hicque ergo valor in ſuperiori illa aequatione ſubſtitui poterit.

§. 228.

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§. 228. Celeb. autem Clairaut concluſit inclinationem mediam ε ex obſeruationibus exquiſitiſſimis $5^\circ\, 8'\, 9''$, ex qua igitur ad quoduis tempus inclinationem veram elicere licebit. Sit enim $\varrho = \varepsilon + \omega$, erit tang $\varrho = \frac{\text{tg}\,\varepsilon + \omega}{1 - \omega\,\text{tg}\,\varepsilon}$ $= \text{tang}\,\varepsilon + \frac{\omega}{\text{cof}\,\varepsilon^2} = V\,\text{tang}\,\varepsilon$, ponendo V pro expreſſione ipſius $\frac{\text{tang}\,\varrho}{\text{tang}\,\varepsilon}$. Hinc erit $\omega = (V-1)\,\text{fin}\,\varepsilon\,\text{cof}\,\varepsilon =$ $\frac{1}{2}(V-1)\,\text{fin}\,2\varepsilon = 0,08915\,(V-1)$: vnde reperitur in minutis ſecundis

$$\begin{aligned}
\varrho = \varepsilon &- 48'' \,\text{cof}\, 2\eta \\
&+ 36 \,\text{cof}\,(2\Phi - 2\pi) \\
&+ 484 \,\text{cof}\,(2\theta - 2\pi) \\
&+ 9 \,\text{cof}\,(4\theta - 4\pi) \\
&- 2 \,\text{cof}\, r \\
&+ 11 \,\text{cof}\,(2\eta - r) \\
&+ 3 \,\text{cof}\,(2\eta + r) \\
&+ 9 \,\text{cof}\,(2\Phi - 2\pi - r) \\
&+ 3 \,\text{cof}\,(2\Phi - 2\pi + r) \\
&- 5 \,\text{cof}\,(2\theta - 2\pi - r) \\
&+ 5 \,\text{cof}\,(2\theta - 2\pi + r) \\
&+ 23 \,\text{cof}\,(2\Phi - 2\pi - 2r) \\
&- 7 \,\text{cof}\,(2\theta - 2\pi - s) \\
&- 3 \,\text{cof}\,(2\theta - 2\pi + s)
\end{aligned}$$

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§. 229. Hic notandum est, etiamsi valor inclinationis mediae i aliquantillum immutetur, aequationes has tamen inde vix alterari, ita vt eae semper eaedem sint mansurae. Perspicuum quoque est in calculo astronomico sufficere tres inaequalitates primores, et reliquas omnes sine errore sensibili praetermitti posse; nisi forte aequatio 23 cos $(2\varphi - 2\pi - 2r)$ retinenda censeatur, quae inter reliquas est maxima. Exprimit autem angulus $2\varphi - 2\pi - 2r$ duplam distantiam apogei Lunae ab eius nodo, a quo angulo quoque locum nodi non mediocriter affici vidimus, cum correctio hinc oriunda pro loco nodi vsque ad $235''$ assurgere possit.

CAPUT

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CAPUT XIV.

INVESTIGATIO INAEQUALITATUM MOTUS LUNAE AB EIUS INCLINATIONE AD ECLIPTICAM ORIUNDARUM.

§. 230.

Ponamus more adhuc vsitato:

$$\int R\,dr = \mathfrak{A}\cos 2\eta + \mathfrak{C}k\cos\gamma + \mathfrak{D}k\cos(2\eta-\gamma) + \mathfrak{P}e\cos\sigma + \mathfrak{Q}e\cos(2\eta-\sigma) + \mathfrak{E}k\cos(2\eta+\gamma) + \mathfrak{R}e\cos(2\eta+\sigma) + \mathfrak{F}f\cos 2\eta + \mathfrak{G}f\cos(2\varphi-2\pi) + \mathfrak{J}fk\cos\gamma + \mathfrak{K}fk\cos(2\eta-\gamma) + \mathfrak{H}f\sin(2\theta-2\pi) + \mathfrak{L}fk\cos(2\eta+\gamma)$$

$$+ \mathfrak{M}fk\cos(2\varphi-2\pi-\gamma) + \mathfrak{S}fk\cos(2\theta-2\pi-\gamma) + \mathfrak{N}fk\cos(2\varphi-2\pi+\gamma) + \mathfrak{T}fk\cos(2\theta-2\pi+\gamma) + \mathfrak{O}fkk\cos(2\varphi-2\pi-2\gamma) + \mathfrak{U}fe\cos(2\theta-2\pi-\sigma) + \mathfrak{V}fe\cos(2\theta-2\pi+\sigma)$$

$$\text{et } v = A\cos 2\eta + Dk\cos(2\eta-\gamma) + Pe\cos\sigma + Qe\cos(2\eta-\sigma) + Ek\cos(2\eta+\gamma) + Re\cos(2\eta+\sigma) + Ff\cos 2\eta + Gf\cos(2\varphi-2\pi) + Jfk\cos\gamma + Kfk\cos(2\eta-\gamma) + Hf\cos(2\theta-2\pi) + Lfk\cos(2\eta+\gamma)$$

$$+ Mfk\cos(2\varphi-2\pi-\gamma) + Sfk\cos(2\theta-2\pi-\gamma) + Nfk\cos(2\varphi-2\pi+\gamma) + Tfk\cos(2\theta-2\pi+\gamma) + Ofkk\cos(2\varphi-2\pi-2\gamma) + Ufe\cos(2\theta-2\pi-\sigma) + Vfe\cos(2\theta-2\pi+\sigma)$$

§. 231.

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§. 231. His valoribus ſubſtitutis in formula §. 52. orietur

$$R = f \sin 2\eta \left(\ldots \quad fk \sin(2\theta - 2\pi - r) \left(-\frac{3M}{2nn} - \frac{3G}{nn} \right.\right.$$

$$f \sin(2\Phi - 2\pi) \left(+\frac{3H}{2nn} \ldots fk \sin(2\theta - 2\pi + r) \left(-\frac{3N}{2nn} - \frac{3G}{nn} \right.\right.$$

$$f \sin(2\theta - 2\pi) \left(-\frac{3G}{2nn} \ldots fe \sin(2\theta - 2\pi - s) \left(+\frac{9G}{4nn} \right.\right.$$

$$fk \sin r \left(+\frac{3K}{2nn} - \frac{3L}{2nn} \quad \ldots fe \sin(2\theta - 2\pi + s) \left(+\frac{9G}{4nn} \right.\right.$$

$$fk \sin(2\eta - r) \left(+\frac{3I}{2nn} \right.$$

$$fk \sin(2\eta - r) \left(+\frac{3I}{2nn} \right.$$

$$fk \sin(2\Phi - 2\pi - r) \left(+\frac{3S}{2nn} + \frac{3H}{nn} \right.$$

$$fk \sin(2\Phi - 2\pi + r) \left(+\frac{3T}{2nn} + \frac{3H}{nn} \right.$$

$$fk \sin(2\Phi - 2\pi - 2r) \left(+\frac{3S}{nn} \right.$$

§. 232. Altera vero aequatio fundamentalis induet formam ſequentem:

$$\frac{ddv}{dt^2} = \text{Praec.} + f \cos 2\eta \, [-\mathfrak{E}F - 2n\mathfrak{F} - 0{,}005156 - 0{,}026307$$
$$+ f \cos(2\Phi - 2\pi) \, [-\mathfrak{E}G - 2n\mathfrak{G} + 0{,}003938 - 1$$
$$+ f \cos(2\theta - 2\pi) \, [-\mathfrak{E}H - 2n\mathfrak{H} + 0{,}052614 + 0{,}002578$$

+

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$+fk\cos r\ [-\mathfrak{C}J-2n\mathfrak{J}-0,00290\ -0,00893$
$-0,00098-0,00278-0,00098$
$+fk\cos(2\eta-r)\ [-\mathfrak{C}K+\tfrac{1}{2}bF-2n\mathfrak{K}+0,02098-0,00470$
$-0,00258-0,01315$
$+fk\cos(2\eta+r)\ [-\mathfrak{C}L+\tfrac{1}{2}bF-2n\mathfrak{L}+0,00634+0,00448$
$-0,00258-0,01315$
$+fk\cos(2\varphi-2\pi-r)\ [-\mathfrak{C}M+\tfrac{1}{2}bG-2n\mathfrak{M}+0,01796+0,00145$
$+0,00197-\tfrac{1}{2}$
$+fk\cos(2\varphi-2\pi+r)\ [-\mathfrak{C}N+\tfrac{1}{2}bG-2n\mathfrak{N}+0,00556+0,00145$
$+0,00197-\tfrac{1}{2}$
$+fkk\cos(2\varphi-2\pi-2r)\ [-\mathfrak{C}O+\tfrac{1}{2}hM+\tfrac{3}{4}G-2n\mathfrak{O}+0,7698+0,0089$
$+0,0010+0,0007-\tfrac{3}{4}$
$+fk\cos(2\theta-2\pi-r)\ [-\mathfrak{C}S+\tfrac{1}{2}bH-2n\mathfrak{S}-0,00896-0,00317$
$+0,02631+0,00129$
$+fk\cos(2\theta-2\pi+r)\ [-\mathfrak{C}T+\tfrac{1}{2}bH-2n\mathfrak{T}+0,00940-0,01049$
$+0,02631+0,00129$
$+fs\cos(2\theta-2\pi-s)\ [-\mathfrak{C}U-2n\mathfrak{U}-0,04806$
$+fs\cos(2\theta-2\pi+s)\ [-\mathfrak{C}V-2n\mathfrak{V}-0,01704$

§. 233. Quoniam manifeſtum eſt, coefficientes F, G, H etc. admodum fore paruos; cum maxima huius generis inaequalitas aliquot minuta prima non excedat, hi iidem coefficientes per nn diuiſi tam euadent parui, vt ſine errore reiici queant. Hoc autem facto quoque litterae germanicae $\mathfrak{J}$, $\mathfrak{G}$, $\mathfrak{H}$ etc. pro nihilo erunt habendae, ex quo ſola poſterior aequatio differentio-differentialis reſoluenda ſupererit; in qua ob eandem rationem terminos ex diuiſione coefficientium per nn oriundos omiſimus, cum in tam operoſo calculo ſufficiat

ciat correctiones inde resultantes proxime saltem determinasse; praesertim cum haec praetermissio vix ad aliquot minuta secunda sit ascensura.

§. 234. Ob eandem rationem licebit in valoribus differentialium $\frac{d\Phi}{dr}$ et $\frac{d\eta}{dr}$ particulas ab inclinatione pendentes negligere, vnde erit: $\frac{dv}{dr} =$

$f \sin 2\eta \; [-2\alpha F = -F']$

$f \sin (2\varphi - 2\pi) \; [-2(\alpha + \frac{1}{n}) G - 0{,}008482\, G = -G']$

$f \sin (2\theta - 2\pi) \; [+Ge' - \frac{2}{n}) H - 0{,}008482\, H = -H']$

$fk \sin r \; [Fd' - Fe' - J = -J']$

$fk \sin (2\eta - r) \; [+Fc' - (2\alpha - 1) K = -K']$

$fk \sin (2\eta + r) \; [+Fc' - (2\alpha + 1) L = -L']$

$fk \sin (2\varphi - 2\pi - r) \; [-2(\alpha + \frac{1}{n}) M + M - 0{,}008482\, M = -M']$

$fk \sin (2\varphi - 2\pi + r) \; [-2(\alpha + \frac{1}{n}) N - N - 0{,}008482\, N = -N']$

$fkk \sin (2\varphi - 2\pi - 2r) \; [-2(\alpha + \frac{1}{n}) O + 2O - 0{,}008482\, O = -O']$

$fk \sin (2\theta - 2\pi - r) \; [+Ge' - \frac{2}{n} H - \frac{2}{n} S + S - 0{,}008482\, S = -S']$

$fk \sin (2\theta - 2\pi + r) \; [+Gd' - \frac{2}{n} H - \frac{2}{n} T - T - 0{,}008482\, T = -T']$

$fe \sin (2\theta - 2\pi - s) \; [+Gr' + \frac{2}{n} H - \frac{1}{n} U - 0{,}008482\, U = -U']$

$fe \sin (2\theta - 2\pi + s) \; [+Gq' + \frac{2}{n} H - \frac{3}{n} V - 0{,}008482\, V = -V']$

§. 235.

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§. 235. Si nunc simili modo denuo differentiemus, prodibit: $\frac{ddv}{dr^2} =$

$f \cos 2\eta \; [-2\alpha F']$

$f \cos(2\varphi - 2\pi) \; [-2(\alpha + \frac{1}{n}) G' - 0{,}008482\, G']$

$f \cos(2\theta - 2\pi) \; [+ G'\omega' - \frac{2}{n} H' - 0{,}008482\, H']$

$fk \cos r \; [F'd' + Fe' - J']$

$fk \cos(2\eta - r) \; [F'e' - (2\alpha - 1) K']$

$fk \cos(2\eta + r) \; [F'c' - (2\alpha + 1) L']$

$fk \cos(2\varphi - 2\pi - r) \; [-2(\alpha + \frac{1}{n}) M' + M' - 0{,}008482\, M']$

$fk \cos(2\varphi - 2\pi + r) \; [-2(\alpha + \frac{1}{n}) N' - N' - 0{,}008482\, N']$

$fkk \cos(2\varphi - 2\pi - 2r) \; [-2(\alpha + \frac{1}{n}) O' + 2O' - 0{,}008482\, O']$

$fk \cos(2\theta - 2\pi - r) \; [G'e' - \frac{2}{n} H' - \frac{2}{n} S' + S' - 0{,}008482\, S']$

$fk \cos(2\theta - 2\pi + r) \; [G'd' - \frac{2}{n} H' - \frac{2}{n} T' - T' - 0{,}008482\, T']$

$fe \cos(2\theta - 2\pi - s) \; [G'f' + \frac{2}{n} H' - \frac{1}{n} U' - 0{,}008482\, U']$

$fe \cos(2\theta - 2\pi + s) \; [G'g' + \frac{2}{n} H' - \frac{3}{n} V' - 0{,}008482\, V']$

§. 236. Hinc autem sequentes eliciuntur valores

$F = 0{,}01273 \ldots l F = 8{,}104833$

$G = 0{,}32213 \ldots l G = 9{,}508032$

$H = 0{,}06976 \ldots l H = 8{,}843590$

$J = -1{,}87800 \ldots l -J = 0{,}273710$

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$K = +0{,}05615 \ldots l\text{-}K = 8{,}749352$
$L = -0{,}00077 \ldots l\text{-}L = 6{,}888904$
$M = -0{,}29638 \ldots l\text{-}M = 9{,}471854$
$N = +0{,}00012 \ldots l\,N = 6{,}089109$
$O = +0{,}32287 \ldots l\,O = 9{,}509034$
$S = +0{,}39091 \ldots l\,S = 9{,}592073$
$T = +0{,}69579 \ldots l\,T = 9{,}842475$
$U = -0{,}07922 \ldots l\text{-}U = 8{,}898830$
$V = -0{,}05141 \ldots l\text{-}V = 8{,}711093$

§. 237. Pro distantia ergo lunae a sole curtata $x = \frac{(1-kk)uu}{1-k\cos r}$ erit

$u =$ Praec.

		Log. coeff.	Valores coeff. integri
$+ 0{,}000072f$	$\cos 2\eta$	5,860017	+ 0,000079
$+ 0{,}001833f$	$\cos(2\Phi - 2\pi)$	7,263216	+ 0,002005
$+ 0{,}000397f$	$\cos(2\theta - 2\pi)$	6,598774	+ 0,000434
$- 0{,}01069fk$	$\cos r$	8,028894	— 0,000634
$+ 0{,}00032fk$	$\cos(2\eta - r)$	6,504536	+ 0,000019
$- 0{,}00000fk$	$\cos(2\eta + r)$	4,644088	— 0,000000
$- 0{,}00169fk$	$\cos(2\Phi - 2\pi - r)$	7,227038	— 0,000100
$+ 0{,}00000fk$	$\cos(2\Phi - 2\pi + r)$	3,834293	+ 0,000000
$+ 0{,}00184fk^2$	$\cos(2\Phi - 2\pi - 2r)$	7,264218	+ 0,000006
$+ 0{,}00223fk$	$\cos(2\theta - 2\pi - r)$	7,347257	+ 0,000132
$+ 0{,}00396fk$	$\cos(2\theta - 2\pi + r)$	7,597659	+ 0,000235
$- 0{,}00045fe$	$\cos(2\theta - 2\pi - s)$	6,654014	— 0,000000
$- 0{,}00029fe$	$\cos(2\theta - 2\pi + s)$	6,466277	— 0,000000

vbi notandum est esse $f = 1{,}093756$, et $l\,f = 0{,}038921$.

§. 238.

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§. 238. Deinde pro motu momentaneo habebitur

$$\frac{d\Phi}{dr} = \text{Praec.}$$

		Log. coeff.	Valores coeff. in numeris.
— 0,000146f	$\cos 2\eta$	6,164934	— 0,000160
— 0,003700f	$\cos(2\Phi-2\pi)$	7,568133	— 0,004046
— 0,000801f	$\cos(2\theta-2\pi)$	6,903691	— 0,000876
+ 0,02157fk	$\cos r$	8,333811	+ 0,001286
— 0,00065fk	$\cos(2\eta+r)$	6,809453	— 0,000038
+ 0,00000fk	$\cos(2\eta+r)$	4,949005	+ 0,000001
+ 0,00340fk	$\cos(2\Phi-2\pi-r)$	7,531955	+ 0,000203
— 0,00000fk	$\cos(2\Phi-2\pi+r)$	4,139210	— 0,000000
— 0,00371fk^2	$\cos(2\Phi-2\pi-2r)$	7,569135	— 0,000012
— 0,00449fk	$\cos(2\theta-2\pi-r)$	7,652174	— 0,000267
— 0,00799fk	$\cos(2\theta-2\pi+r)$	7,902575	— 0,000476
+ 0,00091fe	$\cos(2\theta-2\pi-s)$	6,958931	+ 0,000000
+ 0,00059fe	$\cos(2\theta-2\pi+s)$	6,771194	+ 0,000000

§. 239. Pro correctione longitudinis verae hinc oriunda ponatur,

$$\Phi = \text{Praec.}$$

$$+ \mathfrak{W}f\sin 2\eta + \mathfrak{P}fk\sin r + \mathfrak{W}'fk\sin(2\Phi-2\pi-r)$$
$$+ \mathfrak{S}'f\sin(2\Phi-2\pi) + \mathfrak{K}fk\sin(2\eta-r) + \mathfrak{M}fk\sin(2\Phi-2\pi+r)$$
$$+ \mathfrak{H}'f\sin(2\theta-2\pi) + \mathfrak{L}'fk\sin(2\eta+r) + \mathfrak{O}'fkk\sin(2\Phi-2\pi-2r)$$
$$+ \mathfrak{S}'fk\sin(2\theta-2\pi-r) + \mathfrak{U}'fe\sin(2\theta-2\pi-s)$$
$$+ \mathfrak{T}'fk\sin(2\theta-2\pi+r) + \mathfrak{V}'fe\sin(2\theta-2\pi+s)$$

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eritque

$$2a\,\mathfrak{F}' = -0{,}000146$$
$$0{,}026834\,\mathfrak{G}' = -0{,}003700$$
$$0{,}159358\,\mathfrak{H}' - \mathfrak{G}'a' = -0{,}000801$$
$$\mathfrak{J}' - \mathfrak{F}'d' - \mathfrak{F}'e' = +0{,}02157$$
$$0{,}867476\,\mathfrak{K}' - \mathfrak{F}'c' = -0{,}00065$$
$$2{,}867476\,\mathfrak{L}' - \mathfrak{F}'d' = -0{,}00000$$
$$1{,}026834\,\mathfrak{M}' = +0{,}00340$$
$$3{,}026834\,\mathfrak{N}' = -0{,}00000$$
$$0{,}026834\,\mathfrak{O}' = +0{,}00371$$
$$-0{,}840642\,\mathfrak{S}' - \mathfrak{G}'e' + \frac{2}{n}\mathfrak{H}' = -0{,}00449$$
$$+1{,}159358\,\mathfrak{T}' - \mathfrak{G}'d' + \frac{2}{n}\mathfrak{H}' = -0{,}00799$$
$$+0{,}083920\,\mathfrak{U}' - \mathfrak{G}'r' - \frac{2}{n}\mathfrak{H}' = +0{,}00091$$
$$+0{,}234796\,\mathfrak{V}' - \mathfrak{G}'q' + \frac{2}{n}\mathfrak{H}' = +0{,}00059$$

§. 240. Expeditis igitur his formulis orietur:

		Log. coeff.	coeff. tot. in sec.
$\Phi = \mathrm{Pr.}$	$-0{,}000078f \sin 2\eta$	5,893680	$-18''$
	$-0{,}001825f \sin(2\Phi - 2\pi)$	7,261316	-422
	$-0{,}004800f \sin(2\theta - 2\pi)$	7,681286	-1083
	$+0{,}02154fk \sin r$	8,333246	$+264$
	$-0{,}00074fk \sin(2\eta - r)$	6,867925	-9
	$+0{,}00332fk \sin(2\Phi - 2\pi - r)$	7,520465	$+41$
	$-0{,}13818fk^2 \sin(2\Phi - 2\pi - 2r)$	9,140450	-92
	$+0{,}00446fk \sin(2\theta - 2\pi - r)$	7,649421	$+55$
	$-0{,}00685fk \sin(2\theta - 2\pi + r)$	7,835624	-84
	$+0{,}00310fe \sin(2\theta - 2\pi - s)$	7,491107	$+11$
	$+0{,}00030fi \sin(2\theta - 2\pi + \varepsilon)$	6,474418	-1

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§. 241. Haec omnia satis conuenient cum notis inaequalitatibus motus lunae, nisi quod inaequalitas ab angulo $2\theta - 2\pi$ pendens plane aduersari videatur, cum nullum eius vestigium in tabulis astronomicis occurrat; quod quidem eo magis est mirandum, cum correctio inde oriunda ad $18'$, $3''$ exsurgat. Lubens equidem agnosco, in hoc calculo non omnem curam esse adhibitam, vt hanc aequationem tanquam omnibus numeris absolutam spectare liceat, quoniam ad plurimos terminos, quos formulae nostrae suppeditant, non respexi. Interim tamen calculum repetenti mox patebit, non admodum enormiter esse aberratum, praesertim cum aequatio ab angulo $2\Theta - 2\pi$ pendens, quae pari passu procedit, veritati perquam consentanea prodierit, cum ea reductio lunae ad eclipticam contineatur. Ac si quidem haec inaequalitas ad semissem vsque diminuatur, tamen tanta remanet, vt merito dubitare debeamus, eius effectum ab Astronomis non esse animaduersum; cum eius omissio vix per aliam aequationem compensari queat. Hancobrem, siue omissio terminorum neglectorum sit in causa, sive etiam in calculo numerico error fuerit admissus, quod facile euenire potuit, istam inuestigationem in capite sequenti accuratius suscipiamus.

CAPUT

CAPUT XV.

ACCURATIOR INUESTIGATIO INAEQUALITATUM LUNAE AB INCLINATIONE EJUS ORBITAE PENDENTIUM.

§. 242.

Quoniam praecipuum dubium circa inaequalitatem ab angulo $2\theta - 2\pi$ pendentem verſatur, noſtram inueſtigationem ab iis inaequalitatibus, quae ſimul ab alterutra excentricitate pendent, abſtrahamus. Ponamus ergo:

$$\int R dr = \mathfrak{A}\cos 2\eta + \mathfrak{F}f\cos 2\eta + \mathfrak{G}f\cos(2\Phi - 2\pi) + \mathfrak{H}f\cos(2\theta - 2\pi) + \mathfrak{J}f\cos(4\theta - 4\pi)$$

$$\text{et } v = A\cos 2\eta + Ff\cos 2\eta + Gf\cos(2\Phi - 2\pi) + Hf\cos(2\theta - 2\pi) + Jf\cos(4\theta - 4\pi)$$

Poſitoque $\frac{2\pi F + \mathfrak{F}}{nn} = f'$; $\frac{2\pi G + \mathfrak{G}}{nn} = g'$; $\frac{2\pi H + \mathfrak{H}}{nn} = b'$; $\frac{2\pi J + \mathfrak{J}}{nn} = i'$ erit:

$$\frac{d\Phi}{dr} = \alpha + \frac{1}{n} - \mathfrak{A}'\cos 2\eta - f'f\cos 2\eta - g'f\cos(2\Phi - 2\pi) - b'f\cos(2\theta - 2\pi) - i'f\cos(4\theta - 4\pi)$$

$$\frac{d\eta}{dr} = \alpha - \mathfrak{A}'\cos 2\eta - f'f\cos 2\eta - g'f\cos(2\Phi - 2\pi) - b'f\cos(2\theta - 2\pi) - i'f\cos(4\theta - 4\pi)$$

$$\frac{d\theta}{dr} = \frac{1}{n} \text{ et } \frac{d\pi}{dr} = -0{,}004241 - 0{,}004221\cos 2\eta + 0{,}004241\cos(2\Phi - 2\pi) + 0{,}004241\cos(2\theta - 2\pi)$$

§. 243.

§. 243. His valoribus in formulis principalibus ſubſtitutis habebimus has aequationes:

$$R = -\frac{3G}{2nn} f \sin(2\theta - 2\pi) + \frac{3H}{2nn} f \sin(2\Phi - 2\pi)$$

$$\frac{ddv}{dt^2} = f\cos 2\eta \left\{ -\mathfrak{C}F - 2\kappa\mathfrak{F} \right.$$

$$f\cos(2\Phi - 2\pi) \left\{ \begin{array}{l} -\mathfrak{C}G + \frac{3H}{4nn} - 2\kappa\mathfrak{G} + \frac{\mathfrak{A}\mathfrak{H}}{nn} + \frac{3A\mathfrak{H}}{nn} \\ + \frac{3\mathfrak{A}H}{nn} + \frac{3AH}{nn} \end{array} \right.$$

$$f\cos(2\theta - 2\pi) \left\{ \begin{array}{l} -\mathfrak{C}H + \frac{3G}{4nn} - 2\kappa\mathfrak{H} + \frac{\mathfrak{A}\mathfrak{G}}{nn} + \frac{3A\mathfrak{G}}{nn} \\ + \frac{3\mathfrak{A}G}{nn} + \frac{3AG}{nn} \end{array} \right.$$

$$f\cos(4\theta - 4\pi) \left[-\mathfrak{C}J - 2\kappa\mathfrak{J} \right.$$

$$f\cos 2\eta \left(-0{,}005156 - 0{,}026307 + 0{,}001969 \frac{A}{nn} \right)$$

$$f\cos(2\Phi - 2\pi) \left(+0{,}003938 - 1 - 0{,}052614 \frac{A}{nn} - 0{,}002578 \frac{A}{nn} \right)$$

$$f\cos(2\theta - 2\pi) \left(+0{,}052614 + 0{,}002578 - 0{,}001969 \frac{2A}{nn} + \frac{A}{nn} \right)$$

$$f\cos(4\theta - 4\pi) \left(+0{,}001320 + 0{,}026307 \frac{A}{nn} \right)$$

§. 244. Vel in numeris erit

$$R = 0{,}008540 H f \sin(2\Phi - 2\pi) - 0{,}008540 G f \sin(2\theta - 2\pi)$$

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$$\frac{ddv}{dr^2} =$$

$f \cos 2\eta \; [-1{,}01591\,F - 2\varkappa\,\mathfrak{F} - 0{,}031478$

$f\cos(2\Phi-2\pi)\{[-1{,}01591\,G - 2\varkappa\,\mathfrak{G} + 0{,}000636\,H - 0{,}845648$
$\qquad - 0{,}027110\,\mathfrak{H}$

$f\cos(2\theta-2\pi)\{[-1{,}01591\,H - 2\varkappa\,\mathfrak{H} + 0{,}000636\,G + 0{,}047722$
$\qquad - 0{,}027110\,\mathfrak{G}$

$f\cos(4\theta-4\pi)\,[-1{,}01591\,J - 2\varkappa\,\mathfrak{J} + 0{,}001123$

Nunc autem ex formulis assumtis erit

$$\begin{aligned} R = {} & f \sin 2\eta \; [-2\alpha\,\mathfrak{F} + 0{,}004241\,\mathfrak{G} - 0{,}004241\,\mathfrak{H}] \\ & f\sin(2\Phi-2\pi)[+\mathfrak{A}\,b' - 2{,}026834\,\mathfrak{G} + 0{,}004221\,\mathfrak{H}] \\ & f\sin(2\theta-2\pi)[-\mathfrak{A}\,g' - 0{,}023965\,\mathfrak{G} - 0{,}159358\,\mathfrak{H}] \\ & f\sin(4\theta-4\pi)[+0{,}004241\,\mathfrak{H} - 0{,}318716\,\mathfrak{J}] \end{aligned}$$

at est

$\mathfrak{A}b' = -0{,}00931H - 0{,}00461\mathfrak{H}$; et $\mathfrak{A}g' = -0{,}00931G - 0{,}00461\mathfrak{G}$

vnde fit:

$$\begin{aligned} 1{,}867476\,\mathfrak{F} &= 0{,}004241\,(\mathfrak{G}-\mathfrak{H}) \\ 2{,}026874\,\mathfrak{G} &= -0{,}01785\,H - 0{,}00039\,\mathfrak{H} \\ 0{,}159358\,\mathfrak{H} &= +0{,}01785\,G - 0{,}01935\,\mathfrak{G} \\ 0{,}318716\,\mathfrak{J} &= +0{,}004241\,\mathfrak{H} \end{aligned}$$

§. 245. Tum simili modo differentiando valorem ipsius v ponatur

$$\begin{aligned} F' &= 1{,}867476\,F - 0{,}004241\,G + 0{,}004241\,H \\ G' &= 2{,}026834\,G + 0{,}01091\,H + 0{,}00750\,\mathfrak{H} \\ H' &= 0{,}159358\,H + 0{,}03909\,G - 0{,}00750\,\mathfrak{G} \\ J' &= 0{,}318716\,F - 0{,}004241\,H \end{aligned}$$

vt

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vt ſit $\frac{dv}{dr} =$

$A'\sin 2\eta - F'f\sin 2\eta - G'f\sin(2\Phi-2\pi)$
$- H'f\sin(2\vartheta-2\pi) - J'f\sin(4\vartheta-4\pi)$

eritque

$\frac{ddv}{dr^2} = f\cos 2\eta\ [-2F' + 0{,}004241\,G' + 0{,}004241\,H']$
$f\cos(2\Phi-2\pi)[+A'b' - 2{,}026834\,G' - 0{,}004221\,H']$
$f\cos(2\Phi-2\pi)[+A'g' - 0{,}159358\,H' - 0{,}023965\,G']$
$f\cos(4\vartheta-4\pi)[+0{,}004241\,H' - 0{,}318716\,J']$

ſeu $\frac{ddv}{dr^2} =$

$f\cos 2\eta$ $\{-3{,}48745F + 0{,}01668G - 0{,}00721H - 0{,}00003\mathfrak{G} + 0{,}00007\mathfrak{H}$
$f\cos(2\Phi-2\pi)\ [-4{,}10820G - 0{,}05121H - 0{,}02929\mathfrak{H} + 0{,}00003\mathfrak{G}$
$f\cos(2\vartheta-2\pi)\ [-0{,}02565H - 0{,}08323G - 0{,}01290\mathfrak{G} - 0{,}00018\mathfrak{H}$
$f\cos(4\vartheta-4\pi)\ [-0{,}10158J + 0{,}00016G + 0{,}00202H - 0{,}00003\mathfrak{G}$

§. 246. Hinc pro $\mathfrak{F}$ et $\mathfrak{J}$ ſubſtituris valoribus

$\mathfrak{F} = 0{,}00227(\mathfrak{G}-\mathfrak{H})$ et $\mathfrak{J} = 0{,}01331\,\mathfrak{H}$

habebimus has aequationes

$+2{,}47154F - 0{,}01668G + 0{,}00721H - 0{,}00456\mathfrak{G} + 0{,}00456\mathfrak{H} = 0{,}031478$
$+3{,}09229G + 0{,}05185H + 0{,}00218\mathfrak{H} - 2{,}01799\mathfrak{G} = 0{,}995648$
$-0{,}99026H + 0{,}08387G - 0{,}01421\mathfrak{G} - 2{,}01780\mathfrak{H} = -0{,}047722$
$-0{,}91433J - 0{,}00016G - 0{,}00202H + 0{,}00003\mathfrak{G} - 0{,}02685\mathfrak{H} = -0{,}001123$

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§. 247. Deinde reperitur

$$\mathfrak{G} = -0{,}00881\,H - 0{,}00002\,G$$
$$\mathfrak{H} = +0{,}11201\,G + 0{,}00107\,H$$

qui valores ſubſtituti praebent:

$$+2{,}47154\,F - 0{,}01618\,G + 0{,}00725\,H = 0{,}031478$$
$$+3{,}09256\,G + 0{,}06964\,H = 0{,}995648$$
$$+0{,}99229\,H + 0{,}14239\,G = 0{,}047722$$
$$-0{,}91430\,J - 0{,}00297\,G - 0{,}00205\,H = -0{,}001123$$

vnde tandem reperitur

$$F = +0{,}014830 \ldots l\,F = 8{,}171166$$
$$G = +0{,}321910 \ldots l\,G = 9{,}507734$$
$$H = +0{,}001901 \ldots l\,H = 7{,}278927$$
$$J = +0{,}000180 \ldots l\,J = 6{,}256381$$

atque

$$\mathfrak{F} = -0{,}000080 \ldots l\text{-}\mathfrak{F} = 5{,}903090$$
$$\mathfrak{G} = -0{,}000023 \ldots l\text{-}\mathfrak{G} = 5{,}361728$$
$$\mathfrak{H} = +0{,}036056 \ldots l\,\mathfrak{H} = 8{,}556977$$
$$\mathfrak{J} = +0{,}000480 \ldots l\,\mathfrak{J} = 6{,}681241$$

§. 248. Hinc ergo pro diſtantia $s = \frac{(1-kk)\,au}{1-k\cos r}$ reperitur

		log.coeff.	val.coeff.
$s = Pr.+$	$0{,}000084f \cos 2\eta$	5,926350	+0,000092
$+$	$0{,}001832f \cos(2\Phi - 2\pi)$	7,262918	+0,002004
$+$	$0{,}000011f \cos(2\vartheta - 2\pi)$	5,034111	+0,000012
$+$	$0{,}000001f \cos(4\vartheta - 4\pi)$	4,011565	+0,000001

At

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At pro motu momentaneo erit

		log.coeff.	val.coeff.
$\frac{d\Phi}{dt}$ = Pr.	$-0{,}000169f \cos 2\eta$	6,227887	$-0{,}000185$
	$-0{,}003697f \cos(2\Phi-2\pi)$	7,567849	$-0{,}004043$
	$-0{,}000227f \cos(2\theta-2\pi)$	6,356026	$-0{,}000248$
	$-0{,}000005f \cos(4\theta-4\pi)$	4,698970	$-0{,}000005$

vnde quidem iam patet inaequalitatem ab angulo $2\theta-2\pi$ pendentem multo fore minorem, quam supra inueneramus, in quo non paruum veritatis criterium cernitur.

§. 249. Pro ipsa iam longitudine lunae ponamus:

$$\Phi = \text{Pr.} + \mathfrak{A}' \sin 2\eta + \mathfrak{F}' f \sin 2\eta + \mathfrak{G}' f \sin(2\Phi-2\pi) + \mathfrak{H}' f \sin(2\theta-2\pi) + \mathfrak{J}' f \sin(4\theta-4\pi)$$

atque obtinebimus has aequationes:

$$2\,\mathfrak{F}' - 0{,}004241\,(\mathfrak{G}' + \mathfrak{H}') = -0{,}000169$$
$$2{,}026834\,\mathfrak{G}' + 0{,}004221\,\mathfrak{H}' - 0{,}000227\,\mathfrak{A}' = -0{,}003697$$
$$0{,}159358\,\mathfrak{H}' + 0{,}023965\,\mathfrak{G}' - 0{,}003697\,\mathfrak{A}' = -0{,}000227$$
$$0{,}318716\,\mathfrak{J}' - 0{,}004241\,\mathfrak{H}' = -0{,}000005$$

vnde erit

		log. coeff.	vol.tot.coeff. in min. sec.
Φ = Pr.	$-0{,}000096f \sin 2\eta$	5,984018	$-22''$
	$-0{,}001823f \sin(2\Phi-2\pi)$	7,260310	-411
	$-0{,}000910f \sin(2\theta-2\pi)$	6,959131	-205
	$-0{,}000028f \sin(4\theta-4\pi)$	5,450835	-6

Patet ergo reuera aequationem ab angulo $2\theta-2\pi$ ortam multo essa minorem, quam capite praecedente inueneramus, atque nunc quidem non vltra $205''$ seu $3'$, $25''$ ascendere. Nullum igitur est dubium, quin haec aequatio tabulas lunares ad multo maiorem perfectionem sit

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§. 250. Cum igitur neglectus terminorum, minimorum tantum errorem pepererit in aequatione ab angulo $2\theta - 2\pi$ pendente, operae erit pretium, etiam aequationes insuper ab excentricitate orbitae lunaris pendentes curatius inuestigare, quae quidem alicuius videntur esse momenti. In hunc finem ponamus.

$$\int R\,dr = \mathfrak{A}\cos 2\eta + \mathfrak{C}k\cos r + \mathfrak{D}k\cos(2\eta - r) + \mathfrak{E}k\cos(2\eta + r) + \mathfrak{Z}kk\cos(2\eta - 2r)$$
$$+ \mathfrak{G}f\cos(2\theta - 2\pi) + \mathfrak{H}f\cos(2\theta - 2\pi)$$
$$+ \mathfrak{J}fk\cos r + \mathfrak{M}fk\cos(2\Phi - 2\pi - r) + \mathfrak{S}fk\cos(2\theta - 2\pi - r)$$
$$+ \mathfrak{O}fkk\cos(2\Phi - 2\pi - 2r) + \mathfrak{T}fk\cos(2\theta + 2\pi + r)$$

$$\text{et } v = A\cos 2\eta + Dk\cos(2\eta - r) + Ek\cos(2\eta + r) - 14kk\cos(2\eta - 2r)$$
$$+ Gf\cos(2\Phi - 2\pi) + Hf\cos(2\theta - 2\pi)$$
$$+ Jfk\cos r + Mfk\cos(2\Phi - 2\pi - r) + Sfk\cos(2\theta - 2\pi - r)$$
$$+ Ofkk\cos(2\Phi - 2\pi - 2r) + Tfk\cos(2\theta - 2\pi + r)$$

§. 251. His valoribus in aequationibus nostris principalibus substitutis habebimus:

$$R = fk\sin r \quad (0)$$
$$fk\sin(2\Phi - 2\pi - r) \quad (0{,}00854\,S + 0{,}01708\,H)$$
$$fkk\sin(2\Phi - 2\pi - 2r) \quad (0{,}01708\,S)$$
$$fk\sin(2\theta - 2\pi - r) \quad (-0{,}00854\,M - 0{,}01708\,G)$$
$$fk\sin(2\theta - 2\pi + r) \quad (-0{,}01708\,G)$$

et:

$$\frac{ddv}{dr^2} = fk\cos r \begin{cases} -6J - 2\mathfrak{Z} - 0{,}00290 - 0{,}00893 - 0{,}00278 \\ -0{,}00098 - 0{,}00098 - 0{,}00448\frac{A}{nn} + 0{,}00470\frac{A}{nn} \\ +0{,}01315\frac{A}{nn} + 0{,}01315\frac{A}{nn} + 0{,}0263\frac{D}{nn} \\ +0{,}0263\frac{E}{nn} \end{cases}$$

fk

$$
\begin{aligned}
fk\cos(2\Phi-2\pi-r)\{\;
&-6M-2n\mathfrak{M}+\tfrac{1}{2}bG+0{,}00427S-0{,}00854H\\
&-0{,}02711\mathfrak{S}-0{,}03634S+\frac{\mathfrak{C}}{nn}\mathfrak{G}+\frac{3\mathfrak{C}}{nn}G\\
&+0{,}55422\mathfrak{H}+0{,}51332H-\frac{3AH}{2nn}+0{,}01796\\
&+0{,}00145+0{,}001969-\tfrac{1}{2}+0{,}00896\frac{A}{nn}\\
&+0{,}01049\frac{A}{nn}-0{,}02631\frac{A}{nn}-0{,}00197\frac{A}{nn}\\
&-0{,}00129\frac{A}{nn}-0{,}05261\frac{D}{nn}-0{,}00258\frac{E}{nn}\\
fkk\cos(2\Phi-2\pi-2r)\{\;
&-6O-2n\mathfrak{O}+\tfrac{1}{2}bM+\tfrac{3}{4}\mathfrak{G}-\frac{10}{nn}\mathfrak{H}+\frac{54}{nn}H\\
&-\frac{3DH}{2nn}+0{,}7698+0{,}00898+0{,}00098\\
&+0{,}00072-\tfrac{1}{4}+0{,}00448\frac{A}{nn}+0{,}0052\frac{A}{nn}\\
&-0{,}00064\frac{A}{nn}+0{,}00896\frac{D}{nn}-0{,}01315\frac{A}{nn}\\
&-0{,}02630\frac{D}{nn}+0{,}01949\frac{E}{nn}-0{,}00129\frac{E}{nn}\\
&-0{,}00129\frac{E}{nn}-0{,}00042\\
fk\cos(2\theta-2\pi-r)\{\;
&-6S-2n\mathfrak{S}+\tfrac{1}{2}bH+0{,}00427M-0{,}00854G\\
&-0{,}02711\mathfrak{M}-0{,}03634M+\frac{\mathfrak{C}}{nn}\mathfrak{H}+\frac{3\mathfrak{C}}{nn}H\\
&-0{,}01606\mathfrak{G}-0{,}02844G-\frac{3AG}{2nn}-0{,}00896\\
&-0{,}00327+0{,}026307+0{,}00129-0{,}00179\frac{A}{nn}\\
&-0{,}00145\frac{A}{nn}+\frac{A}{2nn}-0{,}00394\frac{E}{nn}+\frac{E}{nn}
\end{aligned}
$$

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$$fk\cos(2\vartheta-2\pi+r)\begin{cases}-\mathfrak{E}T-2\varkappa\mathfrak{T}+\frac{1}{2}bH+0{,}00854\,G+\frac{\mathfrak{C}}{nn}\mathfrak{H}+\frac{3\mathfrak{C}}{nn}H\\+0{,}55422\mathfrak{S}+0{,}51332G-\frac{3AG}{2nn}+0{,}00940\\-0{,}01049+0{,}026307+0{,}00129-0{,}00556\frac{A}{nn}\\-0{,}00145\frac{A}{nn}-0{,}003938\frac{A}{nn}+\frac{A}{2nn}\\-0{,}0394\frac{D}{nn}+\frac{D}{nn}\end{cases}$$

§. 252. Hae autem formulae intricatae reducuntur ad sequentes

$$\frac{ddv}{dr}=\text{Pr.}+fk\cos r\,(-\mathfrak{E}J-2\varkappa\mathfrak{J}-0{,}01182)$$
$$fk\cos(2\Phi-2\pi-r)\begin{pmatrix}-\mathfrak{E}M-2\varkappa\mathfrak{M}-0{,}03207S+0{,}53619\\-0{,}02711\mathfrak{S}\end{pmatrix}$$
$$fkk\cos(2\Phi-2\pi-2r)\,(-\mathfrak{E}O-2\varkappa\mathfrak{O}+1{,}52115M+0{,}76615)$$
$$fk\cos(2\vartheta-2\pi-r)\begin{pmatrix}-\mathfrak{E}S-2\varkappa\mathfrak{S}-0{,}03207M+0{,}00286\\-0{,}02711\mathfrak{M}\end{pmatrix}$$
$$fk\cos(2\vartheta-2\pi+r)\,(-\mathfrak{E}T-2\varkappa\mathfrak{T}+0{,}38147)$$

§. 253. Nunc eosdem valores ex formulis assumtis eruamus, ac posito more adhuc recepto $\frac{2\varkappa J+\mathfrak{J}}{nn}=i'$, $\frac{2\varkappa M+\mathfrak{M}}{nn}=m'$ etc. erit

$$\frac{d\Phi}{dr}=\alpha+\frac{1}{n}-a'\cos 2\eta-d'k\cos(2\eta-r)-g'f\cos(2\Phi-2\pi)$$
$$-e'k\cos(2\eta+r)-b'f\cos(2\vartheta-2\pi)$$
$$-i'fk\cos r-m'fk\cos(2\Phi-2\pi-r)-s'fk\cos(2\vartheta-2\pi-r)$$
$$-n'fkk\cos(2\Phi-2\pi-2r)-t'fk\cos(2\vartheta-2\pi+r)$$

$d\eta$

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$$\frac{d\eta}{dr} = a - a'\cos 2\eta - c'k\cos r - d'k\cos(2\eta - r) - g'k\cos(2\Phi - 2\pi)$$
$$- e'k\cos(2\eta + r) - b'k\cos(2\theta - 2\pi)$$
$$- i'fk\cos r - m'fk\cos(2\Phi - 2\pi - r) - s'fk\cos(2\theta - 2\pi - r)$$
$$- n'fkk\cos(2\Phi - 2\pi - 2r) - t'fk\cos(2\theta - 2\pi + r)$$

$$\frac{d\theta}{dr} = \frac{1}{n} + \frac{2}{n}k\cos r + \frac{3}{2n}kk\cos 2r \quad \text{atque}$$

$$\frac{d\pi}{dr} =$$

$$-0{,}004241 \quad +0{,}004241\,\mathrm{cs}(2\Phi - 2\pi)$$
$$-0{,}004221\cos 2\eta + 0{,}004241\,\mathrm{cs}(2\theta - 2\pi)$$
$$-0{,}01766k\cos r - 0{,}00996k\cos(2\eta - r) - 0{,}010636kk\,\mathrm{cs}(2\eta - 2r)$$
$$-0{,}00831k\cos(2\eta + r) - 0{,}021273kk\cos 2r$$
$$+0{,}00924k\cos(2\Phi - 2\pi - r) + 0{,}00841k\cos(2\theta - 2\pi - r)$$
$$+0{,}00841k\cos(2\Phi - 2\pi + r) + 0{,}00924k\cos(2\theta - 2\pi + r)$$
$$+0{,}01064kk\cos(2\Phi - 2\pi - 2r)$$

§. 254. His iam valoribus introducendis differentiemus formulas nostras assumtas pro $\int R\,dr$ et v; atque obtinebimus primo:

$$R = \text{Praec.}$$

$$+fk\sin r\left\{\begin{array}{l} +0{,}00924\mathfrak{G} + 0{,}00841\mathfrak{H} - 0{,}004241\mathfrak{M} \\ -0{,}00841\mathfrak{G} - 0{,}00924\mathfrak{H} - 0{,}004241\mathfrak{S} \\ \qquad\qquad\qquad\qquad + 0{,}004241\mathfrak{T} \end{array}\right\}$$

$$+fk\sin(2\Phi - 2\pi - r)\left\{\begin{array}{l} +\mathfrak{A}s' - 0{,}01766\mathfrak{G} - 0{,}00996\mathfrak{H} \\ \qquad - 1{,}026834\mathfrak{M} - 0{,}004221\mathfrak{S} \end{array}\right\}$$

$$+fkk\sin(2\Phi - 2\pi - 2r)\left\{\begin{array}{l} +\mathfrak{D}s' + 32h' - 0{,}02127\mathfrak{G} - 0{,}01064\mathfrak{H} \\ -0{,}026834\mathfrak{O} - 0{,}01766\mathfrak{M} - 0{,}00996\mathfrak{S} \end{array}\right\}$$

$$+fk\sin(2\theta-2\pi-r)\left\{\begin{array}{l}-\mathfrak{A}m'+\mathfrak{D}b'+\mathfrak{G}c'-\frac{2}{n}\mathfrak{H}+0{,}840642\,\mathfrak{S}+\mathfrak{M}a'\\-\mathfrak{E}g'-0{,}00831\,\mathfrak{G}-0{,}01766\,\mathfrak{H}-0{,}004221\,\mathfrak{M}\end{array}\right.$$

$$+fk\sin(2\theta-2\pi+r)\left\{\begin{array}{l}-\mathfrak{D}g'+\mathfrak{E}b'+\mathfrak{G}d'-\frac{2}{n}\mathfrak{H}-1{,}159358\,\mathfrak{T}\\+0{,}00996\,\mathfrak{G}\;\;-0{,}01766\,\mathfrak{H}\end{array}\right.$$

§. 255. Hinc igitur conſequimur iſtas aequationes

$$0{,}00083\,(\mathfrak{G}-\mathfrak{H})-\mathfrak{J}-0{,}004241\,(\mathfrak{M}+\mathfrak{S}-\mathfrak{T})=0$$

$$\mathfrak{A}s'-0{,}01766\,\mathfrak{G}-0{,}00996\,\mathfrak{H}-0{,}004221\,\mathfrak{S}-1{,}026834\,\mathfrak{M}=0{,}01708\,\mathfrak{H}+0{,}00854\,\mathfrak{S}$$

$$\mathfrak{D}s'+32b'-0{,}002127\,\mathfrak{G}-0{,}01064\,\mathfrak{H}-0{,}026834\,\mathfrak{O}-0{,}01766\,\mathfrak{M}-0{,}00996\,\mathfrak{S}=0{,}01708\,\mathfrak{S}$$

$$-\mathfrak{A}m'+\mathfrak{D}b'-\mathfrak{E}g'+\mathfrak{G}c'-0{,}00831\,\mathfrak{G}-\frac{2}{n}\mathfrak{H}-0{,}01766\,\mathfrak{H}+\mathfrak{M}a'-0{,}004221\,\mathfrak{M}+0{,}840642\,\mathfrak{S}=-0{,}01708\,\mathfrak{G}-0{,}00854\,\mathfrak{M}$$

$$-\mathfrak{D}g'+\mathfrak{E}b'+\mathfrak{G}d'-0{,}00996\,\mathfrak{G}-\frac{2}{n}\mathfrak{H}-0{,}01766\,\mathfrak{H}-1{,}159358\,\mathfrak{T}=-0{,}01708\,\mathfrak{G}$$

§. 256. Ponatur nunc vlterius:

$$J'=\mathfrak{J}-0{,}00083\,(G-H)+0{,}004241\,(M+S-T)$$

$$M'=1{,}026834\,M-As'+0{,}01766\,G+0{,}00996\,H+0{,}004221\,S$$

$$O'=0{,}026834\,O-Ds'+14b'+0{,}02127\,G+0{,}01064\,H+0{,}01766\,M+0{,}00996\,S$$

$$S'=-0{,}840642\,S+Am'-Db'+Eg'-Gc'+0{,}00831\,G+\frac{2}{n}H+0{,}01766\,H-Ma'+0{,}004221\,M$$

$$T'=1{,}159358\,T+Dg'-Eb'-Gd'+0{,}00996\,G+\frac{2}{n}H+0{,}01766\,H$$

eritque

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eritque $\frac{ddv}{dr} = \text{Praec.}$

$+fk\cos r\,[+0{,}01766\,(G'+H') - J' + 0{,}004241\,(M'+S'+T')$

$+fk\cos(2\varphi-2\pi-r)\{A's' - 0{,}01766\,G' - 0{,}00996\,H' - 0{,}004221\,S' - 1{,}026834\,M'$

$+fkk\cos(2\varphi-2\pi-2r)\{D's' - 2{,}6b' - 0{,}02127\,G' - 0{,}01064\,H' - 0{,}01766\,M' - 0{,}00996\,S' - 0{,}026834\,O'$

$+fk\cos(2\theta-2\pi-r)\{A'm' + D'b' + E'g' + G'c' + 0{,}840642\,S' - 0{,}00831\,G' - \frac{2}{n}H' - 0{,}01766\,H' + M'a' - 0{,}004221\,M'$

$+fk\cos(2\theta-2\pi+r)\{D'g' + E'b' + G'd' - 0{,}00996\,G - \frac{2}{n}H' - 0{,}01766\,H' - 1{,}159358\,T'$

§. 257. Prioris ordinis aequationes huc reducuntur:

$\mathfrak{J} = -0{,}004241\,(\mathfrak{M}+\mathfrak{S}+\mathfrak{T})$

$1{,}026834\,\mathfrak{M} = -0{,}01785\,S - 0{,}00883\,\mathfrak{S} - 0{,}00039 - 0{,}01766\,\mathfrak{M} + 0{,}00690$

$0{,}026834\,\mathfrak{O} = -0{,}05835\,S - 0{,}03041\,\mathfrak{S}$

$-0{,}840642\,\mathfrak{S} = +0{,}01785\,M - 0{,}01935\,\mathfrak{M} + 0{,}00263$

$1{,}159358\,\mathfrak{T} = +0{,}01247$

Hinc fit

$\mathfrak{J} = +0{,}00007\,S + 0{,}00008\,M + 0{,}00006$ seu $\mathfrak{J} = 0$

$\mathfrak{M} = -0{,}01738\,S + 0{,}00018\,M - 0{,}00035$

$\mathfrak{O} = -2{,}16266\,S + 0{,}02394\,M + 0{,}26092$

$\mathfrak{S} = -0{,}00040\,S - 0{,}02123\,M - 0{,}00313$

$\mathfrak{T} = -0{,}01076$

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§. 258. Porro reperietur

$$J' = J + 0{,}004241\,(M+S-T) - 0{,}00026$$
$$M' = 1{,}026834\,M + 0{,}01935\,S - 0{,}00016\,M + 0{,}00568$$
$$O' = 0{,}026834\,O - 0{,}37652\,S + 0{,}01805\,M + 0{,}01063$$
$$S' = 0{,}840642\,S + 0{,}00013\,S + 0{,}00883\,M - 0{,}00167$$
$$T' = 1{,}159358\,T + 0{,}01023$$

ac succinctius habebitur $\frac{ddv}{dr^2} =$ Praec.

$$+fk\cos r\,[-J' + 0{,}004241(M'+S'+T') + 0{,}01175]$$
$$+fk\cos(2\Phi-2\pi-r)\left\{\begin{matrix}-1{,}026834\,M' - 0{,}00422\,S' - 0{,}02842\,S \\ +0{,}00030\,M - 0{,}01160\end{matrix}\right]$$
$$+fkk\cos(2\Phi-2\pi-2r)\left\{\begin{matrix}-0{,}026834\,O' - 0{,}01766\,M' - 0{,}00354\,M \\ -0{,}01511 - 0{,}00996\,S' + 0{,}33746\,S\end{matrix}\right]$$
$$+fk\cos(2\theta-2\pi-r)\left\{\begin{matrix}+0{,}840642\,S' - 0{,}02396\,M' - 0{,}02843\,M \\ -0{,}01472 + 0{,}00025\,S\end{matrix}\right]$$
$$+fk\cos(2\theta-2\pi+r)\,[-1{,}159358\,T' + 0{,}33856]$$

§. 259. Hinc tandem valores quaesiti eliciuntur

$J = -0{,}81144 \ldots l-J = 9{,}909256$
$M = -1{,}25325 \ldots l-M = 0{,}098046$
$O = -2{,}12630 \ldots l-O = 0{,}327624$
$S = -0{,}13490 \ldots l-S = 9{,}130012$
$T = -0{,}10080 \ldots l-T = 9{,}003441$

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ac $\mathfrak{Z} = -0,00005 \ldots l\mathfrak{Z} = 5,698970$
$\mathfrak{M} = +0,00177 \ldots l\mathfrak{M} = 7,247973$
$\mathfrak{O} = +0,52266 \ldots l\mathfrak{O} = 9,718219$
$\mathfrak{S} = +0,02355 \ldots l\mathfrak{S} = 8,371991$
$\mathfrak{T} = +0,01076 \ldots l\mathfrak{T} = 8,031812$

§. 260. Ex his iam pro distantia Lunae a terra erit

$u =$ Praec.

		Log. coeff.	Valor. coeff. totius
$-0,00462\,fk$	$\cos 2\eta$	7,664440	$-0,000275$
$-0,00773\,fk$	$\cos(2\Phi-2\pi-r)$	7,853230	$-0,000424$
$-0,01210\,fkk$	$\cos(2\Phi-2\pi-2r)$	8,082808	$-0,000039$
$-0,00077\,fk$	$\cos(2\theta-2\pi-r)$	6,885196	$-0,000046$
$-0,00057\,fk$	$\cos(2\theta-2\pi+r)$	6,758625	$-0,000034$

et pro motu momentaneo

$$\frac{d\Phi}{dr} =$$ Praec.

		Log. coeff.	Val. coeff.
$+0,00932\,fk$	$\cos r$	7,969416	$+0,000555$
$+0,01558\,fk$	$\cos(2\Phi-2\pi-r)$	8,192568	$+0,000928$
$+0,02142\,fkk$	$\cos(2\Phi-2\pi-2r)$	8,330819	$+0,000069$
$+0,00142\,fk$	$\cos(2\theta-2\pi-r)$	7,152288	$+0,000085$
$+0,00109\,fk$	$\cos 2\theta-2\pi+r)$	7,037426	$+0,000064$

§. 261. Pro longitudine autem lunae sequentes resolui debent aequationes:

$+0,00932 = \mathfrak{Z}' - 0,01766\,(\mathfrak{S}'+\mathfrak{H}') - 0,004241\,(\mathfrak{M}'+\mathfrak{S}'+\mathfrak{T}')$

$+0,01558 = 1,026834\,\mathfrak{M}' - \mathfrak{M}'s' + 0,01766\,\mathfrak{S}' + 0,00996\,\mathfrak{H}'$
$+0,004221\,\mathfrak{S}'$

$$+0,02142 = 0,026834\,\mathfrak{D}' - \mathfrak{D}'t' + 2,6b' + 0,02127\,\mathfrak{S}'$$
$$+0,01064\,\mathfrak{H}' + 0,01766\,\mathfrak{M}' + 0,00996\,\mathfrak{S}'$$
$$+0,00142 = -0,840642\,\mathfrak{S}' - \mathfrak{A}'m' - \mathfrak{D}'b' - \mathfrak{E}'g' + 0,02114\,\mathfrak{S}'$$
$$+0,16853\,\mathfrak{H}' + 0,02396\,\mathfrak{M}'$$
$$+0,00109 = 1,159358\,\mathfrak{T}' - \mathfrak{D}'g' - \mathfrak{E}'b' - 0,35615\,\mathfrak{S}'$$
$$+0,16850\,\mathfrak{H}'$$

hincque prodit pro longitudine vera:

		Log. coeff.	Val. coeff. in min. sec.
Φ = Pr.	$+0,00932\,fk \sin r$	7,969416	+115
	$+0,01521\,fk \sin(2\Phi - 2\pi - r)$	8,182130	+187
	$+0,79079\,fkk \sin(2\Phi - 2\pi - 2r)$	9,898060	+529
	$-0,00121\,fk \sin(2\theta - 2\pi - r)$	7,083939	— 15
	$-0,00082\,fk \sin(2\theta - 2\pi + r)$	6,913527	— 10

§. 262. Ob inclinationem ergo orbitae lunaris ad eclipticam omnes correctiones huc redeunt, vt ſit

I. Pro diſtantia lunae a terra:

u = Praec.

	Log. coeff.	Val. coeff.
$+0,000084f \cos 2n$	5,926350	+0,000092
$+0,001832f \cos(2\Phi - 2\pi)$	7,262918	+0,002004
$+0,000011f \cos(2\theta - 2\pi)$	5,034111	+0,000012
$+0,000001f \cos(4\theta - 4\pi)$	4,011565	+0,000001
$-0,00462fk \cos r$	7,664440	— 0,000275
$-0,00773fk \cos(2\Phi - 2\pi - r)$	7,853230	— 0,000424
$-0,01210fkk \cos(2\Phi - 2\pi - 2r)$	8,082808	— 0,000039
$-0,00077fk \cos(2\theta - 2\pi - r)$	6,885196	— 0,000046
$+0,00057fk \cos(2\theta - 2\pi + r)$	1,758625	— 0,000034

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II. Pro motu momentaneo:

$\frac{d\Phi}{dt}$ = Praec.

	Log. coeff.	Val. coeff.
— 0,000169f cos 2η	6,227887	— 0,000185
— 0,003697f cos $(2\Phi-2\pi)$	7,567849	— 0,004043
— 0,000227f cos $(2\theta-2\pi)$	6,356026	— 0,000248
— 0,000005f cos $(4\theta-4\pi)$	4,698970	— 0,000005
+ 0,00932fk cos r	7,969416	+ 0,000555
+ 0,01558fk cos $(2\Phi-2\pi-r)$	8,192568	+ 0,000928
+ 0,02142fkk cos $(2\Phi-2\pi-2r)$	8,330819	+ 0,000069
+ 0,00142fk cos $(2\theta-2\pi-r)$	7,152288	+ 0,000085
+ 0,00109fk cos $(2\theta-2\pi+r)$	7,037426	+ 0,000064

III. Pro longitudine lunae vera

	Log. coeff.	Val. coeff. in min. sec.
Φ = Pr. — 0,000096f sin 2η	5,984018	— 22″
— 0,001823f sin $(2\Phi-2\pi)$	7,260310	— 411
— 0,000910f sin $(2\theta-2\pi)$	6,959131	— 205
— 0,000028f sin $(4\theta-4\pi-r)$	5,450835	— 6
+ 0,00932fk sin r	7,969416	+ 115
+ 0,01521fk sin $(2\theta-2\pi-r)$	8,182130	+ 187
+ 0,79079fkk sin $(2\Phi-2\pi-2r)$	9,898060	+ 529
— 0,00121fk sin $(2\theta-2\pi-r)$	7,083939	— 15
— 0,00082fk sin $(2\theta-2\pi+r)$	6,913527	— 10

CAPUT

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CAPUT XVI.

EXPOSITIO INAEQUALITATUM LUNAE HACTENUS INUENTARUM.

§. 263.

Quas igitur inuenimus hactenus lunae inaequalitates eae primum, si originem earum spectemus, ad sex classes reducuntur. Quatenus enim luna in motu suo a regulis Keplerianis, in quibus quidem motum apogei compectimur, recedit, eius errores vel primo a solo lunae aspectu, seu eius distantia a sole pendent, seu quod eodem redit, per angulum η tantum definiuntur, quibus variatio lunae continetur. Ad secundam classem refero eas lunae inaequalitates, quae insuper ab excentricitate eius orbitae pendent. Tertia classis eas complectitur inaequalitates, quae ab excentricitate orbitae solis ortum trahunt. Quarta vero eas, quae per vtramque excentricitatem coniunctim determinantur. Quintae porro classi annumeramus eas inaequalitates, quae parallaxin solis inuoluunt, atque errores quatuor ante memoratorum generum implicant. Sexta denique classis suppeditat eas inaequalitates, quae praeterea ab inclinatione orbitae lunaris ad ecliptican pendent.

§. 264. Quodsi vero ad vsum harum inaequalitatum attendamus, prouti eae, ad lunam accommodari debent, tum eae in quinque classes commodissime distribuuntur. Primo enim perpendendae sunt eae inaequali-

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qualitates, quarum ope vera distantia lunae a terra determinatur, vt inde porro tam lunae diameter apparens, quam eius parallaxis horizontalis assignari possit. Secundo loco formulae erunt collocandae illae, quae motui momentaneo definiendo inseruiunt, ex quibus deinceps motus lunae horarius accurate exhiberi poterit. Tertium locum occupabunt eae inaequalitates, quae veram longitudinem lunae ad eclipticam relatam praebent. Quarto vero positio lineae nodorum lunae, seu longitudo nodi ascendentis; ac quinto vera inclinatio orbitae lunaris ad eclipticam inueniri debebit; vt deinde vera lunae latitudo concludi possit. Manifestum autem est, has inaequalitates plurimum inter se permisceri, ita vt vix vllum habeatur genus, cuius inaequalitates non a reliquis generibus pendeant; cui tamen incommodo facile medela adhibetur.

§. 265. Quanquam numerus inaequalitatum, quas sumus consecuti, tantopere increuit, vt calculus sine maxima molestia expediri nequeat, tamen iam monui, non omnes inaequalitates, quibus motus Lunae perturbatur, esse definitas, sed potius earum numerum omnino esse infinitum. Facile quidem intelligitur, plerasque has praetermissas inaequalitates nullius fere esse momenti, atque sine notabili errore iis superstederi posse: verum tamen sunt inter eas nonnullae, quae ad plura minuta secunda assurgere videntur, quarum argumenta supra iam innui; ex quo omnino operae esset pretium in eas omni cura inquirere. Sed earum inuestigatio tam est lubrica et incerta, vt leuissima omissio in calculo facta eas

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maxime afficiat. Cum igitur in calculo plurimos terminos reiicere cogamur, iſtam inueſtigationem fruſtra plane ſuſciperemus, quamdiu ſcilicet rem ſine appropinquatione exequi non licet. Cuius defectus eximium habemus exemplum in inaequalitatibus poſtremo loco inuentis, quae ſtatim atque in negligendo minus largi fueramus, mirum quantum prodierunt immutatae; ac nullum plane eſt dubium, ſi calculum adhuc accuratius proſequi liceret, quin valores inuenti notabilem inſuper mutationem ſint ſubiturae. Imprimis autem aequatio ab angulo $2\Phi - 2\pi - 2r$ ſeu a dupla diſtantia apogei a nodo pendens, eſt ſuſpecta, ac minime pro certa haberi poteſt, cum leuiſſima circumſtantia eam magnopere perturbare valeat.

§. 266. Si enim in cauſam inquiramus, cur analyſis poſterior tam diuerſos valores pro his inaequalitatibus ſuppeditauerit, primo quidem ſtatim patet, neglectum litterarum germanicarum $\mathfrak{Z}$, $\mathfrak{M}$, $\mathfrak{O}$, etc. in calculo priori potiſſimum hoc diſcrimen produxiſſe: ingens enim valor litterae $\mathfrak{O}$ imprimis aequationem ab angulo $2\Phi - 2\pi - 2r$ pendentem tantopere auxit. Praeterea vero etiam non parum augmenti haec aequatio inde eſt nacta, quod in calculo poſteriori rationem quoque habuimus termini $\cos(2\eta - 2r)$, qui tam in valore $\frac{d\Phi}{dr}$ quam $\frac{d\eta}{dr}$ ineſſe eſt deprehenſus; vnde tuto colligere licet, ſi alios quoque terminos ſimiles veluti $2\theta - 2\pi - 2r$, etſi per ſe ſint minimi, in calculum introduxiſſemus, coefficien-

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efficientes terminorum $2\Phi - 2\pi - 2r$ non mediocrem inde mutationem ſubituros fuiſſe. Quamobrem plus hinc colligere non poſſumus, niſi inaequalitatem Lunae ab angulo hoc $2\Phi - 2\pi - 2r$ pendentem minime eſſe contemnendam, etiamſi fortaſſe tanta non ſit, quam inuenimus. Vera autem eius quantitas certius ex obſeruationibus quam ex Theoria colligi poſſe videtur.

§. 267. Quoniam vero hae inaequalitates omnes ad anomaliam Lunae veram referuntur, antequam eas ad vſum adhibere liceat, modum tradi conveniet ad quodvis tempus propoſitum anomaliam Lunae veram determinandi. Cognita autem excentricitate orbitae lunaris k et motu anomaliae mediae, inde ad quodvis tempus facile anomalia media p colligitur. Verum ex anomalia media p et excentricitate k anomalia vera r definiri debet ope huius aequationis $dp = \frac{(1-kk)^{\frac{3}{2}}\, dr}{(1-k\cos r)^2}$; vnde quidem non difficulter, ſi nota eſſet anomalia vera r, viciſſim inveniri poſſet anomalia media p. Calculo enim peracto, ſi breuitatis gratia ponatur

$$\delta = \frac{1-\sqrt{(1-kk)}}{k} = \tfrac{1}{2}k + \frac{1.1}{2.4}k^3 + \frac{1.1.3}{2.4.6}k^5 + \text{etc.}$$

reperitur:

$$p = r + 2k\sin r + 2\delta(k-\tfrac{1}{2}\delta)\sin 2s + 2\delta\delta(k-\tfrac{3}{3}\delta)\sin 3s + 2\delta^3(k-\tfrac{3}{4}\delta)\sin 4s \text{ etc.}$$

cuius ſeriei progreſſio primo intuitu patet. Cum autem k ac proinde δ ſit valde parvum, erit ſatis exacte:

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$$p = r + 2k\sin r + (\tfrac{3}{4}kk + \tfrac{1}{8}k^4 + \tfrac{3}{64}k^6)\sin 2r + (\tfrac{1}{3}k^3 + \tfrac{1}{8}k^5)\sin 3r$$
$$+(\tfrac{5}{32}k^4 + \tfrac{3}{32}k^6)\sin 4r + \tfrac{3}{40}k^5\sin 5r + \tfrac{7}{192}k^6\sin 6r$$

§. 268. Data ergo anomalia media Lunae p, eius anomalia vera r elici debebit ex hac aequatione

$$r = p - 2k\sin r - (\tfrac{3}{4}kk + \tfrac{1}{3}k^4 + \tfrac{3}{64}k^6)\sin 2r - (\tfrac{1}{3}k^3 + \tfrac{1}{8}k^5)\sin 3r$$
$$-(\tfrac{5}{32}k^4 + \tfrac{3}{32}k^6)\sin 4r - \tfrac{3}{40}k^5\sin 5r - \tfrac{7}{192}k^6\sin 6r$$

cuius quidem ope, si cognita fuerit excentricitas k, calculus non difficulter expedietur. Quoniam enim termini sinus inuoluentes sunt admodum parui, in iis statim poni poterit $r = p$, vnde valor verior pro r eruetur, qui deinde iterum in his terminis adhibitus, iustiorem valorem pro r suppeditabit. Atque hoc modo post aliquot operationes verus tandem valor pro anomalia vera r obtinebitur. Interim tamen, quo iste calculus facilius perfici queat, aequatio haec ita potest transformari, vt loco sinuum anomaliae verae r, sinus anomaliae mediae p introducantur; id quod sequenti modo praestabitur.

§. 269 Ponatur breuitatis gratia:

$$\tfrac{3}{4} + \tfrac{1}{8}kk + \tfrac{3}{64}k^4 = \alpha;\quad \tfrac{1}{3} + \tfrac{1}{8}kk = \mathfrak{G};\quad \tfrac{5}{3} + \tfrac{3}{32}kk = \gamma$$
$$\tfrac{3}{40} = \delta \quad \text{et} \quad \tfrac{7}{192} = \varepsilon$$

vt sit:

$$r = p - 2k\sin r - \alpha kk\sin 2r - \mathfrak{G}k^3\sin 3r - \gamma k^4\sin 4r - \delta k^5\sin 5r - \varepsilon k^6\sin 6r$$

ac ponatur denuo

$$2k\sin r + \alpha kk\sin 2r + \mathfrak{G}k^3\sin 3r + \gamma k^4\sin 4r + \delta k^5\sin 5r + \varepsilon k^6\sin 6r = R$$

et

et cum sit $r = p - R$ habebitur:

$\sin r = (1 - \frac{1}{2}RR + \frac{1}{24}R^4)\sin p - (R - \frac{1}{6}R^3 + \frac{1}{120}R^5)\cos p$

$\sin 2r = (1 - 2RR + \frac{2}{3}R^4)\sin 2p - (2R - \frac{4}{3}R^3)\cos 2p$

$\sin 3r = (1 - \frac{9}{2}RR)\sin 3p - (3R - \frac{9}{2}R^3)\cos 3p$

$\sin 4r = (1 - 8RR)\sin 4p - 4R\cos 4p$

$\sin 5r = \sin 5p - 5R\cos 5p$

$\sin 6r = \sin 6p$

negligendo scilicet terminos, qui ipsius k potestates sexta altiores continent.

§. 270. Euolutio autem huius calculi sit maxime prolixa, si quidem ad sextam potestatem ipsius k ascendere velimus. Facile autem expedietur, si ad quartam subsistamus, tum autem reperietur:

$$r = p - (2k - \tfrac{1}{4}k^3)\sin p + (\tfrac{5}{4}kk - \tfrac{5}{8}\tfrac{1}{4}k^4)\sin 2p - \tfrac{1}{12}\tfrac{3}{2}k^3\sin 3p - \tfrac{7}{120}k^4\sin 4p$$

quae expressio satis accurate pro quauis anomalia media p conuenientem anomaliam veram indicabit. Calculus autem, si excentricitas k constet, facili negotio absoluetur. Formula haec quoque ita repraesentari poterit, vt sit

$$r = p - 2k(1 - \tfrac{1}{8}kk)\sin p + \tfrac{5}{4}kk(1 - \tfrac{1}{3}\tfrac{1}{0})kk\sin 2p - \tfrac{1}{12}\tfrac{3}{2}k^3\sin 3p - \tfrac{7}{120}k^4\sin 4p$$

Hinc igitur tabulam construi conueniet, vnde pro quavis anomalia media proposita ipsi respondens anomalia vera excerpi queat.

§. 271. Cum autem inuenta fuerit anomalia vera r, longitudo Lunae regula Kepleriana inuenienda, quam

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ſupra (206) per ζ indicauimus, ita exprimetur, vt ſit

$$\zeta = C + 1{,}0085272\,p$$
$$-1{,}0085272\left(\begin{array}{l} 2k\sin r + \frac{3}{4}kk(1+\frac{1}{6}kk+\frac{1}{2}\frac{1}{5}k^4)\sin 2r + \frac{1}{3}k^3(1+\frac{3}{8}kk)\sin 3r \\ +\frac{5}{3}\frac{5}{2}k^4(1+\frac{3}{3}kk)\sin 4r + \frac{3}{4}\frac{3}{0}k^5\sin 5r + \frac{7}{1}\frac{7}{9}\frac{7}{2}k\sin 6r \end{array}\right)$$

vbi $C+1{,}0085272\,p$ exhibet longitudinem Lunae mediam; quae ſi vocetur $= \xi$, atque in coefficientium partibus minimis pro k ſcribatur valor proximus 0,0545, erit

		log. coeff.	val. in min. ſec.
$\zeta = \xi$	$-2{,}0170544\,k \sin r$	0,304718	$22675'' = 6^\circ,17',55''$
	$-0{,}756770\,kk \sin 2r$	9,878964	$464 = 7,44$
	$-0{,}336551\,k^3 \sin 3r$	9,527051	$11\frac{1}{2}$
	$-0{,}15786\,k^4 \sin 4r$	9,198282	$\frac{5}{7}$

vnde patet ſuperfluum futurum fuiſſe, ſi ſuperiores expreſſiones vltra quartam poteſtatem ipſius k extendere voluiſſemus.

CAPUT

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CAPUT XVII.

INUESTIGATIO ELEMENTORUM MOTUS LUNAE.

§. 272.

Inuentis iam per Theoriam hisce inaequalitatibus, quibus motus Lunae perturbatur, antequam eas ad computum aſtronomicum accommodare liceat, elementa, quae in eas ingrediuntur, per obſeruationes determinari oportet. Primo ſcilicet ad datam epocham cum longitudo Lunae media, tum eius anomalia media, ac locus nodi medius conſtitui debebit, vt eaedem res inde ad quoduis aliud tempus aſſignari queant. Deinde quoque ex obſeruationibus verus valor excentricitatis lunaris colligi debet, a quo potiſſimum quantitas praecipuarum inaequalitatum pendet. Excentricitas autem orbitae ſolaris pro ſatis certa haberi poterit, cum ſit $\epsilon = 0{,}0168$. Lunae vero excentricitas tam prope iam conſtat, vt inde ſine errore ad quamlibet anomaliam mediam vera ſatis exacte aſſignari poſſit. Etſi enim in anomalia vera error aliquot minutorum primorum committitur, inaequalitates Lunae inde non vltra aliquot minuta ſecunda afficiuntur.

§. 273. Quodſi autem ſtatim quasuis Lunae obſeruationes ad hunc finem adhibere velimus, ob tam ingentem inaequalitatum numerum, inueſtigatio elementorum maxime moleſta redderetur. Quocirca ex obſer-

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obseruationibus eas eligi conueniet, pro quibus numerus inaequalitatum multo fiat minor; dum scilicet distantia Lunae a sole seu angulus η datum obtinet valorem. Commodissimae ergo erunt eae obseruationes, quae in ipsis momentis coniunctionis vel oppositionis sunt institutae. Accuratas itaque obseruationes eclipsium lunarium ad hoc negotium adhibebo, quoniam praeter haec tempora, vera vel coniunctionis vel oppositionis momenta non satis certo ex obseruationibus colligi licet.

§. 274. Momento autem oppositionis verae Lunae et Solis, longitudo Lunae sex signis distat a longitudine solis, ita vt sit $\vartheta = \varphi \pm 180°$, ideoque angulus $\eta = 180°$. Posito autem pro η hoc valore longitudo Lunae vera φ ex media ξ per sequentes formulas definietur, in quas formulae hactenus inuentae abeunt:

$$\varphi = \xi - 2{,}0170544k\sin r - 0{,}756770kk\sin 2r - 0{,}33655k^3\sin 3r$$
$$+0{,}0101460 \quad +0{,}004200$$
$$+0{,}4202260 \quad -0{,}573280$$
$$+0{,}0049920 \quad +0{,}003180$$
$$-0{,}0052860 \quad +0{,}150830.$$
$$-0{,}000860 \quad -0{,}000002$$
$$+1{,}1959\nu$$
$$-0{,}0757\nu$$
$$+0{,}00932f$$

$$+(0{,}201385+0{,}021889-0{,}016368-0{,}3959\nu+4{,}1738\nu)e\sin s$$
$$+(0{,}06645 - 0{,}02332 + 0{,}00840)ee\sin 2s$$
$$+(0{,}74760 - 0{,}81430 - 0{,}01420)ek\sin(r-s)$$
$$+(-0{,}61850 - 0{,}23960 - 0{,}00610)ek\sin(r+s)$$

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$$-(0{,}002823 + 0{,}000910)f\sin(2\varphi - 2\pi)$$
$$-0{,}000028 f\sin(4\varphi - 4\pi)$$
$$+(0{,}01521 - 0{,}00121)fk\sin(2\varphi - 2\pi - r)$$
$$+0{,}79079 fkk\sin(2\varphi - 2\pi - 2r)$$

§. 275. Cum igitur sit $f = 1{,}09375$, et $\nu = \frac{1}{288}$, erit has formulas colligendo:

$$\varphi = \xi - 1{,}572993k\sin r - 1{,}17186kk\sin 2r - 0{,}3365k^3\sin 3r$$
$$+0{,}21998e\sin s + 0{,}05123ee\sin 2s - 0{,}0809ek\sin(r-s)$$
$$-0{,}8642ek\sin(r+s)$$
$$-0{,}002989\sin(2\varphi - 2\pi) + 0{,}01531k\sin(2\varphi - 2\pi - r)$$
$$-0{,}000031\sin(4\varphi - 4\pi)$$
$$+0{,}86493kk\sin(2\varphi - 2\pi - 2r)$$

et cum sit $e = 0{,}0168$, erit hoc valore substituto:

$$\varphi = \xi - 1{,}572993k\sin r - 1{,}17186kk\sin 2r - 0{,}3365k^3\sin 3r$$
$$+0{,}003697\sin s + 0{,}000014\sin 2s - 0{,}001359k\sin(r-s)$$
$$-0{,}014523k\sin(r+s)$$
$$-0{,}002989\sin(2\varphi - 2\pi) + 0{,}01531k\sin(2\varphi - 2\pi - r)$$
$$-0{,}000031\sin(2\varphi - 4\pi)$$
$$+0{,}86493kk\sin(2\varphi - 2\pi - 2r)$$

§. 276. Assumta iam hypothesi quapiam non nimis a vero aberrante, vnde ad datum quoduis tem-

pus definiri poſſit tam longitudo lunae, quam eius anomalia media, ex qua praeterea ope excentricitatis proxime cognitae anomalia vera aſſignari queat: haec elementa correctione indigebunt, quam ex obſeruationibus elici oporteat. Ponamus ergo longitudinem mediam ex tabulis deſumtam augeri debere m minutis ſecundis. Tum vero excentricitas ſuppoſita, quae ſit $= 0,0545$, augeri debeat $\frac{n}{10000}$, vt ſit $k = 0,0545 + \frac{n}{10000}$; ipſa vero anomalia vera tabularis, quae ſit $= v$, augmentum requirat μ minutorum ſecundorum, vt ſit $r = v + \mu''$: eritque ſin $r = $ ſin $v + \mu''$ cof v; ſin $2r = $ ſin $2v + 2\mu''$ cof $2v$; et ſin $3r = $ ſin $3v$ in terminis enim minimis haec correctio praetermitti poterit.

§. 277. Quod ſi haec omnia in minuta ſecunda conuertantur, prodibit longitudo lunae vera

$$\Phi = \text{Long. med.} + m''$$
$$- 17682'' \text{ ſin } v - 32,445\, n'' \text{ ſin } v - 0,085728\, \mu'' \text{ cof } v$$
$$- 7187'' \text{ ſin } 2v - 2,635\, n'' \text{ ſin } 2v - 0,006962\, \mu'' \text{ cof } 2v$$
$$- 11'' \text{ ſin } 3v + 762'' \text{ ſin } s + 3'' \text{ ſin } 2s$$
$$- 15'' \text{ ſin } (r-s) - 163'' \text{ ſin } (r+s)$$
$$- 616'' \text{ ſin } (2\Phi - 2\pi) + 172'' \text{ ſin } (2\Phi - 2\pi - r)$$
$$- 6'' \text{ ſin } (4\Phi - 4\pi) + 530 \text{ ſin } (2\Phi - 2\pi + 2r)$$

Cum

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Cum autem postremus terminus sit suspectus, loco eius coefficientis 530 malumus ponere coefficientem indefinitum 100γ, atque ex obseruationibus valorem ipsius γ indagare. Deinde sit error anomaliae verae i minutorum primorum, vt calculus commodior reddatur, atque ob $\mu = 60i$, neglectis terminis minimis erit:

$$\varphi = \text{Long. med.} + m''$$
$$-17682'' \sin v - 32,445\, m'' \sin v + 5,143\, i'' \cos v$$
$$- 718'' \sin 2v - 2,635\, n \sin 2v + 0,417\, i \cos 2v$$
$$+ 762 \sin s - 15 \sin (r-s) - 163 \sin (r+s)$$
$$- 616 \sin (2\varphi - 2\pi) + 172'' \sin (2\varphi - 2\pi - r)$$
$$+ 100\gamma \sin (2\varphi - 2\pi - 2r)$$

§. 278. Oblata autem obseruatione eclipsis lunae, quaeratur primum momentum medium huius eclipsis, pro quo colligatur longitudo solis, itemque longitudo nodi ascendentis. Punctum autem soli oppositum nondum erit longitudo lunae vera in ecliptica; verumtamen longitudo lunae pro hoc momento eclipsis medio inueniri poterit ope sequentis tabellae.

Subtrahatur longitudo nodi a longitudine ſolis, et aequatio tabulae ſecundum titulos adſcriptos applicetur puncto ſoli oppoſito in ecliptica.

	{ O Sign. VI. Sign. }	
gr.	ſubtrahe	
0	0′, 0″	30
1	0, 32	29
2	1, 6	28
3	1, 39	27
4	2, 12	26
5	2, 45	25
6	3, 17	24
7	3, 49	23
8	4, 21	22
9	4, 53	21
10	5, 24	20
11	5, 56	19
12	6, 26	18
	adde	gr.
	{ V Sign. XI. Sign. }	

§. 279. Quanquam autem hoc momento, ad quod lunae longitudinem hinc colligimus, non vera lunae oppoſitio exiſtit, ſed luna ſecundum longitudinem a puncto ſoli oppoſito diſtat particula, quam haec tabula monſtrat; tamen tuto pro hoc momento ex formula noſtra longitudinem lunae inueſtigare poterimus, viſuri, quam

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quam exacte ea conueniat cum longitudine eius ad hoc tempus ex obſeruatione concluſa. Cum enim luna hoc tempore nunquam vltra 5′ a vero oppoſitionis loco diſtet, ſi formula noſtra generali vti vellemus, foret angulus η minor 5 minutis primis; vnde facile perſpicitur, diſcrimen in loco lunae inde oriundum vix vnquam 12″ eſſe ſuperaturum. Quoniam itaque medium cuiusque eclipſis momentum ipſum tam accurate definiri nequit, vt non error dimidii minuti primi ſit pertimeſcendus, ſuperfluum ſane foret in calculo ad iſtiusmodi minutias attendere.

§. 280. Hanc ob cauſam quoque ex calculo, quem inibo, non ſummam praeciſionem expectari conueniet; quia ipſae obſeruationes, quibus vtar, non plenae accurationis ſunt capaces. Plus igitur me non effecturum confido, quam vt ſatis prope tam excentricitatem orbitae lunaris, quam longitudinem et anomaliam lunae mediam ad datam epocham definiam. Quod cum fuerit factum maiori confidentia theoriam ad quasuis alias obſeruationes transferre licebit; quae ſi nullis erroribus fuerint inquinatae, non admodum erit difficile reliquas elementorum correctiones, quibus formulae noſtrae ſunt innixae, inde concludere. Imprimis autem hic calculus veram excentricitatem orbitae lunaris ſatis exacte manifeſtabit, vt deinceps accuratius pro quauis anomalia media conuenientem anomaliam veram definire valeamus. Hunc igitur in finem nonnullas eclipſes lunares Pariſiis inſtitutas calculo ſubiiciam.

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§. 281. Primae igitur eclipſis medium contigiſſe reperio Pariſiis A. 1712. Jan. 23^d, 7^h; $55'$, $16''$ temp. medio. Pro quo momento colligitur:

Longitudo ſolis θ . . .	10^s, $3°$, $0'$, $54''$
Anomalia vera ſolis s . .	6, 24, 25, 13
Deinde ex tabulis meis:	
Longitudo lunae media .	4, 7, 18, 55
Anomalia lunae media .	2, 0, 18, 20
Anomalia lunae vera $v =$	1, 25, 6, 27
Longitudo nodi vera $\pi =$	9, 24, 34, 32
Diſt. nodi a ſole $\theta - \pi =$	0, 8, 26, 22
Hinc aequatio loci lunae .	— — 4, 33
Ergo longitudo lunae vera $\odot =$	4, 2, 56, 21

§. 282. Hinc calculus ſequenti modo inſtituetur:

$v =$	1, 25, 6, 27	;	$\sin v = +\sin 55°, 6', 27''$ $\cos v = +$
$2v =$	3, 20, 12, 54	;	$\sin 2v = +\sin 69, 47, 6$ $\cos 2v = -$
$s =$	6, 24, 25, 13	;	$\sin s = -\sin 24, 25, 13$
$v - s =$	7, 0, 41, 14	;	$\sin = -\sin 30, 41, 14$
$v + s =$	8, 19, 31, 40	;	$\sin = -\sin 79, 31, 46$
$\odot - \pi =$	6, 8, 21, 49		
$2\odot - 2\pi =$	0, 16, 43, 38	;	$\sin = +\sin 16, 43, 38$
$r =$	1, 25, 6, 27		
$2\odot - 2\pi - r =$	10, 21, 37, 11	;	$\sin = -\sin 38, 22, 49$
$2\odot - 2\pi - 2r =$	8, 26, 30, 44	;	$\sin = -\sin 86, 30, 44$

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+	9,91393	+	9,9139	+	9,7575
—	4,24753	—	1,5111	—	0,7104
—	4,16146	—	1,4250n	—	0,4679i
+	9,9724	+	9,9724	—	9,5385
—	2,8561	—	0,4208	—	9,6201
—	2,8285	—	0,3932n	+	9,1586i
—	9,6163	—	9,7078	—	9,9927
+	2,8819	—	1,1761	—	2,2122
—	2,4982	+	0,8839	+	2,2049
+	9,4588	—	9,7930		
—	2,7896	+	2,2355	—	99,8y
—	2,2484	—	2,0285		

aeq. +	aeq. —	— 26,6n
+ 8	— 14493	— 2,5n
+ 160	— 674	— 2,9i
+ 168	— 315	+ 0,1i
— 15766	— 177	— 99,8y
— 15598	— 107	
— 259′, 58″	— 15766	
— 4°, 19, 58	aequatio	

Long. med. 4,7,18,55 + m
aeq. −4,19,58

4,2,58,57 − 29,1n − 2,8i − 99,8y = 4,2,56,21
4,2,56,21

Ergo 0 = 2,36 − 29,1n − 2,8i − 99,8y + m

§. 283.

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§. 283. Secundae eclipsi medium contigit: Parisiis A. 1713. Dec. 1^d, 15^b, 26^l, 34^h temp. med. Pro quo momento colligitur

Longitudo Solis $\odot$	$=$	8, 9, 53, 40
Anomalia vera Solis s	$=$	5, 1, 46, 43
Longitudo Lunae media		2, 5, 2, 26
Anomalia Lunae media		9, 12, 27, 42
Anomalia Lunae vera v	$=$	9, 18, 24, 49
Longitudo nodi π	$=$	8, 17, 46, 10
Distantia Solis a nodo		11, 22, 7, 30
Aequatio loci Lunae		$+$ 4, 17
Longitudo Lunae vera Φ	$=$	2, 9, 57, 57

§. 284. Hinc calculus sequens instituatur:

v	$=$ 9, 18, 24, 49	; $\sin v = -\sin 71, 35, 11$
		$\cos v = +$
$2v$	$=$ 7, 6, 49	; $\sin 2v = -\sin 36, 49$
		$\cos 2v = -$
s	$=$ 5, 1, 46	; $\sin s = +\sin 28, 14$
$v - s$	$=$ 4, 16, 39	; $\sin = +\sin 43, 21$
$v + s$	$=$ 2, 20, 11	; $\sin = +\sin 80, 11$
$\Phi - \pi$	$=$ 11, 22, 12	
$2\Phi - 2\pi$	$=$ 11, 14, 24	; $\sin = -\sin 15, 36$
r	$=$ 9, 18, 25	
$2\Phi - 2\pi - r$	$=$ 1, 25, 59	; $\sin = +\sin 55, 59$
$2\Phi - 2\pi - 2r$	$=$ 4, 7, 34	; $\sin = +\sin 52, 26$

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— 9,97717	— 9,9772	+ 9,4996
— 4,24753	— 1,5111	— 0,7104
+ 4,22470	+ 1,4883n	— 0,2100i
— 9,7776	— 9,7776	— 9,9034
— 2,8561	— 0,4208	— 9,6201
+ 2,6337	+ 0,1984n	+ 9,5235i
+ 9,6749	+ 9,8366	+ 9,9936
+ 2,8819	— 1,1761	— 2,2122
+ 2,5568	— 1,0127	— 2,2058
— 9,4296	+ 9,9185	+ 79, 2y
— 2,7866	+ 2,2355	
+ 2,2192	+ 2,1530	

aeq. aff.	aequat.	
+ 16776	— 10	+ 30, 8n
+ 430	— 161	+ 1, 6n
+ 360	— 171	— 1, 6i
+ 166		+ 0, 3i
+ 143		
+ 17875		
— 171		
+ 17704		
+ 2961′,4″	aequatio	

Long. media = 2, 5, 2, 26 + m
aeq. + 4, 55, 4 .
Long. vera 2, 9, 57, 30, + m
obs. 2, 9, 57, 57
Ergo o = — 0′,27″ + m + 32, 4n — 1,3i + 79, 2y

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§. 285. Tertiae eclipſis medium contigit Pariſiis A. 1717 Mart. 26^d, 15^h, $21'$, $20''$ temp. med. Pro quo tempore colligitur

Longitudo ſolis vera $\odot$ =	0°,	6°,	19′,	56″,
Anomalia ſolis vera s =	8,	28,	0,	17
Longitudo media lunae . .	6,	1,	37,	2
Anomalia media lunae . .	8,	24,	7,	21
Anomalia lunae vera v =	9,	0,	19,	10
Longitudo nodi vera π =	6,	13,	30,	22
Diſtantia nodi a ſole . .	5,	22,	49,	29
aeq. pro loco lunae		+	3,	57
Ergo longitudo lunae vera $\mathfrak{C}$ =	6,	6,	23,	53

§. 286. Calculus igitur ita ſe habebit

v = 9, 0, 19, 10	; ſin v =	− ſin 89°, 40′, 50″
	cof. =	+
$2v$ = 6, 0, 38,	; ſin $2v$ =	− ſin 0°, 38′
	cof =	−
s = 8, 28, 0	; ſin s =	− ſin 88°, 0′
$u - s$ = 0, 2, 19	; ſin =	+ ſin 2°, 19
$v + s$ = 5, 28, 19	; ſin =	+ ſin 1, 41
$\mathfrak{C} - \pi$ = 5, 22, 53		
$2\mathfrak{C} - 2\pi$ = 11, 15, 46	; ſin =	− ſin 14, 14
r = 9, 0, 19		
$2\mathfrak{C} - 2\pi - r$ = 2, 15, 27	; ſin =	+ ſin 75, 27
$2\mathfrak{C} - 2\pi - 2r$ = 5, 15, 8	; ſin =	+ ſin 14, 52

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— 9,99999	— 10,0000	+ 7,7425
— 4,24753	— 1,5111	— 0,7104
+ 4,24752	+ 1,5111n	— 8,4526i

— 8,0435	— 8,0435	— 9,9999
— 2,8561	— 0,4208	— 9,6201
+ 0,8996	+ 8,4643n	+ 9,6200i

— 9,9997	+ 8,6066	+ 8,4680
+ 2,8819	— 1,1761	— 2,2122
— 2,8816	— 9,7827	— 0,6802

— 9,3907	+ 9,9858	+ 25,65y
— 2,7893	+ 2,2355	
+ 2,1893	+ 2,2213	

aeq. aff.	aeq. neq.	
+ 17682	— 762	+ 32, 4n
+ 8	— 1	+ 0, 0n
+ 152	— 4	— 0, 0i
+ 166	— 767	+ 0, 2i
+ 18008	+ 18008	
	+ 17241	
	+ 287,21	
	+ 4,47,21	aequatio

Long. media ☾ = 6, 1, 37, 2
aeq. + 4, 47, 21
Long. ☽ vera = 6, 6, 24, 23
obſ. 6, 6, 23, 53
Ergo $0 = +30 + m + 32{,}4n + 0{,}2i + 25{,}65y$

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§. 287. Quartae eclipſis medium erat Pariſiis A. 1718 Sept. 9^d, 8^b, $1'$, $1''$ temp. medio Pro quo tempore colligitur

Longitudo ſolis vera $\theta =$	5, 16, 40, 58
Anomalia ſolis vera $s =$	2, 8, 19, 59
Longitudo lunae media	11, 17, 25, 16
Anomalia lunae media	0, 10, 41, 28
Anomalia lunae vera $v =$	0, 9, 36, 52
Longitudo nodi vera $\pi =$	5, 15, 59, 35
Diſtantia nodi a ſole	0, 0, 41, 23
aeq. pro loco lunae	— 22
Longitudo lunae obſ. $\varphi =$	11, 16, 40, 36

§. 288. Calculus ergo ſequens habebitur.

$v =$	0, 9, 36, 52	; ſin $v =$ + ſin 9, 36, 52
		cof = +
$2v =$	0, 19, 14	; ſin $2v =$ + ſin 19, 14
		cof = +
$s =$	2, 8, 20	; ſin $s =$ + ſin 68, 20
$v - s =$	10, 1, 17	; ſin = − ſin 58, 43
$v + s =$	2, 17, 57	; ſin = + ſin 77, 57
$\varphi - \pi =$	0, 0, 41	
$2\varphi - 2\pi =$	0, 1, 22	; ſin = + ſin 1, 22
$r =$	0, 9, 37	
$2\varphi - 2\pi - r =$	11, 21, 45	; ſin = − ſin 8, 15
$2\varphi - 2\pi - 2r =$	11, 12, 8	; ſin = − ſin 17, 52

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+ 9,22274	+ 9,2227	+ 9,9938
— 4,24753	— 1,5111	— 0,7104
— 3,47027	— 0,7338 *n*	— 0,7042 *i*
+ 9,5177	+ 9,5177	+ 9,9750
— 2,8561	— 0,4208	— 9,6201
— 2,3738	— 9,9385 *n*	— 9,5951 *i*
+ 9,9682	— 9,9318	+ 9,9903
+ 2,8819	— 1,1761	— 2,2122
+ 2,8501	+ 1,1079	— 2,2025
+ 8,3775	— 9,1568	— 30,68 *y*
— 2,7896	+ 2,2355	
— 1,1671	— 1,3923	

aeq. aff.	aeq. neg.	
+ 708	— 2953	— 5, 4 *m*
+ 13	— 237	— 0, 8 *n*
+ 721	— 159	— 5, 1 *i*
— 3389	— 15	— 0, 4 *i*
— 2668	— 25	
aeq. = — 44′, 28″	— 3389	

Long. ☾ med. 11, 17, 25, 16
aeq. — 44, 28
Long. vera 11, 16, 40, 48
obs. 11, 16, 40, 36

Ergo $o = +12 + m - 6,2n - 5,5i - 30,68y$

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§. 289. Quintae eclipſis medium erat:
Pariſiis A. 1719 Aug. 29^d, 8^b, 33^l, $19''$ temp. med.
Pro quo tempore colligitur:

Longitudo ſolis vera $\theta =$	5, 5, 47, 14
Anomalia ſolis vera $s =$	1, 27, 25, 24
Longitudo lunae media .	11, 2, 9, 40
Anomalia lunae media . .	10, 15, 59, 25
Anomalia lunae vera $v =$	10, 20, 5, 19
Longitudo nodi vera $\pi =$	4, 27, 44, 39
Diſtantia nodi a ſole $\equiv$	0, 8, 2, 35
Aequ. pro loco lunae . .	— — 4, 22
Long. lunae obſeruata . .	11, 5, 42, 52

§. 290. Calculus ergo ita ſe habebit:

$v =$ 10, 20, 5, 19	;	ſin $=$ − ſin 39, 54, 41 coſ $=$ +
$2v =$ 9, 10, 11	;	ſin $2v =$ − ſin 79, 49 coſ $=$ +
$v =$ 10, 20, 5		
$s =$ 1, 27, 25	;	ſin $s =$ + ſin 57, 25
$v-s =$ 8, 22, 40	;	ſin $=$ − ſin 82, 40
$v+s =$ 0, 17, 30	;	ſin $=$ + ſin 17, 30
$\Phi-\pi =$ 0, 7, 58		
$2\Phi-2\pi =$ 0, 15, 16	;	ſin $=$ + ſin 15, 56
$r =$ 10, 20, 5		
$2\Phi-2\pi-r =$ 1, 25, 15	;	ſin $=$ + ſin 55, 51
$2\Phi-2\pi-2r =$ 3, 5, 46	;	ſin $=$ + ſin 84, 14

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—	9, 80726	−	9, 8073	+	9, 8849
—	4, 24753	−	1, 5111	−	0, 7104
+	4, 05479	+	1, 3184n	−	0, 5953i
—	9, 9931	−	9, 9931	+	9, 2475
—	2, 8561	−	0, 4208	−	9, 6201
+	2, 8492	+	0, 4139n	−	8, 8676i
+	9, 9256	−	9, 9964	+	9, 4781
+	2, 8819	−	1, 1761	−	2, 2122
+	2, 8075	+	1, 1725	−	1, 6903
+	9, 4386	+	9, 9178	+	99, 5y
—	2, 7896	+	2, 2355		
—	2, 2282	+	2, 1533		

aeq. aff.	aeq. neg.		
+ 11345	− 49	+	20, 8n
+ 707	− 170	+	2, 6n
+ 642	− 219	−	3, 9i
+ 15	+ 12851	−	0, 1i
+ 142	+ 12632		
+ 12851	210, 32		
	+ 3, 30, 32	aequatio	

Long. lunae med. 11, 2, 9, 40
aeq. + 3, 30, 32
Long. lunae vera 11, 5, 40, 12
obs. 11, 5, 42, 52

$0 = -2,40 + n + 23,4n - 4,0i + 99,5y$

§. 291.

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§. 291. Sextae eclipſis medium erat Pariſiis A. 1722. Jun. $28^d, 13^h, 58', 41''$ temp. med. Pro quo tempore habetur:

Longitudo ſolis vera $\vartheta =$	3, 6, 51, 7
Anomalia ſolis vera $s =$	11, 28, 26, 56
Longitudo lunae media .	9, 9, 31, 50
Anomalia lunae media .	4, 28, 8, 18
Anomalia lunae vera $v =$	4, 24, 39, 53
Longitudo nodi vera $\pi =$	3, 2, 36, 2
Diſtantia nodi a ſole . .	0, 4, 15, 5
Aequatio loci lunae . .	— 2, 20
Longitudo lunae obſeruata	9, 6, 48, 47

§. 292. Calculus ergo ita ſe habebit

$v =$	4, 24, 39, 53 ;	$\sin v = +\sin 35°, 20', 7''$
		$\cos v = -$
$2v =$	9, 19, 20 ;	$\sin 2v = -\sin 70, 40$
		$\cos 2v = +$
$r =$	4, 24, 40	
$s =$	11, 28, 27 ;	$\sin s = -\sin 1, 33$
$r - s =$	4, 26, 13 ;	$\sin = +\sin 33, 47$
$r + s =$	4, 23, 7 ;	$\sin = +\sin 36, 53$
$\Phi - \pi =$	0, 4, 13	
$2\Phi - 2\pi =$	0, 8, 26 ;	$\sin = +\sin 8, 26$
$r =$	4, 24, 40	
$2\Phi - 2\pi - r =$	7, 13, 46 ;	$\sin = -\sin 43, 46$
$2\Phi - 2\pi - 2r =$	2, 19, 6 ;	$\sin = +\sin 79, 6$

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+	9,76220	+	9,7622	−	9,9116
−	4,24753	−	1,5111	−	0,7104
−	4,00973	−	1,2733n	+	0,6220i
−	9,9748	−	9,9748	+	9,5199
−	2,8561	−	0,4208	−	9,6201
+	2,8309	+	0,3956n	−	9,1400i
−	8,4321	+	9,7451	+	9,7783
+	2,8819	−	1,1761	−	2,2122
−	1,3140	−	0,9212	−	1,9905
+	9,1663	−	9,8399	+	98,2y
−	2,7896	+	2,2355		
−	1,9559	−	2,0754		

aeq. aff.	aeq. neg.		
+ 678	− 10227	− 18,8n	
− 10563	− 21	+ 2,5n	
− 9885	− 8		
164,45	− 98	+ 4,2i	
aeq. = − 2,44,45	− 90	− 0,1i	
	− 119		
	−101563		

Long. lun. med. 9, 9, 31, 50
− 2, 44, 45
Long. lun. obs. 9, 6, 47, 5
9, 6, 48, 47

Ergo $0 = -1,42 + m - 16,3n + 4,1i + 98,9y$

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§. 293. Septimae eclipsis medium obseruatum est Parisiis A. 1724 Oct. 31^d, 15^h, $34'$, $17''$ temp. med. Pro quo tempore colligitur

Longitudo solis vera $\odot$ =	7,	8,	56,	1
Anomalia solis vera s =	4,	0,	29,	44
Longitudo lunae media	1,	9,	23,	59
Anomalia lunae media	5,	22,	38,	2
Anomalia lunae vera v =	5,	21,	46,	51
Longitudo nodi vera	1,	16,	36,	22
Distantia nodi a sole	5,	22,	19,	39
aequatio loci lunae	+		4,	10
Long. lunae obseruata	1,	9,	0,	11

§. 294. Calculus ergo ita ineatur:

v =	5, 21, 46, 51	; $\sin v$ = + $\sin$ 8°, 13′, 9″
		cos = −
$2v$ =	11, 13, 34,	; $\sin 2v$ = − $\sin$ 16°, 26′
r =	5, 21, 47	cos = +
s =	4, 0, 30	; $\sin s$ = + $\sin$ 59°, 30′
$r-s$ =	1, 21, 17	; $\sin$ = + $\sin$ 51, 17
$r+s$ =	9, 22, 17	; $\sin$ = − $\sin$ 67, 43
$\Phi-\pi$ =	5, 22, 24	
$2\Phi-2\pi$ =	11, 14, 48	; $\sin$ = − $\sin$ 15, 12
r =	5, 21, 47	
$2\Phi-2\pi-r$ =	5, 23, 1	; $\sin$ = + $\sin$ 6, 59
$2\Phi-2\odot-2r$ =	0, 1, 14	; $\sin$ = + $\sin$ 1, 14

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+	9, 15520	+	9, 1552	−	9, 9955
−	4, 24753	−	1, 5111	−	0, 7104
−	3, 40273	−	0, 6663*n*	+	0, 7059*i*
−	9, 4516	−	9, 4516	+	9, 9819
−	2, 8561	−	0, 4208	−	9, 6201
+	2, 3077	+	9, 8724*n*	−	9, 6020*i*
+	9, 9353	+	9, 8922	−	9, 9663
+	2, 8819	−	1, 1761	−	2, 2122
−	2, 8172	−	1, 0683	+	2, 1785
−	9, 4186	+	9, 0849	+	2, 15*y*
−	2, 7896	+	2, 2355		
+	2, 2082	+	1, 3024		

aeq. aff.	aeq. neg.	
+ 203	− 2528	− 4, 6*n*
+ 657	− 11	− 7, 5*n*
+ 151	− 2539	
+ 161	+ 1193	+ 5, 1*i*
+ 21	− 1346	− 0, 4*i*
+ 1193	− 22, 26	aequatio

Long. ☽ med.	1, 9, 23, 59
	− 22, 26
Long. calc.	1, 9, 1, 33
Long. obs.	1, 9, 0, 11

$$o = + 1,22 + m - 3,9n + 4,7i + 2,15y$$

§. 295. Octauae eclipſis medium obſeruatum eſt Pariſiis A. 1729. Febr. 13^d, 9^b, $6'$, $56''$ temp. med. Pro quo tempore colligitur:

Longitudo ſolis vera $\theta =$	10^s, 25°, $13'$, $23''$
Anomalia ſolis vera $s =$	7, 16, 43, 34
Longitudo lunae media .	5, 0, 5, 27
Anomalia lunae media .	3, 18, 53, 24
Anomalia lunae vera $v =$	3, 12, 54, 9
Longitudo nodi vera $\pi =$	10, 24, 4, 30
Diſtantia nodi a ſole $=$	0, 1, 8, 53
aequatio pro long. lunae .	— 0, 37
Longitudo lunae obſeruata	4, 25, 12, 46

§. 296. Calculus ergo ita ſe habebit:

$v =$ 3, 12, 54, 9	;	$\sin v = +\sin 77^\circ, 5', 51''$
		$\cos v = -$
$2v =$ 6, 25, 48	;	$\sin 2v = -\sin 25, 48$
$r =$ 3, 12, 54		$\cos 2v = -$
$s =$ 7, 16, 44	;	$\sin s = -\sin 46, 44$
$r-s =$ 7, 26, 10	;	$\sin = -\sin 56, 10$
$r+s =$ 10, 29, 38	;	$\sin = -\sin 30, 22$
$\Phi-\pi =$ 0, 1, 8,		
$2\Phi-2\pi =$ 0, 2, 16,	;	$\sin = +\sin 2, 16$
$r =$ 3, 12, 54		
$2\Phi-2\pi-r =$ 8, 19, 22	;	$\sin = -\sin 79, 22$
$2\Phi-2\pi-2r =$ 5, 6, 28	;	$\sin = +\sin 23, 32$

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+ 9,98889	+ 9,9889	− 9,3488
− 4,24757	− 1,5111	− 0,7104
− 4,23642	− 1,5000n	+ 0,0502 i
− 9,6387	− 9,6387	− 9,9544
− 2,8561	− 0,4208	− 9,6201
+ 2,4948	+ 0,0595n	+ 9,5745 i
− 9,8622	− 9,9194	− 9,7037
+ 2,8819	− 1,1761	− 2,2122
− 2,7441	+ 1,0955	+ 1,9159
+ 8,5971	− 9,9925	+ 39, 9y
− 2,7896	+ 2,2355	
− 1,3867	− 2,2280	

aeq. affi.	aeq. neg.		− 31, 7n
+ 312	− 17235	+	1, 1n
+ 12	− 555		
+ 82	− 24	+	1, 1 i
+ 406	− 169	+	0, 3 i
	− 17983		
	+ 406		
	− 17577		
	− 292, 57		
	− 4,52,57	aequatio	

Long. lunae media	=	5, 0, 5, 27
aeq.		− 4, 52, 57
Long. lunae calc.		4, 25, 12, 30
Long. lunae obſ.		4, 25, 12, 46

$$o = -16 + m - 30,6n + 1,4i + 39,9y$$

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§. 297. Nonae eclipſis medium obſeruatum eſt Pariſiis A. 1729. Aug. 8^d, 13^h, $14'$, $14''$ temp. med. Pro quo tempore reperitur.

Longitudo ſolis vera $\odot$	$= 4, 16, 17, 29$
Anomalia ſolis vera s	$= 1, 7, 47, 12$
Longitudo lunae media	$10, 11, 23, 57$
Anomalia lunae media	$8, 10, 36, 19$
Anomalia lunae vera v	$= 8, 16, 34, 40$
Longitudo nodi vera π	$= 10, 14, 58, 21$
Diſtantia nodi a ſole	$6, 1, 19, 8$
Aequatio pro loco lunae	$- 43$
Long. lunae obſeruata	$10, 16, 16, 46$

§. 298. Calculus ergo ita inſtituetur:

v	$= 8, 16, 34, 40$	; $\sin v = -\sin 76, 34, 40$
		$\cos v = -$
$2v$	$= 5, 3, 9$	; $\sin 2v = +\sin 26, 51$
		$\cos 2v = -$
r	$= 8, 16, 35$	
s	$= 1, 7, 47$	; $\sin s = +\sin 37, 47$
$r - s$	$= 7, 8, 48$	; $\sin = -\sin 38, 48$
$r + s$	$= 9, 24, 22$	; $\sin = -\sin 65, 38$
$\odot - \pi$	$= 6, 1, 19$	
$2\odot - 2\pi$	$= 0, 2, 38$	; $\sin = +\sin 2, 38$
r	$= 8, 16, 35$	
$2\odot - 2\pi - r$	$= 3, 16, 3$	; $\sin = +\sin 73, 57$
$2\odot - 2\pi - 2r$	$= 6, 29, 28$	; $\sin = -\sin 29, 28$

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— 9,93797	— 9,9880	— 9,3655
— 4,24755	— 1,5111	— 0,7104
+ 4,23550	+ 1,4991n	+ 0,0759i
+ 9,6548	+ 9,6548	— 9,9505
— 2,8561	— 0,4208	— 9,6201
— 2,5109	— 0,0756n	+ 9,5706i
+ 9,7872	— 9,7970	— 9,9595
+ 2,8819	— 1,1761	— 2,2122
+ 2,6691	+ 0,9731	+ 2,1717
+ 8,6622	+ 9,9827	— 49, 2y
— 2,7896	+ 2,2355	
— 1,4518	+ 2,2182	

aeq. aff.	aeq. neg.	+ 31, 6n
+ 17199	— 324	— 1, 2n
+ 467	— 28	
+ 9	— 352	+ 1, 2i
+ 148	+ 17988	+ 0, 4i
+ 165	+ 17636	
+ 17988	+ 293, 56	
	+ 4, 53, 56	aequatio

Long. ☽ med. = 10, 11, 23, 57
aeq. + 4, 53, 56
Long. ☽ calc. = 10, 16, 17, 53
Long. ☽ obs. 10, 16, 16, 46

0 = + 1, 7 + n + 30, 4n + 1, 6i — 49, 2y

§. 299.

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§. 299. Decimae eclipſis medium obſeruatum eſt Pariſiis A. 1731. Jun. $19^d, 13^h, 55', 13''$ temp. med. Pro quo tempore colligitur

Longitudo ſolis vera $\odot$ =	2^s,	$28°$,	$5'$,	$41''$
Anomalia ſolis vera s =	11,	19,	48,	47
Longitudo lunae media .	9,	1,	45,	1
Anomalia lunae media .	4,	15,	9,	43
Anomalia lunae vera v =	4,	10,	34,	21
Longitudo nodi vera π =	9,	8,	6,	38
Diſtantia nodi a ſole . .	5,	19,	59,	3
Aequatio pro loco lunae .	+	5,	24	
Longitudo lunae obſeruata	8,	28,	11,	5

§. 300. Calculus ergo ita inſtituetur

v =	4, 10, 34, 21	;	$\sin v$ = + $\sin$ 49, 25, 39
			$\cos v$ = −
$2v$ =	8, 21, 9	;	$\sin 2v$ = − $\sin$ 81, 9
			$\cos 2v$ = −
r =	4, 10, 34		
s =	11, 19, 49	;	$\sin s$ = − $\sin$ 10, 11
$r-s$ =	4, 20, 45	;	$\sin$ = + $\sin$ 39, 15
$r+s$ =	4, 0, 23	;	$\sin$ = + $\sin$ 59, 37
$\Phi-\pi$ =	5, 20, 4		
$2\Phi-2\pi$ =	11, 10, 8	;	$\sin$ = − $\sin$ 29, 52
r =	4, 10, 34		
$2\Phi-2\pi-r$ =	6, 29, 34	;	$\sin$ = − $\sin$ 29, 34
$2\Phi-2\pi-2r$ =	2, 19, 0	;	$\sin$ = + $\sin$ 79, 0

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+ 9,88057	+ 9,8806	− 9,8131
− 4,24753	− 1,5111	− 0,7104
− 4,12810	− 1,3917n	+ 0,2100i
− 9,9948	− 9,9948	− 9,1871
− 2,8561	− 0,4208	− 9,6201
+ 2,8509	+ 0,4156n	+ 8,8072i
− 9,2475	+ 9,8012	+ 9,9358
+ 2,8819	− 1,1761	− 2,2122
− 2,1294	− 0,9773	− 2,1480
− 9,5313	− 9,6932	+ 98, 1y
− 2,7896	+ 2,2355	
+ 2,3209	− 1,9287	

aeq. aff.	aeq. neg.	
+ 709	− 13431	− 24, 8n
+ 209	− 135	+ 2, 6n
+ 918	− 9	+ 3, 3i
− 13801	− 141	+ 0, 1i
− 12883	− 85	
− 214, 43	− 13801	
aeq. − 3, 34, 43		

Long. lunae media	9, 1, 45, 1
aeq.	− 3, 34, 43
Long. lunae calc.	8, 28, 10, 18,
Long. lunae obſ.	8, 28, 11, 5

Ergo $o = -47'' + m - 22,2n + 3,4i + 98,1y$

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§. 301. Eclipsis undecimae medium observatum est Parisiis A. 1732 Dec. $1^d, 9^h, 48', 23''$ temp. med. Pro quo tempore colligitur:

Longitudo solis vera θ =	8, 10, 3, 6
Anomalia solis vera s =	5, 1, 29, 50
Longitudo lunae media .	2, 6, 8, 19
Anomalia lunae media . .	7, 19, 24, 12
Anomalia lunae vera v =	7, 24, 19, 39
Longitudo nodi vera π =	8, 10, 41, 14
Distantia nodi a sole ==	11, 29, 21, 52
Aequ. pro loco lunae . .	+ 21
Long. lunae observata . .	2, 10, 3, 27

§. 302. Calculus ergo ita se habebit:

v = 7, 24, 19, 39	; $\sin v$ = $-\sin$ 54, 19, 39
	$\cos v$ = −
$2v$ = 3, 18, 39	; $\sin 2v$ = $+\sin$ 71, 21
	cos = −
r = 7, 24, 20	
s = 5, 1, 30	; $\sin s$ = $+\sin$ 28, 30
$r-s$ = 2, 22, 50	; sin = $+\sin$ 82, 50
$r+s$ = 0, 25, 50	; sin = $+\sin$ 25, 50
$\Phi-\pi$ = 11, 29, 22	
$2\Phi-2\pi$ = 11, 28, 44	; sin = $+\sin$ 1, 16
r = 7, 24, 40	
$2\Phi-2\pi-r$ = 4, 4, 4	; sin = $+\sin$ 55, 56
$2\Phi-2\pi-2r$ = 8, 9, 24	; sin = $+\sin$ 69, 24

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—	9,90975	− 9,9097	− 9,7657	
—	4,24753	− 1,5111	− 0,7104	
+	4,15728	+ 1,4208n	+ 0,4761i	
+	9,9766	+ 9,9766	− 9,5048	
—	2,8561	− 0,4208	− 9,6201	
—	2,8327	− 0,3974n	+ 9,1249i	
+	9,6787	+ 9,9969	+ 9,6444	
+	2,8819	− 1,1761	− 2,2122	
+	2,5606	− 1,1730	− 1,8566	
—	8,3445	+ 9,9182	− 93,6y	
—	2,7896	+ 2,2355		
+	1,1341	+ 2,1537		

aeq. aff.	aeq. neq.	+ 26,4n
+ 14364	− 680	− 2,5n
+ 364	− 15	+ 3,0i
+ 14	− 72	+ 0,1i
+ 142	− 767	
+ 14884	+ 14884	
	+ 14117	
	+ 235,17	
	+ 3,55,17	aequatio

Long. lunae media 2, 6, 8, 19
aeq. + 3, 55, 17
Long. lunae calc. 2, 10, 3, 36
obs. 2, 10, 3, 27

$$o = +9 + m + 23,9n + 3,1i - 93,6y$$

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§. 303. Eclipsis duodecimae medium obseruatum est Parisiis A. 1736 Mart. 26^d, 12^b, $14'$, $36''$ temp. med. Pro quo tempore colligitur

Longitudo solis vera $\theta =$	0°,	6°,	35′,	42″
Anomalia solis vera $s =$	8,	27,	58,	24
Longitudo lunae media	6,	4,	5,	0
Anomalia lunae media	7,	3,	25,	43
Anomalia lunae vera $v =$	7,	7,	2,	56
Longitudo nodi vera $\pi =$	6,	6,	24,	31
Distantia nodi a sole	6,	0,	11,	11
aeq. pro long. lunae	——			6
Longitudo lunae obs.	6,	6,	35,	36

§. 304. Calculus ergo ita instituatur.

$v =$	7,	7,	2, 56	; sin $v =$	$-$ sin 57, 2, 56
				cos $=-$	
$2v =$	2,	14,	6	; sin $2v =$	$+$ sin 74, 6
				cos $=+$	
$r =$	7,	7,	3		
$s =$	8,	27,	58	; sin $s =$	$-$ sin 87, 58
$r - s =$	10,	9,	5	; sin $=$	$-$ sin 50, 55
$r + s =$	4,	5,	1	; sin $=$	$+$ sin 54, 59
$\Phi - \pi =$	6,	0,	11		
$2\Phi - 2\pi =$	0,	0,	22	; sin $=$	$+$ sin 0, 22
$r =$	7,	7,	3		
$2\Phi - 2\pi - r =$	4,	23,	19	; sin $=$	$+$ sin 36, 41
$2\Phi - 2\pi - 2r =$	9,	16,	16	; sin $=$	$-$ sin 73, 44

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— 9,77995	— 9,7799	— 9,9021
— 4,24753	— 1,5111	— 0,7104
+ 4,02748	+ 1,2910n	+ 0,6125i
+ 9,9831	+ 9,9831	+ 9,4377
— 2,8561	— 0,4208	— 9,6201
— 2,8392	— 0,4039n	— 9,0578i
— 9,9997	— 9,8900	+ 9,9133
+ 2,8819	— 1,1761	— 2,2122
— 2,8816	+ 1,0661	— 2,1255
+ 7,8061	+ 9,7763	— 96, 0y
— 2,7896	+ 2,2355	
— 0,5957	+ 2,0118	

aeq. aff.	aeq. neg.	+ 19, 6n
+ 10653	— 691	— 2, 5n
+ 12	— 761	
+ 103	— 133	+ 4, 1i
+ 10768	— 4	— 0, 1i
— 1589	— 1589	
+ 9179		
+ 152, 59		
aeq. = + 2,32,59		

Long. ☾ med. 6, 4, 5, 0
aeq. + 2, 32, 59
Long. ☽ calc. 6, 6, 37, 59
obs. 6, 6, 35, 36

$$0 = +2',23'' + n + 17,1n + 4,0i - 96,0y$$

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§. 305. Eliplis decima tertiae medium obſeruatum eſt Pariſiis A. 1736 Sept. 19^d, 14^h, $59'$, $36''$ temp. med. Pro quo tempore colligitur

Longitudo ſolis vera $\theta =$	5^s, 27°, $21'$, $39''$,
Anomalia ſolis vera $s =$	2, 18, 43, 51
Longitudo lunae media . .	11, 27, 48, 53
Anomalia lunae media . .	0, 7, 25, 42
Anomalia lunae vera $v =$	0, 6, 40, 44
Longitudo nodi vera .	5, 27, 15, 4
Diſtantia nodi a ſole . .	0, 9, 6, 35
aeq. pro long. lunae	— 4
Longitudo lunae obſeruata	11, 27, 21, 35

§. 306. Calculus ergo ita inſtituetur

$v =$ 0, 6, 40, 44	; $\sin v = +$ ſin 6, 40, 50
	$\cos v = +$
$2v =$ 0, 13, 21	; $\sin 2v = +$ ſin 13, 21
	$\cos 2v = +$
$r =$ 0, 6, 41	
$s =$ 2, 18, 44	; ſin $s = +$ ſin 88, 44
$r - s =$ 9, 17, 57	; ſin $= -$ ſin 72, 3
$r + s =$ 2, 25, 25	; ſin $= +$ ſin 85, 25
$\Phi - \pi =$ 0, 0, 6	
$2\Phi - 2\pi =$ 0, 0, 12	; ſin $= +$ ſin 0, 12
$r =$ 0, 6, 41	
$2\Phi - 2\pi - r =$ 11, 23, 31	; ſin $= -$ ſin 6, 29
$2\Phi - 2\pi - 2r =$ 11, 16, 50	; ſin $= -$ ſin 13, 10

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+	9, 06561	+	9,0656	+	9, 9970
—	4, 24753	—	1,5111	—	0, 7104
—	3, 31314	—	0, 5767n	—	0, 7074i
+	9, 3634	+	9,3634	+	9, 9881
—	2, 8561	—	0, 4208	—	9, 6201
—	2, 2195	—	9, 7842n	—	9, 6082i
+	9, 9915	—	9, 9783	+	9, 9986
+	2, 8819	—	1, 1751	—	2, 2122
+	2, 8734	+	1, 1534	—	2, 2108
+	7, 5429	—	9, 0527	—	22, 6y
—	2, 7896	+	2, 2355		
—	0, 3325	—	1, 2882		

aeq. aff.	aeq. neg.		
+ 747	— 2057	—	3, 8n
+ 14	— 166	—	0, 6n
+ 761	— 2	—	5, 1 i
— 2406	— 19	—	0, 4 i
— 1645	— 2406		
— 27′,25″	aequatio		

Long. ☽ med. 11, 27, 48, 53
aeq. — 27, 25
Long. ☽ calc. 11, 27, 21, 28
Long. ☽ obl. 11, 27, 21, 35

$\bullet = -0,7'' + m - 4,4n - 5,5i - 22,6y$

§. 307.

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§. 307. Ex his ergo tredecim eclipsibus nacti sumus aequationes, ex quibus cum tabularum, quibus sum vsus, correctiones, tum verus valor aequationis ab angulo $2\mathfrak{D} - 2\pi - 2r$ pendentis definiri debebit:

Aequationes autem inde ortae sunt sequentes.

I. $0 = +156'' + m - 29,1n - 2,8i - 99,8y$

II. $0 = -27 + m + 32,4n - 1,3i + 79,2y$

III. $0 = +30 + m + 32,4n + 0,2i + 25,6y$

IV. $0 = +12 + m - 6,2n - 5,5i - 30,7y$

V. $0 = -160 + m + 23,4n - 4,0i + 99,5y$

VI. $0 = -102 + m - 16,3n + 4,1i + 98,2y$

VII. $0 = +82 + m - 3,9n + 4,7i + 2,1y$

VIII. $0 = -16 + m - 30,6n + 1,4i + 39,9y$

IX. $0 = +67 + m + 30,4n + 1,6i - 49,2y$

X. $0 = -47 + m - 22,2n + 3,4i - 98,1y$

XI. $0 = +9 + m + 23,9n + 3,1i - 93,6y$

XII. $0 = +143 + m + 17,1n + 4,0i - 96,0y$

XIII. $0 = -7 + m - 4,4n - 5,5i - 22,6y$

§. 308.

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§. 308. Hic ſtatim commode euenit, vt errores calculi ab obſeruationibus infra tria minuta prima ſubſiſtant, qui autem infra ſesquiminutum primum deprimuntur, ſimul ac litterae y valor tribuitur vnitati fere aequalis. Hincque ergo cognoſcimus valorem ipſius y, quem quinario maiorem inueneramus, merito nobis fuiſſe ſuſpectum, cum iam perſpiciamus, eum vnitatem ſuperare non poſſe. Quamobrem ponamus $y = 1$, ſeu in formula noſtra pro longitudine lunae ſcribamus terminum $100''$ ſin $(2\Phi + 2\varpi - 2r)$. Quod autem ad litteras m, n et i attinet, tentanti mox patebit, quoscunque ipſis valores tribuamus, errores inde non admodum poſſe diminui; interim tamen decem circiter minutis ſecundis diminuentur, ſi ponatur $y = \frac{1}{2}$; $n = \frac{1}{2}$; $i = -3$ et $m = -4$; quo facto errores vix vnum minutum primum ſuperabunt.

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CAPUT XVIII.

CONSTITUTIO ELEMENTORUM PRO TABULIS LUNARIBUS.

§. 309.

Tabulae autem, quibus in praecedenti calculo sum vsus, praebent pro meridiano Parisino ad epocham 1701 seu ad meridiem diei vltimi anni 1700 tempore medio

Longitudinem Lunae mediam $5^s, 20^o, 19', 47''$
et Anomaliam Lunae mediam 6, 13, 26, 51

Hinc accuratius habebimus haec elementa pro eodem tempore eodemque loco: scilicet

Longitudinem Lunae mediam $5^s, 20^o, 19', 43''$
Anomaliam Lunae mediam 6, 13, 24, 0
unde Longitudo Apogei 11, 6, 55, 43

§. 310. Si haec elementa comparemus cum Tabulis astronomicis Cel. Cassini et Monnierii, reperiemus pro eodem tempore et loco

	Cassini	Monnier
Long. mediam Lunae	5, 20, 18, 19	5, 20, 19, 28
Anom. mediam Lunae	6, 13, 10, 48	6, 13, 13, 2
Long. Apogei	11, 7, 7, 27	11, 7, 6, 26

Hic quidem longitudo media satis conuenit cum ea, quam ex obseruationibus concluſimus; verum anomalia media inuenta superat Cassinianam $13', 12''$, Monnierianam autem $11'$, quod discrimen satis est notabile. Verum

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Verum si perpendamus motum lunae a tam multis variisque inaequalitatibus perturbari, mirum sane non est, anomaliam mediam per solas obseruationes accuratius definiri non potuisse; praesertim cum error 15′ in anomalia media commissus in loco lunae ad summum errorem 1′, 45″ gignere valeat.

§. 311. Excentricitatem autem orbitae lunaris, quam statueram = 0,0545 iam $\frac{1}{20000}$ vel 0,00005 augeri oportet, ita vt nunc sit excentricitatis valor $k = 0,05455$; qui a supra assumto tam parum discrepat, vt anomalia vera inde ex media collecta pro satis exacta haberi possit: aequationes autem ab excentricitate pendentes aliquod augmentum capient, quod nunc quidem diligentius definiri oportet. Primum ergo formulam pro longitudine lunae inuentam hinc corrigamus; deinde vero etiam formulas pro distantia lunae a terra, pro eius motu momentaneo, et pro loco nodi veraque inclinatione orbitae lunaris ad ecliptican hinc euoluamus.

§. 312. Ante omnia autem oportebit formulam exhibere, cuius ope ex data quauis anomalia lunae media p elicere liceat, conuenientem anomaliam veram r. Ac substituto quidem pro k vero eius valore nunc invento, coefficientibusque in minuta secunda conuersis, formula supra (§. 306) exhibita sequentem induet formam:

$$r = p - 22495'' \sin p + 766'' \sin 2p - 36'' \sin 3p$$
$$4,352086 \qquad 2,884229 \qquad 1,55630$$

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Huius ergo formulae ope haud difficulter tabula computabitur, quae ad singulos anomaliae mediae gradus exhibeat valores anomaliae verae.

§. 313. Inuenta autem anomalia vera v, si habeatur quoque anomalia vera solis s, vna cum angulo η et longitudinibus Φ, θ, π saltem proxime, formula longitudinem veram Φ datae mediae ξ respondentem exhibens, sequenti modo habebitur expressa:

		log. coeff.	
$\Phi = \xi$	$-$ 22466″ sin v	4,351535	I
	$-$ 462 sin $2v$	2,66456	
	$-$ 11 sin $3v$	1,0518	
	$+$ 701 sin s	2,84572	II
	$+$ 4 sin $2s$	0,602	
	$+$ 141 sin $(v-s)$	2,1492	III
	$-$ 118 sin $(v+s)$	2,0719	IV
	$-$ 175 sin η	2,2430	V
	$+$ 2115 sin 2η	3,32531	
	$+$ 4 sin 3η	0,602	
	$-$ 8 sin 4η	0,903	
	$+$ 59 sin $(\eta-v)$	1,7708	VI
	$+$ 352 sin $(2\eta-2v)$	2,5465	
	$-$ 2729 sin $(2\eta-v)$	3,67477	VII
	$-$ 93 sin $(4\eta-2v)$	1,9685	
	$+$ 56 sin $(2\eta+v)$	1,7482	VIII
	$+$ 59 sin $(4\eta-v)$	1,7708	IX
	$-$ 49 sin $(\eta+s)$	1,6902	X
	$-$ 76 sin $(2\eta-s)$	1,8808	XI
	$-$ 57 sin $(2\eta+s)$	1,7559	XII
	$+$ 154 sin $(2\eta-v+s)$	2,1875	XIII

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+	45 sin $(2\eta-r-s)$	1,6532	XIV
—	411 sin $(2\varphi-2\pi)$	2,6138	XV
—	205 sin $(2\theta-2\pi)$	2,3117	XVI
—	6 sin $(4\theta-4\pi)$	0,778	
+	187 sin $(2\varphi-2\pi-r)$	2,2718	XVII
+	80 sin $(2\varphi-2\pi-2r)$	1,9031	XVIII
—	15 sin $(2\theta-2\pi-r)$	1,176	XIX
—	10 sin $(2\theta-2\pi+r)$	1,000	XX

§. 314. Inaequalitates has ita disposui, vt eas, quae vna tabula comprehendi possunt, coniunctim exposuerim, quo facilius calculus expediri queat. Hinc igitur patet omissis iis inaequalitatibus, quae 10″ non superant, locum lunae per viginti inaequalitates corrigi debere, antequam vera eius longitudo obtineatur.

§. 315. Haec autem expressio adhuc isto defectu laborat, quod pleraeque inaequalitates ipsam lunae longitudinem veram φ, quae tamen demum quaeritur, involuant, ideoque calculus, cum longitudo lunae etiamnunc est incognita, commode expediri non possit. Quoniam tamen sufficit longitudinem lunae proxime tantum nosse, cum longitudo media per quatuor priores inaequalitates fuerit correcta, ea pro sequentibus inaequalitatibus loco longitudinis verae vsurpari poterit, sicque tandem longitudo lunae multo exactior reperietur. Quo facto si accuratior desideretur, omnes inaequalitates post 4 priores denuo ad calculum reuocari conueniet, iisque euolutis longitudo lunae vera prodibit, quae nulla amplius correctione indigebit. Interim

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tamen ne calculum per se satis taedioſum bis repetere opus sit, non difficulter hanc expreſſionem ita transformare licet, vt locus lunae per quatuor tantum priores inaequalitates correctus sine errore in sequentibus loco φ adhiberi possit.

§. 316. Cum autem longitudo lunae iam per obseruationes fuerit cognita, haec expreſſio sine vlla immutatione ad calculum accommodabitur, vt hoc modo consensus theoriae cum veritate exploretur. In inaequalitatibus enim determinandis pro littera φ vbique longitudo lunae obseruata introducetur, calculoque peracto patebit, quantum locus lunae per calculum definitus etiamnunc discrepet ab eius loco vero obseruato. Atque si hoc modo plurimae obseruationes calculo subiiciantur, ex aberrationibus a veritate non solum elementa, quibus haec formula innititur, accuratius definire licebit, sed etiam inaequalitates, quae nondum satis certae videntur, inde emendari poterunt. Quin etiam nouae inaequalitates, quas per Theoriam determinare non licuerat, hoc modo forte certius colligi poterunt.

§. 317. Antequam autem huiusmodi calculi specimen exhiberi queat, necesse est vt aequationem pro loco nodi vero inueniendo ad calculum accommodemus. Formulae autem supra (219) exhibitae, si pro r substituamus valorem inuentum $r = p - 2k \sin r - \frac{3}{4} kk \sin 2r$, pars: Const. $- 0,004053\,p$ indicabit longitudinem nodi mediam. Hincque longitudo nodi vera erit $= \pi$

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		Log.coeff.
π = Long.med.	$- 107'' \sin r$	2, 0294
	$- 6 \sin 2r$	0, 778
	$+ 551 \sin s$	2, 7411
	$- 453 \sin 2\eta$	2, 6561
	$- 129 \sin (2\eta - r)$	2, 1106
	$- 33 \sin (2\eta + r)$	1, 518
	$+ 55 \sin (2\eta - 2r)$	1, 740
	$+ 420 \sin (2\Phi - 2\pi)$	2, 6232
	$+ 98 \sin (2\Phi - 2\pi - r)$	1, 991
	$+ 30 \sin (2\Phi - 2\pi + r)$	1, 477
	$+ 235 \sin (2\Phi - 2\pi - 2r)$	2, 3711
	$+ 5426 \sin (2\theta - 2\pi)$	3, 73448
	$+ 75 \sin (4\theta - 4\pi)$	1, 875
	$- 53 \sin (2\theta - 2\pi - r)$	1, 724
	$+ 53 \sin (2\theta - 2\pi + r)$	1, 724
	$- 90 \sin (2\theta - 2\pi - s)$	1, 954
	$- 32 \sin (2\theta - 2\pi + s)$	1, 505

§. 318. In hoc calculo plerasque inaequalitates omittere licet, siquidem tantum longitudinem lunae investigare sit propositum: manifestum enim est, etiamsi in loco nodi error plurium minutorum primorum committatur, inde vix errorem aliquot minutorum secundorum in longitudinem lunae redundare. Quodsi vero eclipsis cuiuspiam omnia phaenomena diligenter definire velimus, tum locum nodi exactissime cognitum esse oportet. Praeterea vero pro latitudine assignanda vera inclinatio orbitae lunaris ad eclipticam ex media ε accuratissime erit definienda ope huius formulae:

$\varepsilon = \varepsilon$

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		Log coeff.
$\varrho = \varepsilon$	$- 2^N \cos r$	0, 30
	$- 48 \cos 2\eta$	1, 681
	$+ 11 \cos(2\eta - r)$	1, 041
	$+ 3 \cos(2\eta + r)$	0, 48
	$+ 36 \cos(2\Phi - 2\pi)$	1, 556
	$+ 9 \cos(2\Phi - 2\pi - r)$	0, 95
	$+ 3 \cos(2\Phi - 2\pi + r)$	0, 48
	$+ 23 \cos(2\Phi - 2\pi - 2r)$	1, 362
	$+ 484 \cos(2\theta - 2\pi)$	2, 6848
	$+ 9 \cos(4\theta - 4\pi)$	0, 95
	$- 5 \cos(2\theta - 2\pi - r)$	0, 70
	$+ 5 \cos(2\theta - 2\pi - + r)$	0, 70
	$- 7 \cos(2\theta - 2\pi - s)$	0, 84
	$- 3 \cos(2\theta - 2\pi + s)$	0, 48

Tabula autem pro distantia lunae a terra, vnde eius parallaxis et diameter apparens definiatur, ex formulis supra exhibitis facile construetur.

ADDI-

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ADDITAMENTUM

CONTINENS ALIAS METHODOS INUESTIGANDI MOTUS LUNAE INAEQUALITATES.

Qui methodum ante descriptam accuratius euoluerit, eam quidem in se spectatam satis bonam atque plerisque lunae inaequalitatibus definiendis aptam deprehendet; interim tamen fateri cogor, eam non solum maxime esse operosam, sed etiam ita comparatam, vt plures inaequalitates, quae tamen motum lunae imprimis afficere videntur, non satis exacte exhibeat, et quasi in dubio relinquat. Causa huius incertitudinis manifesto in hoc est sita, quod omnes inaequalitates ita inter se sunt connexae, vt nullius valor verus accurate definiri possit, quin simul reliquae inaequalitates omnes fuerint cognitae. Cum igitur eiusmodi methodo approximandi sim vsus, vt primo quasdam inaequalitates tanquam cognitas assumserim, ex quibus deinceps reliquas definiuerim, probe notandum est ab his inuentis iterum priores, quae erant assumtae, leuem quandam mutationem pati; quae si statim ab initio nota fuisset, etiam reliquarum valores aliquantillum mutati prodiissent: at quaedam inaequalitates adeo sunt lubricae, vt facta vel minima mutatione in iis, a quibus pendent, inde non exiguam alterationem trahant. Huc imprimis pertinet motus apogei, cuius inuestigatio omnes omnino inaequa-

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litates implicat, ita vt ſine harum cognitione neutiquam accurate definiri queat.

Cum igitur haec methodus iſtis tantis incommodis ſit obnoxia, aliam maxime diuerſam tentaui viam, quae ab iis eſſet libera, etiamſi negare nequeam, etiam hanc ſuis non carere incommodis, quae tamen prorſus alius ſunt generis. Ex quo confido his duabus diuerſis methodis combinandis haud exiguum fructum in veram motuum lunarium cognitionem eſſe redundaturum. Praecipuum autem diſcrimen verſatur in electione anomaliae, quae in ſuperiore methodo non ita eſt aſſumta, vt diſtantia lunae a terra fieret vel maxima vel minima, ſi anomalia vel $= 0$ vel $= 180°$ ſtatuatur: neque enim differentiale diſtantiae dx euaneſcit, quando ſinus anomaliae in nihilum abit, ſed praeterea etiamnunc ab elongatione ſolis a luna ſeu angulo η pendet. Ita ſecundum hanc methodum neque apogaeum lunae neque perigaeum ibi ſtatuitur, ubi angulus, quem motus lunae directio cum radio vectore facit, eſt rectus; ſed plerumque in alia puncta incidunt, quae ab iis locis, vbi luna terrae vel eſt proxima, vel ab ea maxime remota, notabiliter ſint diuerſa. Etſi autem in hoc calculo non verae lineae abſidum poſitio conſideratur, hinc tamen methodus minime vitioſa eſt reputanda; propterea quod non eſt quaeſtio, quo nomine quaepiam orbitae lunaris puncta appellentur, dummodo cunctae inaequalitates recte exprimantur. Sed quoniam circa has ipſas inaequalitates nonnulla grauiora dubia ſunt orta, haud abs re fore arbitror, et alteram methodum hic proponere.

I. Sit

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I.

Sit igitur vt ante: Maſſa ſolis $= \odot$; terrae $= \mathfrak{t}$ et lunae $= \mathfrak{D}$; atque vis attractiua terrae in diſtantia d vt $\frac{1}{dd} - \frac{1}{bb}$; manente vi ſolis quadrato diſtantiae exacte proportionali. Tum vero ſit

Longitudo lunae $= \varphi$; latitudo $= \psi$; et diſtantia curtata $= x$

Longitudo ſolis $= \theta$; eiusque a terra diſtantia $= y$

Longitudo nodi aſcendentis lunae $= \pi$ et inclinatio ad eclipticam $= \varrho$

ac ponatur breuitatis ergo elongatio lunae a ſole $\varphi - \theta = \eta$ et diſtantia $\sqrt{(xx \text{ ſec. } \psi^2 - 2xy \cos \eta + yy)} = z$.

Quibus poſitis ſupra §. 20. vidimus motum lunae his quatuor aequationibus contineri:

I. $2dxd\varphi + xdd\varphi = -\frac{1}{2} dt^2 . \odot \left(\frac{y}{z^3} - \frac{1}{yy}\right) \sin \eta$

II. $ddx - xd\varphi^2 = -\frac{1}{2} dt^2 (\mathfrak{t} + \mathfrak{D}) \cos \psi^3 \left(\frac{1}{xx} - \frac{1}{bb}\right)$
$- \frac{1}{2} dt^2 . \odot \left(\frac{x - y \cos \eta}{z^3} + \frac{\cos \eta}{yy}\right)$

III. $d\pi = -\frac{1}{2} dt^2 . \odot \left(\frac{y}{z^3} - \frac{1}{yy}\right) \frac{\sin (\varphi - \pi) \sin (\theta - \pi)}{xd\varphi}$

IV. $dl \tang \varrho = \frac{d\pi}{\tang (\varphi - \pi)}$, et $\tang \psi = \tg \varrho \cos (\varphi - \pi)$

vbi elementum temporis dt ſumtum eſt pro conſtante.

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II.

Quatenus hic motus solis ingreditur, is pro regulari atque regulis Kepleri conformi haberi poterit: habebimus ergo

$$2dydθ + ydd\theta = 0 \text{ et } ddy - yd\theta^2 = -\tfrac{1}{2}dt^2.\frac{\odot+\delta}{yy}$$

vnde si ponamus orbitae solaris:

semiparametrum $= c$; excentricitatem $= e$ et anomaliam veram $= u$

erit $y = \frac{c}{1-e\cos u}$; $du = d\theta = \frac{dt}{yy}\sqrt{\tfrac{1}{2}c(\odot+\gamma)}$

Sit a semiaxis transuersus orbitae solis, ac tempore $= t$ sol motu medio absoluat angulum $= \omega$, quo pro mensura temporis t vtamur: erit ergo $d\omega = \frac{dt}{aa}\sqrt{\tfrac{1}{2}a(\odot+\delta)}$

ideoque $\frac{1}{2}dt^2 = \frac{a^3 d\omega^2}{\odot+\delta}$. At est $a = \frac{c}{1-ee}$. Hinc ergo fit

$$du = d\theta = \frac{adω}{yy}\sqrt{ac} = \frac{aadω}{yy}\sqrt{(1-ee)} = \frac{dω(1-e\cos u)^2}{(1-ee)\sqrt{(1-ee)}}$$

sicque tam du quam $d\theta$ per elementum $d\omega$ loco temporis introductum expressimus. Quia autem massa solis $\odot$ massam terrae δ tam enormiter excedit, sine errore pro $\frac{1}{2}dt^2$ scribi poterit $\frac{a^3 d\omega^2}{\odot}$, eruntque nostrae aequationes pro luna:

I. $$2dx d\Phi + x dd\Phi = -a^3 d\omega^2 \left(\frac{y}{z^3} - \frac{1}{yy}\right) \sin\eta$$

II. $$ddx - x d\Phi^2 = -\frac{a^3(\delta+\mathfrak{D})d\omega}{\odot}\cos\psi^3\left(\frac{1}{xx}-\frac{1}{bb}\right) - a^3 d\omega^2\left(\frac{x-y\cos\eta}{z^3}+\frac{\cos\eta}{yy}\right)$$

III. dz

III. $d\pi = -\frac{a^3 d\omega^2}{x d\Phi}\left(\frac{y}{z^3} - \frac{1}{yy}\right) \sin(\Phi-\pi) \sin(\theta-\pi)$

IV. $d.l\,\text{tang}\,\varrho = \frac{d\pi}{\text{tg}(\Phi-\pi)}$; atque ob $\text{tg}\,\psi = \text{tg}\,\varrho\,\text{cf}(\Phi-\pi)$, habebitur proxime $\text{cof}\psi^3 = 1 - \frac{3}{4}\text{tg}\,\varrho^2 - \frac{3}{4}\text{tg}\,\varrho^2\,\text{cf}\,2(\Phi-\pi)$.

III.

Incipiamus a duabus aequationibus prioribus, ac ponamus breuitatis gratia

$$a^3\left(\frac{y}{z^3} - \frac{1}{yy}\right)\sin\eta = M \quad \text{et}$$

$$\frac{a^3(\mathfrak{D}+\mathfrak{D})}{\odot}\cos\psi^3\left(\frac{1}{xx} - \frac{1}{bb}\right) + a^3\left(\frac{x - y\cos\eta}{z^3} + \frac{\cos\eta}{yy}\right) = \frac{A}{xx} + N$$

quandoquidem haec posterior expressio terminum involuit formae $\frac{A}{xx}$ prae ceteris incomparabiliter maiorem; atque habebimus has duas aequationes:

$$2dx\,d\Phi + x\,ddd\Phi = -M d\omega^2 \quad \text{et} \quad ddx - x\,d\Phi^2 = -\frac{A d\omega^2}{xx} - N d\omega^2$$

quarum prior per $2x^3 d\Phi$ multiplicata ob $d\omega$ constans habebit integrale:

$$x^4 d\Phi^2 = -2 d\omega^2 \int M x^3\, d\Phi$$

Tum prior multiplicata per $2x\,d\Phi$ addatur ad posteriorem per $2dx$ multiplicatam, eritque, aggregatum:

$$2x\,dx\,d\Phi^2 + 2xx\,d\Phi\,ddd\Phi + 2dx\,ddx = -2Mx\,d\omega^2 d\Phi - \frac{2A\,d\omega^2 dx}{xx} - 2N\,d\omega^2 dx$$

Cuius integrale erit:

$$dx^2 + xx\,d\Phi^2 = +\frac{2A\,d\omega^2}{x} - 2d\omega^2 \int (M x\,d\Phi + N\,dx)$$

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IV.

Ponantur formulae integrales, quae in his expressionibus insunt:

$$-\int M x^3 d\Phi = P \quad \text{et} \quad -\int (M x d\Phi + N dx) = Q$$

vt habeamus has duas aequationes:

$$x^4 d\Phi^2 = 2P d\omega^2 \quad \text{et} \quad dx^2 + xx d\Phi^2 = \frac{2A d\omega^2}{x} + 2Q d\omega^2$$

vnde cum sit $xx d\Phi^2 = \frac{2P d\omega^2}{xx}$ erit

$$dx^2 = 2d\omega^2 \left(Q + \frac{A}{x} - \frac{P}{xx}\right) \quad \text{et} \quad dx = + d\omega \sqrt{2}\left(Q + \frac{A}{x} - \frac{P}{xx}\right)$$

sicque differentiale dx per $d\omega$ exprimitur. Deinde vero habetur

$$d\Phi = \frac{d\omega}{xx} \sqrt{2P}$$

estque per hypothesin:

$$dP = -M x d\omega \sqrt{2P} \quad \text{et} \quad dQ = -\frac{M d\omega}{x} \sqrt{2P} + N d\omega \sqrt{2}\left(Q + \frac{A}{x} - \frac{P}{xx}\right)$$

vbi quidem signorum ambiguorum inferius locum habere statuamus, quia motum ab apogeo numerare in animo est, ita vt hinc exeundo distantia x minuatur.

V.

Cum igitur differentiale dx in apogeo et perigeo euanescat, necesse est vt his locis formula irrationalis $\sqrt{\left(Q + \frac{A}{x} - \frac{P}{xx}\right)}$ in nihilum abeat, in reliquis autem locis valorem sortiatur realem. Commodissime ergo haec formula per sinum cuiuspiam anguli v exhibebitur, qui cum in apogeo euanescat, in perigeo autem duobus rectis

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ctis aequalis fiat, anomaliam lunae referet: idque ſenſu vero, ita vt diſtantia x in apogeo prodeat maxima, in perigeo vero minima. Sit igitur vt formam motus regularis ſequamur:

ſemilatus rectum orbitae lunaris $= p$
excentricitas orbitae $= q$
et anomalia vera lunae $= v$

eritque hinc per eandem legem diſtantia $x = \frac{p}{1 - q \cos v}$.

Verum hic quantitates p et q, quae in motu regulari eſſent conſtantes, nunc pro variabilibus ſunt habendae, earumque variabilitas per variabilitatem quantitatum P et Q, quae in motu regulari itidem ſunt conſtantes, determinari debebit.

VI.

Subſtituamus ergo valorem aſſumtum $x = \frac{p}{1 - q \cos v}$ in formula irrationali $V\left(Q + \frac{A}{x} - \frac{P}{xx}\right)$, quae abibit in

$$\frac{1}{p} V (Qpp + Ap (1 - q \cos v) - P (1 - q \cos v)^2)$$

et euoluta dabit

$$\frac{1}{p} V (Qpp + Ap - P - Apq \cos v + 2Pq \cos v - Pqq \cos v^2)$$

quae vt reducatur ad formam $V \sin v$, ſtatuatur

primo $2P - Ap = 0$
tum vero $Qpp + Ap - P = Pqq$

ac noſtra formula fiet $= \frac{1}{p} V Pqq \sin v^2 = \frac{q \sin v}{p} V P$, habebimusque

dx

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$$dx = -\frac{q\,d\omega \sin v}{p}\sqrt{2P} \quad \text{et}$$

$$dQ = -\frac{M\,d\omega}{x}\sqrt{2P} + \frac{N q\,d\omega \sin v}{p}\sqrt{2P}$$

VII.

Cum iam ſit $2P - Ap = o$; erit $P = \frac{1}{2}Ap$: quo valore in altera formula ſubſtituto orietur:

$$Qpp + \tfrac{1}{2}Ap = \tfrac{1}{2}Apqq \quad \text{ſeu} \quad Q = -\frac{A}{2p}(1-qq)$$

Sumantur nunc differentialia; eritque
$dP = -Mx\,d\omega\sqrt{2P} = \frac{1}{2}A\,dp$, quae ob $2P = Ap$ abit in hanc

$$-Mx\,d\omega\sqrt{Ap} = \tfrac{1}{2}A\,dp, \text{ ſiue } dp = -\frac{2Mx\,d\omega}{A}\sqrt{Ap}$$

vel etiam $\sqrt{Ap} = -\int M x\,d\omega$
Simili modo erit

$$dQ = +\frac{A\,dp(1-qq)}{2pp} + \frac{Aq\,dq}{p} = -\frac{Mx\,d\omega(1-qq)}{pp}\sqrt{Ap} + \frac{Aq\,dq}{p}$$

ideoque

$$\frac{Aq\,dq}{p} = M\,d\omega\left(\frac{x(1-qq)}{pp} - \frac{1}{x}\right)\sqrt{Ap} + \frac{Nq\,d\omega \sin v}{p}\sqrt{Ap}$$

At eſt $\frac{x(1-qq)}{pp} - \frac{1}{x} = \frac{x}{pp}\left(1-qq-\frac{pp}{xx}\right) = \frac{x}{pp}(1-qq-1+2q\cos v - qq\cos v^2)$

ſiue $\frac{x(1-qq)}{pp} - \frac{1}{x} = \frac{qx}{pp}(2\cos v - q - q\cos v^2)$

Hinc ergo colligitur:

$$dq = \frac{Mx\,d\omega}{Ap}(2\cos v - q - q\cos v^2)\sqrt{Ap} + \frac{N\,d\omega \sin v}{A}\sqrt{Ap} \text{ ſiue}$$

$$dq = d\omega\left(\frac{M}{A}(2\cos v - \frac{q\sin v^2}{1-q\cos v}) + \frac{N}{A}\sin v\right)\sqrt{Ap}$$

VIII.

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VIII.

Inuenta iam relatione differentialium dx, dp et dq ad differentiale temporis $d\omega$ ſcilicet:

$$dx = -\frac{q\,d\omega \sin v}{q}\sqrt{Ap}\,; \quad dp = -\frac{2M x\,d\omega}{A}\sqrt{Ap}$$

et $dq = d\omega\left(\frac{M}{A}(2\cos v - \frac{q\sin v^2}{1-q\cos v}) + \frac{N}{A}\sin v\right)\sqrt{Ap}$

ſupereſt, vt quoque relationem elementi anomaliae dv definiamus. Cum igitur ſit

$x = \frac{p}{1-q\cos v}$, erit $1 - q\cos v = \frac{p}{x}$; hincque differentiando

$$q\,dv\sin v = dq\cos v + \frac{dp}{x} - \frac{p\,dx}{xx};$$

ſubſtituantur valores pro dq, dp et dx inuenti; ac diuiſione facta per $q\sin v$ prodibit

$$dv = \frac{d\omega}{xx}\sqrt{Ap} - \frac{d\omega}{q}\left(\frac{M}{A}(2\sin v + \frac{q\sin v\cos v}{1-q\cos v}) - \frac{N}{A}\cos v\right)\sqrt{Ap}$$

Pro elemento autem longitudinis $d\Phi$ ob $2P = Ap$, ex antecedentibus habemus:

$$d\Phi = \frac{d\omega}{xx}\sqrt{Ap} = \frac{d\omega(1-q\cos v)^2}{pp}\sqrt{Ap}.$$

IX.

Ex his formulis ſtatim ſe offert motus apogei; cum enim longitudo apogei ſit $= \Phi - v$, erit eius differentiale pro tempuſculo $d\omega$:

$$d\Phi - dv = \frac{d\omega}{q}\left(\frac{M}{A}(2\sin v + \frac{q\sin v\cos v}{1-q\cos v}) - \frac{N}{A}\cos v\right)\sqrt{Ap}$$

cuius ergo integrale praebebit verum motum apogei cum omnibus inaequalitatibus, quibus perturbatur. Vnde

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quidem perſpicitur, quod per ſe eſt manifeſtum, ſi quantitates M et N euaneſcerent, motum apogei fore nullum, ſeu apogeum perpetuo in loco fixo eſſe permanſurum. Deinde etiam iuuabit notaſſe has formulas:

$$d.\ q\cos v = -q\,d\Phi \sin v + \frac{2M}{A}\,d\omega\sqrt{Ap}$$

$$d.\ q\sin v = +q\,d\Phi\cos v + d\omega\left(\frac{N}{A}-\frac{M}{A}\cdot\frac{q\sin v}{1-q\cos v}\right)\sqrt{Ap}$$

Tandem quoque habemus ex motu ſolis $du = d\theta = \frac{d\omega(1-e\cos u)^2}{(1-ee)\sqrt{(1-ee)}}$ ideoque

$$d\eta = d\Phi - d\theta = d\omega\left(\frac{(1-q\cos v}{pp}\sqrt{Ap} - \frac{(1-e\cos u)^2}{(1-ee)\sqrt{(1-ee)}}\right)$$

X.

Inuentis nunc omnium differentialium relationibus ad elementum temporis $d\omega$, euoluamus valores litterarum M et N, ac primo quidem cum ſit $z = \sqrt{(yy-2xy\cos\eta + xx\,\text{ſec.}\,\psi^2)}$; quoniam quantitas z nonniſi in terminis minimis occurrit, pro ſec. ψ tuto unitas ſcribi poterit, et quia y tantopere excedit x, erit proxime

$$\frac{1}{z^3} = \frac{1}{y^3} + \frac{3x}{y^4}\cos\eta + \frac{3xx}{2y^5}(5\cos\eta^2 - 1) \quad \text{ſiue}$$

$$\frac{1}{z^3} = \frac{1}{y^3} + \frac{3x}{y^4}\cos\eta + \frac{3xx}{4y^5}(3+5\cos 2\eta)$$

Ideoque hinc habebitur:

$$\frac{y}{z^3} - \frac{1}{yy} = \frac{3x}{y^3}\cos\eta + \frac{3xx}{4y^4}(3+5\cos 2\eta)$$

Vnde

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Vnde obtinemus:

$$M = \alpha^3 \left(\frac{3x}{2y^3} \sin 2\eta + \frac{3xx}{8y^4} (\sin \eta + 5 \sin 3\eta) \right)$$

$$N = \frac{\alpha^3 (\mathfrak{G} + \mathfrak{D})}{\odot} \cos \psi^3 \left(\frac{1}{xx} - \frac{1}{bb} \right) - \frac{A}{xx}$$
$$- \alpha^3 \left(\frac{x}{2y^3} (1 + 3\cos 2\eta) + \frac{3xx}{8y^4} (3\cos \eta + 5\cos 3\eta) \right)$$

XI.

Cum sit proxime $\cos \psi^3 = 1 - \frac{3}{4} \tan \varrho^2 - \frac{3}{4} \tan \varrho^2 \cos 2(\Phi - \pi)$, eius valor vnitate erit minor, atque ex parte constante, et parte variabili constabit, quae illa multo erit minor. Ponatur ergo

$\cos \psi^3 = \lambda + \Pi$; vt sit $\Pi = 1 - \lambda - \frac{3}{4} \tan \varrho^2 - \frac{3}{4} \tan \varrho^2 \cos 2(\Phi - \pi)$

vbi λ denotat partem constantem vnitate proxime aequalem, Π vero partem variabilem.

Erit ergo:

$$N = \frac{\lambda \alpha^3 (\mathfrak{G} + \mathfrak{D})}{\odot} \left(\frac{1}{xx} - \frac{1}{bb} \right) - \frac{A}{xx} + \frac{\alpha^3 (\mathfrak{G} + \mathfrak{D})}{\odot} \Pi \left(\frac{1}{xx} - \frac{1}{bb} \right)$$
$$- \alpha^3 \left(\frac{x}{2y^3} (1 + 3\cos 2\eta) + \frac{3xx}{8y^4} (3\cos \eta + 5\cos 3\eta) \right)$$

Statuatur nunc $A = \frac{\lambda \alpha^3 (\mathfrak{G} + \mathfrak{D})}{\odot}$; vt fiat

$$N = - \frac{A}{bb} + A\Pi \left(\frac{1}{xx} - \frac{1}{bb} \right)$$
$$- \alpha^3 \left(\frac{x}{2y^3} (1 + 3\cos 2\eta) + \frac{3xx}{8y^4} (3\cos \eta + 5\cos 3\eta) \right)$$

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ac ponatur breuitatis gratia: $p = b(1+\xi)$

erit $\frac{\sqrt{Ap}}{pp} = \sqrt{\frac{A}{b^3(1+\xi)^3}} = (1-\frac{3}{2}\xi+\frac{15}{8}\xi^2)\sqrt{\frac{A}{b^3}}$

ob ξ prae 1 vehementer paruum, ſitque porro:

$$\sqrt{\frac{A}{b^3}} = \sqrt{\frac{\lambda a^3(\mathfrak{S}+\mathfrak{D})}{\odot}} = m,$$

atque habebitur $d\Phi = mdw\,(1-\frac{3}{2}\xi+\frac{15}{8}\xi^2)\,(1-q\cos v)^2$

XII.

Subſtituantur nunc pro x et y valores $\frac{p}{1-q\cos v}$ et $\frac{c}{1-e\cos u}$, eritque

$$M = a^3\left(\frac{3p(1-e\cos u)^3}{2c^3(1-q\cos v)}\sin 2\eta + \frac{3pp(1-e\cos u)^4}{8c^4(1-q\cos v)^2}(\sin\eta+5\sin 3\eta)\right)$$

$$N = -\frac{A}{bb} + A\Pi\left(\frac{(1-q\cos v)^2}{pp} - \frac{1}{bb}\right)$$

$$-a^3\left(\frac{p(1-e\cos u)^3}{2c^3(1-q\cos v)}(1+3\cos 2\eta)+\frac{3pp(1-e\cos u)^4}{8c^4(1-q\cos v)^2}(3c\eta+5c3\eta)\right)$$

vbi quidem quoque terminus $\frac{A\Pi}{bb}$ prae termino $\frac{A}{bb}$ omitti poteſt. Nunc vt hinc valores $\frac{M}{A}\sqrt{Ap}$ et $\frac{N}{A}\sqrt{Ap}$ commode exprimantur, erit

$$\frac{a^3b}{Ac^3}\sqrt{Ab} = \frac{a^3}{mc^3} = \frac{1}{m(1-ee)^3} = \frac{1+3ee}{m}$$

quoniam in his terminis minimis pro $1-ee$ ſcribere licet 1. Tum vero ſit $\frac{b}{c} = n$, eritque n fractio valde parua.

XIII.

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XIII.

Factis ergo his ſubſtitutionibus, ob $p = b\,(1+\xi)$ habebimus:

$$\frac{M}{A}\,VAp = \frac{3\,(1+3ee)}{2m}\,\frac{(1-e\cos u)^3}{1-q\cos v}\,(1+\tfrac{3}{2}\xi)\,\sin 2\eta$$
$$+\,\frac{3n}{8m}\,\frac{(1-e\cos u)^4}{(1-q\cos v)^2}\,(1+\tfrac{5}{2}\xi)\,(\sin\eta+5\sin 3\eta)$$

Pro altera valore $\frac{N}{A}VAp$ ſtatuatur terminus minimus:

$$\frac{VAb}{bb} = V\,\frac{\lambda a^3 b\,(\mathfrak{S}+\mathfrak{D})}{\odot b^4} = i\;;\;\text{eritque}$$

$$\frac{N}{A}VAp = -\,\frac{(1+3ee)}{2m}\,\frac{(1-e\cos u)^3}{1-q\cos v}\,(1+\tfrac{3}{2}\xi)\,(1+3\cos 2\eta)$$
$$-\,\frac{3n}{8m}\,\frac{(1-e\cos u)^4}{(1-q\cos v)^2}\,(1+\tfrac{5}{2}\xi)\,(3\cos\eta+5\cos 3\eta)$$
$$+\,m\,(1-q\cos v)^2\,(1-\tfrac{3}{2}\xi)\,\Pi - i$$

vbi notari oportet, terminos per n multiplicatos ratione praecedentium eſſe minimos; tum vero quantitates ξ et Π atque multo magis i eſſe fractiones prae vnitate fere euaneſcentes.

XIV.

Quoniam hi ipſi termini quantitates M et N involuentes ſunt valde parui, in iis ſine errore altiores poteſtates vtriusque excentricitatis q et e negligi poſſunt. In terminis ergo primis ſimpliciter per m diuiſis excentricitates tantum ad duas dimenſiones intro-

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ducantur, in terminis autem per $\frac{n}{m}$ multiplicatis penitus omittantur, quia fractio n iam fere quadrato excentricitatis q aequiualet. In termino autem littera minima Π affecto, quia is per numerum m satis magnum, vtpote 13 fere, est multiplicatus, excentricitas q vnius dimensionis retineatur.

His obseruatis habebimus:

$$\frac{M}{A}\nu Ap = \begin{cases} +\frac{3}{2m}(1+\frac{2}{3}ee+\frac{1}{2}qq)\sin 2\eta + \frac{3q}{4m}\sin(2\eta-v) \\ +\frac{3q}{4m}\sin(2\eta+v) - \frac{9e}{4m}\sin(2\eta-u) - \frac{9e}{4m}\sin(2\eta+u) \\ +\frac{39q}{8m}\sin(2\eta-2v) + \frac{39q}{8m}\sin(2\eta+2v) \\ +\frac{9ee}{8m}\sin(2\eta-2u) + \frac{9ee}{8m}\sin(2\eta+2u) \\ -\frac{9eq}{8m}\sin(2\eta-v+u) - \frac{9eq}{8m}\sin(2\eta+v-u) \\ -\frac{9eq}{8m}\sin(2\eta-v-u) - \frac{9eq}{8m}\sin(2\eta+v+u) \\ +\frac{9}{4m}\xi\sin 2\eta + \frac{9}{8m}q\xi\sin(2\eta-v) + \frac{9}{8m}q\xi\sin(2\eta+v) \\ -\frac{27}{8m}e\xi\sin(2\eta-u) - \frac{27}{8m}e\xi\sin(2\eta+u) \\ +\frac{3n}{8m}\sin\eta + \frac{15n}{8m}\sin 3\eta \end{cases}$$

$$\frac{N}{A}\nu Ap =$$

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$$\frac{N}{A}\sqrt{A}p = \begin{cases} -\frac{1}{2m}(1+\frac{3}{2}ee+\frac{1}{2}qq) - \frac{3}{2m}(1+\frac{3}{2}ee+\frac{1}{2}qq)\cos 2\eta \\ -\frac{q}{2m}\cos v + \frac{3e}{2m}\cos u \\ -\frac{3q}{4m}\cos(2\eta - v) - \frac{3q}{4m}\cos(2\eta + v) \\ +\frac{9e}{4m}\cos(2\eta - u) + \frac{9e}{4m}\cos(2\eta + u) \\ -\frac{qq}{4m}\cos 2v + \frac{3eq}{4m}\cos(v-u) + \frac{3eq}{4m}\cos(v+u) \\ -\frac{3ee}{4m}\cos 2u \\ -\frac{3qq}{8m}\cos(2\eta - 2v) - \frac{3qq}{8m}\cos(2\eta + 2v) \\ -\frac{9ee}{8m}\cos(2\eta - 2u) - \frac{9ee}{8m}\cos(2\eta + 2u) \\ +\frac{9eq}{8m}\cos(2\eta - v + u) + \frac{9eq}{8m}\cos(2\eta + v - u) \\ +\frac{9eq}{8m}\cos 2\eta - v - u) + \frac{9eq}{8m}\cos(2\eta + v + u) \\ -\frac{3}{4m}\xi - \frac{9}{4m}\xi\cos 2\eta - \frac{3q}{8m}\xi\cos v \\ -\frac{9q}{8m}\xi\cos(2\eta - v) - \frac{9q}{8m}\xi\cos(2\eta + v) \\ +\frac{9e}{4m}\xi\cos u + \frac{27e}{8m}\xi\cos(2\eta - u) \\ +\frac{27e}{8m}\xi\cos(2\eta + u) - \frac{9n}{8m}\cos\eta - \frac{15n}{8m}\cos 3\eta \\ +m\Pi - 2mq\Pi\cos v - \frac{3}{2}m\xi\Pi - \text{&c.} \end{cases}$$

XV.

XV.

Quaeramus igitur valores euolutos noſtrorum differentialium ad elementum temporis applicatorum: ac primo quidem habebimus:

$$\frac{d\Phi}{d\omega} = m(1+\tfrac{1}{2}qq) - 2mq\cos v + \tfrac{1}{2}mqq\cos 2v - \tfrac{3}{2}m(1+\tfrac{1}{2}qq)\xi + 3mq\xi\cos v - \tfrac{3}{4}mqq\xi\cos 2v + \tfrac{15}{8}m\xi\xi$$

$$\frac{du}{d\omega} = \frac{d\theta}{d\omega} = 1 + 2ee - 2e\cos u + \tfrac{1}{2}ee\cos 2u;$$ vnde concludimus

$$\frac{d\eta}{d\omega} = m(1+\tfrac{1}{2}qq) - 1 - 2ee - 2mq\cos v + 2e\cos u + \tfrac{1}{2}mqq\cos 2v - \tfrac{1}{2}ee\cos 2u - \tfrac{3}{2}m(1+\tfrac{1}{2}qq)\xi + 3mq\xi\cos v - \tfrac{3}{4}mqq\xi\cos 2v + \tfrac{15}{8}m\xi\xi$$

Deinde cum ſit $\frac{dp}{d\omega} = -2x.\frac{M}{A}\sqrt{A}p = -\frac{2b(1+\xi)}{1-q\cos v}.\frac{M}{A}\sqrt{A}p$, ob $p = b(1+\xi)$ erit $\frac{d\xi}{d\omega} =$

$$\left(-2(1+\tfrac{1}{2}qq) - 2q\cos v - qq\cos 2v - 2\xi - 2q\xi\cos v\right)\frac{M}{A}\sqrt{A}p$$

ac valorem pro $\frac{M}{A}\sqrt{A}p$ inuentum ſubſtituendo obtinebimus ſequentes formulas:

$$\frac{d\xi}{d\omega} =$$

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$$\frac{d\xi}{d\omega} = \begin{cases} -\frac{3}{m}(1+\frac{9}{2}ee+\frac{3}{2}qq)\sin 2\eta - \frac{3q}{m}\sin(2\eta-v) - \frac{3q}{m}\sin(2\eta+v) \\ +\frac{9e}{2m}\sin(2\eta-u) + \frac{9e}{2m}\sin(2\eta+u) \\ -\frac{9qq}{4m}\sin(2\eta-2v) - \frac{9qq}{4m}\sin(2\eta+2v) \\ -\frac{9ee}{4m}\sin(2\eta-2u) - \frac{9ee}{4m}\sin(2\eta+2u) \\ +\frac{9eq}{2m}\sin(2\eta-v+u) + \frac{9eq}{2m}\sin(2\eta+v-u) \\ +\frac{9eq}{2m}\sin(2\eta-v-u) + \frac{9eq}{2m}\sin(2\eta+v+u) \\ -\frac{15}{2m}\xi\sin 2\eta - \frac{15q}{2m}\xi\sin(2\eta-v) - \frac{15q}{2m}\xi\sin(2\eta+v) \\ +\frac{45e}{4m}\xi\sin(2\eta-u) + \frac{45e}{4m}\xi\sin(2\eta+u) \\ -\frac{3n}{4m}\sin\eta - \frac{15n}{4m}\sin 3\eta \end{cases}$$

XVI.

Porro cum sit $\frac{q\sin v^2}{1-q\cos v} = \frac{q-q\cos 2v}{2(1-q\cos v)} =$

$\frac{1}{2}q - \frac{1}{2}q\cos 2v + \frac{1}{4}qq\cos v - \frac{1}{4}qq\cos 3v$; erit

$$\frac{dq}{d\omega} = (2\cos v - \tfrac{1}{2}q + \tfrac{1}{2}q\cos 2v - \tfrac{1}{4}qq\cos v + \tfrac{1}{4}qq\cos 3v)\frac{M}{A}\sqrt{Ap} + \sin v.\frac{N}{A}\sqrt{Ap}$$

Facta ergo substitutione valorum pro $\frac{M}{A}\sqrt{Ap}$ et $\frac{N}{A}\sqrt{Ap}$ inuentorum, habebitur:

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$$\frac{dq}{dw} = \left\{\begin{array}{l}
+\frac{9}{4m}(1+\frac{9}{8}ee+\frac{5}{12}qq)\sin(2\eta-v)+\frac{3}{4m}(1+\frac{9}{8}ee+\frac{3}{4}qq)\\
\quad\sin(2\eta+v)-\frac{1}{2m}(1+\frac{9}{8}ee+\frac{1}{4}qq)\sin v\\
+\frac{3q}{4m}\sin 2\eta+\frac{3q}{2m}\sin(2\eta-2v)+\frac{3q}{4m}\sin(2\eta+2v)-\frac{q}{4m}\sin 2v\\
-\frac{27e}{8m}\sin(2\eta-v-u)-\frac{27e}{8m}\sin(2\eta-v+u)-\frac{9e}{8m}\sin(2\eta+v-u)\\
-\frac{9e}{8m}\sin(2\eta+v+u)+\frac{3e}{4m}\sin(v-u)+\frac{3e}{4m}\sin(v+u)\\
+\frac{15qq}{16m}\sin(2\eta-3v)+\frac{9qq}{16m}\sin(2\eta+3v)-\frac{qq}{8m}\sin 3v\\
-\frac{9eq}{8m}\sin(2\eta-u)-\frac{9eq}{8m}\sin(2\eta+u)-\frac{9eq}{4m}\sin(2\eta-2v+u)\\
-\frac{9eq}{4m}\sin(2\eta-2v-u)-\frac{9eq}{8m}\sin(2\eta+2v-u)-\frac{9eq}{8m}\sin(2\eta+2v+u)\\
+\frac{27ee}{16m}\sin(2\eta-v-2u)+\frac{27ee}{16m}\sin(2\eta-v+2u)\\
+\frac{9ee}{16m}\sin(2\eta+v-2u)+\frac{9ee}{16m}\sin(2\eta+v+2u)\\
+\frac{3eq}{8m}\sin(2v-u)+\frac{3eq}{8m}\sin(2v+u)\\
-\frac{3ee}{8m}\sin(v-2u)-\frac{3ee}{8m}\sin(v+2u)\\
+\frac{27}{8m}\xi\sin(2\eta-v)+\frac{9}{8m}\xi\sin(2\eta+v)-\frac{3}{4m}\xi\sin v\\
\qquad\qquad+\frac{9q}{8m}
\end{array}\right.$$

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$$\left\{\begin{array}{l}
+\frac{9q}{8m}\xi\sin 2\eta+\frac{9q}{4m}\xi\sin(2\eta-2v)+\frac{9q}{8m}\xi\sin(2\eta+2v)\\
-\frac{3q}{8m}\xi\sin 2v+\frac{9e}{8m}\xi\sin(v-u)+\frac{9e}{8m}\xi\sin(v+u)\\
-\frac{81e}{16m}\xi\sin(2\eta-v-u)-\frac{81e}{16m}\xi\sin(2\eta-v+u)\\
-\frac{27e}{16}\xi\sin(2\eta+v-u)-\frac{27e}{16m}\xi\sin(2\eta+v+u)\\
+\frac{15n}{16m}\sin(\eta-v)-\frac{3n}{16m}\sin(\eta+v)\\
+\frac{45n}{16m}\sin(3\eta-v)+\frac{15n}{16m}\sin(3\eta+v)\\
+m\Pi\sin v-mq\Pi\sin 2v-\frac{3}{2}m\xi\Pi\sin v-i\sin v
\end{array}\right.$$

XVII.

Deinde cum ſit $\frac{q\sin v\cos v}{1-q\cos v}=\frac{q\sin 2v}{2(1-q\cos v)}=$
$=\frac{1}{2}q\sin 2v+\frac{1}{4}qq\sin v+\frac{1}{4}qq\sin 3v$; erit pro motu elementari apogei:

$$\frac{q(d\Phi-dv)}{d\omega}=(2\sin v+\tfrac{1}{2}q\sin 2v+\tfrac{1}{4}qq\sin v+\tfrac{1}{4}qq\sin 3v)\frac{M}{A}VAp-\cos v.\frac{N}{A}VAp$$

ac facta ſubſtitutione obtinebitur:

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$$\frac{q(d\Phi - dv)}{d\omega} = \left\{ \begin{array}{l} + \frac{9}{4m}\left(1+\frac{9}{8}ee+\frac{7}{12}qq\right)\cos(2\eta - v) - \frac{3}{4m}\left(1+\frac{9}{8}ee+\frac{7}{12}qq\right) \\ \qquad \cos(2\eta + v) + \frac{1}{2m}\left(1+\frac{9}{8}ee+\frac{3}{4}qq\right)\cos v \\ + \frac{3q}{4m}\cos 2\eta + \frac{3q}{2m}\cos(2\eta - 2v) - \frac{3q}{4m}\cos(2\eta + 2v) \\ + \frac{q}{4m}\cos 2v + \frac{q}{4m} - \frac{3e}{4m}\cos(v-u) - \frac{3e}{4m}\cos(v+u) \\ - \frac{27e}{8m}\cos(2\eta - v - u) - \frac{27e}{8m}\cos(2\eta - v + u) \\ + \frac{9e}{8m}\cos(2\eta + v - u) + \frac{9e}{8m}\cos(2\eta + v + u) \\ + \frac{15qq}{16m}\cos(2\eta - 3v) - \frac{9qq}{16m}\cos(2\eta + 3v) + \frac{qq}{8m}\cos 3v \\ - \frac{9eq}{8m}\cos(2\eta - u) - \frac{9eq}{8m}\cos(2\eta + u) - \frac{3eq}{4m}\cos u \\ - \frac{3eq}{8m}\cos(2v - u) - \frac{3eq}{8m}\cos(2v + u) \\ - \frac{9eq}{4m}\cos(2\eta - 2v + u) - \frac{9eq}{4m}\cos(2\eta - 2v - u) \\ + \frac{9eq}{8m}\cos(2\eta + 2v - u) + \frac{9eq}{8m}\cos(2\eta + 2v + u) \\ + \frac{27ee}{16m}\cos(2\eta - v - 2u) + \frac{27ee}{16m}\cos(2\eta - v + 2u) \\ - \frac{9ee}{16m}\cos(2\eta + v - 2u) - \frac{9ee}{16m}\cos(2\eta + v + 2u) \\ + \frac{3ee}{8m}\cos(v - 2u) + \frac{3ee}{8m}\cos(v + 2u) \end{array} \right.$$

$$+ \frac{27}{8m}$$

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$$\left|\begin{array}{l}
+\frac{27}{8m}\xi\cos(2\eta-v)-\frac{9}{8m}\xi\cos(2\eta+v)+\frac{3}{4m}\xi\cos v\\
+\frac{9q}{8m}\xi\cos 2\eta+\frac{9q}{4m}\xi\cos(2\eta-2v)-\frac{9q}{8m}\xi\cos(2\eta+2v)\\
+\frac{3q}{8m}\xi+\frac{3q}{8m}\xi\cos 2v\\
-\frac{81e}{16m}\xi\cos(2\eta-v-u)+\frac{27e}{16m}\xi\cos(2\eta+v-u)\\
-\frac{81e}{16m}\xi\cos(2\eta-v+u)+\frac{27e}{16m}\xi\cos(2\eta+v+u)\\
-\frac{9e}{8m}\xi\cos(v-u)-\frac{9e}{8m}\xi\cos(v+u)\\
+\frac{15n}{16m}\cos(\eta-v)+\frac{3n}{16m}\cos(\eta+v)\\
+\frac{45n}{16m}\cos(3\eta-v)-\frac{15n}{16m}\cos(3\eta+v)\\
-m\Pi\cos v+mq\Pi+mq\Pi\cos 2v+\frac{3}{2}m\xi\Pi\cos v+i\sin v
\end{array}\right.$$

XVIII.

Euoluamus ſimili modo valorem differentialium $d\varpi$ et $d\varphi$, et cum ſit $\frac{y}{z^3}-\frac{1}{yy}=\frac{3x}{y^3}\sin\eta+\frac{3xx}{4y^4}(3+5\cos 2\eta)$ et $d\Phi=\frac{d\omega}{xx}\sqrt{Ap}$; erit

$$d\varpi=-\frac{a^3x\,d\omega}{\sqrt{Ap}}\left(\frac{3x}{y^3}\cos\eta+\frac{3xx}{4y^4}(3+5\cos 2\eta)\right)\sin(\theta-\pi)\sin(\Phi-\pi)$$

Subſtitutis autem valoribus $x=\frac{p}{1-q\sin v}$, $y=\frac{c}{1-e\sin u}$; $p=b(1+\xi)$; $\sqrt{Ap}=m\sqrt{b^3p}=mbb(1+\frac{1}{2}\xi)$, $\frac{a^3}{c^3}=\frac{1}{(1-ee)^3}=1+3ee$ et $\frac{b}{c}=n$, erit

$$d\varpi=-\frac{d\omega\sin(\theta-\pi)\sin(\Phi-\pi)}{m}\left[\frac{3(1+\frac{3}{2}\xi)(1+3ee)(1-e\sin u)^3}{(1-q\cos v)^2}\sin\eta+\frac{1}{4}n(3+5\cos 2\eta)\right]$$

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Neglectis igitur terminis, qui nullum valorem ſenſibilem continent, habebimus

$$\frac{d\pi}{d\omega} = \left\{\begin{array}{l}
-\frac{3}{4m}(1+\frac{9}{2}ee+\frac{3}{2}qq)-\frac{3}{4m}(1+\frac{9}{2}ee+\frac{3}{2}qq)\cos 2\eta \\
+\frac{3}{4m}(1+\frac{9}{2}ee+\frac{3}{2}qq)\cos 2(\Phi-\pi)+\frac{3}{4m}(1+\frac{9}{2}ee+\frac{3}{2}qq)\cos 2(\theta-\pi) \\
-\frac{3q}{2m}\cos v-\frac{3q}{4m}\cos(2\eta-v)-\frac{3q}{4m}\cos(2\eta+v) \\
+\frac{9e}{4m}\cos u+\frac{9e}{8m}\cos(2\eta-u)+\frac{9e}{8m}\cos(2\eta+u) \\
+\frac{3q}{4m}\cos(2\Phi-2\pi-v)+\frac{3q}{4m}\cos(2\Phi-2\pi+v) \\
+\frac{3q}{4m}\cos(2\theta-2\pi-v)+\frac{3q}{4m}\cos(2\theta-2\pi+v) \\
-\frac{9e}{8m}\cos(2\Phi-2\pi-u)-\frac{9e}{8m}\cos(2\Phi-2\pi+u) \\
-\frac{9e}{8m}\cos(2\theta-2\pi-u)-\frac{9e}{8m}\cos(2\theta-2\pi+u) \\
-\frac{9}{8m}\xi-\frac{9}{8m}\xi\cos 2\eta \\
+\frac{9}{8m}\xi\cos(2\Phi-2\pi)+\frac{9}{8m}\xi\cos(2\theta-2\pi) \\
-\frac{11n}{16m}\cos\eta+\frac{3n}{8m}\cos(\Phi+\theta-2\pi)-\frac{5n}{16m}\cos 3\eta \\
+\frac{5n}{16m}\cos(3\Phi-\theta-2\pi)+\frac{5n}{16m}\cos(3\theta-\Phi-2\pi)
\end{array}\right.$$

XIX.

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XIX.

Simili autem modo praecedentem valorem per tang $(\Phi-\pi)$ diuidendo prodibit differentiale logarithmi tangentis inclinationis ϱ, erit enim

$$\frac{d.l\tang\varrho}{d\omega} = \left\{\begin{array}{l}
+\frac{3}{4m}(1+\frac{9}{2}ee+\frac{3}{2}qq)\sin 2\eta - \frac{3}{4m}(1+\frac{9}{2}ee+\frac{3}{2}qq) \\
\qquad \sin 2(\Phi-\pi) - \frac{3}{4m}(1+\frac{9}{2}ee+\frac{3}{2}qq)\sin 2(\theta-\pi) \\
+\frac{3q}{4m}\sin(2\eta-v) + \frac{3q}{4m}\sin(2\eta+v) \\
-\frac{9e}{8m}\sin(2\eta-u) - \frac{9e}{8m}\sin(2\eta+u) \\
-\frac{3q}{4m}\sin(2\Phi-2\pi-v) - \frac{3q}{4m}\sin(2\Phi-2\pi+v) \\
-\frac{3q}{4m}\sin(2\theta-2\pi-u) - \frac{3q}{4m}\sin(2\theta-2\pi+u) \\
+\frac{9e}{8m}\sin(2\Phi-2\pi-u) + \frac{9e}{8m}\sin(2\Phi-2\pi+u) \\
+\frac{9e}{8m}\sin(2\theta-2\pi-u) + \frac{9e}{8m}\sin(2\theta-2\pi+u) \\
+\frac{9}{8m}\xi\sin 2\eta - \frac{9}{8m}\xi\sin(2\Phi-2\pi) - \frac{9}{8m}\xi\sin(2\theta-2\pi) \\
+\frac{n}{16m}\sin\eta - \frac{3n}{8m}\sin(\Phi+\theta-\pi) + \frac{5n}{16m}\sin 3\eta \\
-\frac{5n}{16m}\sin(3\Phi-\theta-2\pi) - \frac{5n}{16m}\sin(3\theta-\Phi-2\pi)
\end{array}\right.$$

XX.

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XX.

Quo iam facilius has formulas admodum complicatas euoluere queamus, quadruplicis generis terminos distingui conuenit. Primum scilicet genus eos complectitur terminos, qui tantum ab excentricitate orbitae lunaris pendent, neque excentricitatem solis, neque parallaxin solis seu literam $\varkappa$, neque inclinationem orbitae lunaris seu litteram Π inuoluunt. Ad secundum genus refero terminos, qui ad primum genus insuper excentricitatem solis adiungunt. Ad tertium autem eos, qui praeterea parallaxin solis seu litteram $\varkappa$ inducunt. In quarto autem eas inaequalitates, quae insuper ab obliquitate orbitae lunaris proueniunt, complexurus sum. Ab inaequalitatibus ergo primi generis exordiar, ideoque cum excentricitatem solis e, tum eius parallaxin, tum quoque obliquitatem orbitae lunaris reiiciam

INUESTIGATIO INAEQUALITATUM LUNAE PRIMI GENERIS.

XXI.

Neglectis ergo excentricitate solis cum eius parallaxi et obliquitate orbitae lunaris, has habebimus aequationes:

$$\frac{d\xi}{d\omega} =$$

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$$\frac{d\xi}{d\omega} = \begin{cases} -\frac{3}{m}(1+\frac{3}{2}qq)\sin 2\eta - \frac{3q}{m}\sin(2\eta-v) - \frac{3q}{m}\sin(2\eta+v) \\ -\frac{9qq}{4m}\sin(2\eta-2v) - \frac{9qq}{4m}\sin(2\eta+2v) \\ -\frac{15}{2m}\xi\sin 2\eta - \frac{15q}{2m}\xi\sin(2\eta-v) - \frac{15q}{2m}\xi\sin(2\eta+v) \end{cases}$$

$$\frac{dq}{d\omega} = \begin{cases} +\frac{9}{4m}(1+1\frac{5}{2}qq)\sin(2\eta-v) + \frac{3}{4m}(1+\frac{3}{4}qq)\sin(2\eta+v) \\ -\frac{1}{2m}(1+\frac{3}{4}qq)\sin v - i\sin v \\ +\frac{3q}{4m}\sin 2\eta + \frac{3q}{2m}\sin(2\eta-2v) + \frac{3q}{4m}\sin(2\eta+2v) \\ -\frac{q}{4m}\sin 2v \\ +\frac{15qq}{16m}\sin(2\eta-3v) + \frac{9qq}{16m}\sin(2\eta+3v) - \frac{qq}{8m}\sin 3v \\ +\frac{27}{8m}\xi\sin(2\eta-v) + \frac{9}{8m}\xi\sin(2\eta+v) - \frac{3}{4m}\xi\sin v \\ +\frac{9q}{8m}\xi\sin 2\eta + \frac{9q}{4m}\xi\sin(2\eta-2v) + \frac{9q}{8m}\xi\sin(2\eta+2v) \\ -\frac{3q}{8m}\xi\sin 2v \end{cases}$$

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$$\frac{q(d\Phi-dv)}{d\omega} = \left\{\begin{array}{l} +\frac{9}{4m}(1+\frac{7}{12}qq)\cos(2\eta-v)-\frac{3}{4m}(1+\frac{1}{4}qq)\cos(2\eta+v) \\ +\frac{1}{2m}(1+\frac{3}{4}qq)\cos v + i\cos v \\ +\frac{3q}{4m}\cos 2\eta + \frac{3q}{2m}\cos(2\eta-2v) - \frac{3q}{4m}\cos(2\eta+2v) \\ +\frac{q}{4m}\cos 2v + \frac{q}{4m} \\ +\frac{15qq}{16m}\cos(2\eta-3v) - \frac{9qq}{16m}\cos(2\eta+3v) + \frac{qq}{8m}\cos 3v \\ +\frac{27}{8m}\xi\cos(2\eta-v) - \frac{9}{8m}\xi\cos(2\eta+v) + \frac{3}{4m}\xi\cos v \\ +\frac{9q}{8m}\xi\cos 2\eta + \frac{9q}{4m}\xi\cos(2\eta-2v) - \frac{9q}{8m}\xi\cos(2\eta+2v) \\ +\frac{3q}{8m}\xi + \frac{3q}{8m}\xi\cos 2v \end{array}\right.$$

$$\frac{d\Phi}{d\omega} = m(1+\tfrac{1}{2}qq) - 2mq\cos v + \tfrac{1}{2}mqq\cos 2v - \tfrac{3}{2}m(1+\tfrac{1}{2}qq)\xi + 3mq\xi\cos v - \tfrac{3}{4}mqq\xi\cos 2v + \tfrac{15}{8}m\xi\xi$$

$$\frac{d\eta}{d\omega} = m(1+\tfrac{1}{2}qq) - 1 - 2mq\cos v + \tfrac{1}{2}mqq\cos 2v - \tfrac{3}{2}m(1+\tfrac{1}{2}qq)\xi + 3mq\xi\cos v - \tfrac{3}{4}mqq\xi\cos 2v + \tfrac{15}{8}m\xi\xi$$

XXII.

Hic autem primo patet valores litterarum ξ et q fine cognitis rationibus $\frac{d\eta}{d\omega}$ et $\frac{dv}{d\omega}$ definiri non poſſe, has autem viciſſim ipſas quantitates ξ et q inuoluere. Cum autem ad valores ξ et q inueniendos non opus ſit rationes

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tiones $\frac{d\eta}{d\omega}$ et $\frac{dv}{d\omega}$ eo praecisionis gradu nosse, quo ipsi illi valores desiderantur; patet si valores ξ et q prope tantum veri constent, iis in rationibus $\frac{d\eta}{d\omega}$ et $\frac{dv}{d\omega}$ adhibitis, eosdem multo exactiores repertum iri. Cum igitur, si motus esset regularis, foret $\xi = o$ et $q =$ constanti, hinc primam hypothesin constituamus. Sit ergo

$$\xi = o \quad \text{et} \quad q = g$$

et neglectis terminis, qui ob harum litterarum errores affici possent, vtpote valde paruis prae reliquis, habebimus proxime

$$\frac{d\Phi}{d\omega} = m(1+\tfrac{1}{2}gg) - 2mg\cos v\,; \quad \frac{d\eta}{d\omega} = (1+\tfrac{1}{2}gg) - 1 - 2mg\cos v$$

$$\text{et } \frac{d\Phi - dv}{d\omega} = \frac{9}{4mg}(1+\tfrac{7}{12}gg)\cos(2\eta - v) - \frac{3}{4mg}(1+\tfrac{1}{4}gg)\cos(2\eta + v)$$

$$+ \frac{1}{2mg}(1+\tfrac{3}{4}gg)\cos v + \frac{i}{g}\cos v + \frac{1}{4m}$$

ideoque

$$\frac{dv}{d\omega} = m(1+\tfrac{1}{2}gg) - \frac{1}{4m} - (2mg + \frac{1}{2mg} + \frac{3g}{8m} + \frac{i}{g})\cos v$$

$$- \frac{9}{4mg}(1+\tfrac{7}{12}gg)\cos(2\eta - v) + \frac{3}{4mg}(1+\tfrac{1}{4}gg)\cos(2\eta + v)$$

XXIII.

Ponamus ad has formulas abbreuiandas:

$$m(1+\tfrac{1}{2}gg) - 1 = \alpha\,; \qquad 2mg = \gamma$$

$$m(1+\tfrac{1}{2}gg) - \frac{1}{4m} = \mathfrak{G}\,; \quad 2mg + \frac{1}{2mg} + \frac{3g}{8m} + \frac{i}{g} = \delta$$

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et neglectis quadratis gg in reliquis terminis, habebimus has formulas simpliciores:

$$\frac{d\eta}{d\omega} = \alpha - \gamma \cos v$$

$$\frac{dv}{d\omega} = \mathfrak{C} - \delta \cos v - \frac{9}{4mg}\cos(2\eta - v) + \frac{3}{4mg}\cos(2\eta + v)$$

Tum vero pro valoribus ξ et q propius inueniendis has aequationes:

$$\frac{d\xi}{d\omega} = -\frac{3}{m}(1+\tfrac{3}{2}gg)\sin 2\eta - \frac{3g}{m}\sin(2\eta - v) - \frac{3g}{m}\sin(2\eta + v)$$

$$\frac{dq}{d\omega} = +\frac{9}{4m}(1+1\tfrac{5}{12}gg)\sin(2\eta - v) + \frac{3}{4m}(1+\tfrac{3}{4}gg)\sin(2\eta + v)$$

$$- \frac{1}{2m}(1+\tfrac{1}{4}gg)\sin v - i\sin v$$

$$+\frac{3g}{4m}\sin 2\eta + \frac{3g}{2m}\sin(2\eta - 2v) + \frac{3g}{4m}\sin(2\eta + 2v) - \frac{g}{4m}\sin 2v$$

XXIV.

Fingamus ergo primo:

$$\xi = \mathfrak{A}\cos 2\eta + \mathfrak{B}\cos(2\eta - v) + \mathfrak{C}\cos(2\eta + v)$$

vbi notandum est terminos binos posteriores, vti in differentiali, multo esse minores primo. Quare cum etiam in differentialibus $d\eta$ et dv duplicis generis termini occurrant, quorum posteriores prae primis sint valde parui, in differentiatione solius primi termini totum differentialis $d\eta$ valorem pono, in duobus vero reliquis tantum valorem principalem; sic prodibit

$$\frac{d\xi}{d\omega} = -2\alpha\mathfrak{A}\sin 2\eta + \gamma\mathfrak{A}\sin(2\eta - v) + \gamma\mathfrak{A}\sin(2\eta + v)$$

$$-(2\alpha - \mathfrak{C})\mathfrak{B} \quad -(2\alpha + \mathfrak{C})\mathfrak{C}$$

Collato ergo hoc differentiali cum forma proposita obtinetur: $\mathfrak{A} =$

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$$\mathfrak{A} = \frac{3}{2ma}(1+\tfrac{3}{2}gg)$$

$$(2a-\mathfrak{C})\mathfrak{B} = \gamma\mathfrak{A} + \frac{3g}{m} \quad \text{ergo} \quad \mathfrak{B} = \frac{3(\gamma+2ag)}{2ma(2a-\mathfrak{C})}$$

$$(2a+\mathfrak{C})\mathfrak{C} = \gamma\mathfrak{A} + \frac{3g}{m} \quad \text{ergo} \quad \mathfrak{C} = \frac{3(\gamma+2ag)}{2ma(2a+\mathfrak{C})}$$

XXV.

Simili modo fingatur:

$$q = g + A\cos(2\eta-v) + B\cos(2\eta+v) + C\cos v$$
$$+ D\cos 2\eta + E\cos(2\eta-2v) + F\cos(2\eta+2v) + G\cos 2v$$
$$+ H\cos 4\eta + J\cos(4\eta-2v) + K\cos(4\eta+2v)$$

vbi linea prior continet terminos multo maiores, quam binae inferiores. Hinc ergo fit differentiando secundum regulam supra datam:

$$\frac{dq}{dw} = -(2a-\mathfrak{C})A\sin(2\eta-v) - (2a+\mathfrak{C})B\sin(2\eta+v) - \mathfrak{C}C\sin v$$
$$+\left(\tfrac{1}{2}(2\gamma-\delta)A + \tfrac{1}{2}(2\gamma+\delta)B + \frac{3}{2mg}C - 2\alpha D\right)\sin 2\eta$$
$$+\left(\tfrac{1}{2}(2\gamma-\delta)A - \frac{9}{8mg}C - 2(\alpha-\mathfrak{C})E\right)\sin(2\eta-2v)$$
$$+\left(\tfrac{1}{2}(2\gamma+\delta)B - \frac{3}{8mg}C - 2(\alpha+\mathfrak{C})F\right)\sin(2\eta+2v)$$
$$+\left(\tfrac{1}{2}\delta C - \frac{3A+9B}{8mg} - 2\mathfrak{C}G\right)\sin 2v$$
$$+\left(\frac{3A+9B}{8mg} - 4\alpha H\right)\sin 4\eta$$
$$+\left(-\frac{9A}{8mg} - 2(2\alpha-\mathfrak{C})J\right)\sin(4\eta-2v)$$
$$+\left(-\frac{3B}{8mg} - 2(2\alpha+\mathfrak{C})K\right)\sin(4\eta+2v)$$

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Hincque elicientur sequentes coefficientium valores:

$$(2\alpha-\mathfrak{G})\,A = -\frac{9}{4m}(1+\tfrac{5}{12}gg)$$

$$(2\alpha+\mathfrak{G})\,B = -\frac{3}{4m}(1+\tfrac{3}{4}gg)$$

$$\mathfrak{G}\,C = \frac{1}{2m}(1+\tfrac{1}{4}gg)+i$$

$$2\alpha\,D = \tfrac{1}{2}(2\gamma-\delta)A+\tfrac{1}{2}(2\gamma+\delta)B+\frac{3}{2mg}C-\frac{3g}{4m}$$

$$2(\alpha-\mathfrak{G})\,E = \tfrac{1}{2}(2\gamma-\delta)A-\frac{9}{8mg}C-\frac{3g}{2m}$$

$$2(\alpha+\mathfrak{G})\,F = \tfrac{1}{2}(2\gamma+\delta)A-\frac{3}{8mg}C-\frac{3g}{4m}$$

$$2\mathfrak{G}\,G = -\frac{3A+9B}{8mg}+\tfrac{1}{2}\delta C+\frac{g}{4m}$$

$$4\alpha\,H = \frac{3A+9\beta}{8mg};\quad 2(2\alpha-\mathfrak{G})\,J = -\frac{9}{8mg}A$$

$$2(2\alpha+\mathfrak{G})\,K = -\frac{3}{8mg}B$$

XXVI.

Cum igitur his inuentis valoribus fit multo verius:

$\xi = \mathfrak{A}\cos 2\eta$ et $q = g + A\cos(2\eta-v)+B\cos(2\eta+v)+C\cos v$

vbi terminos minores data opera adhuc omitto, quia fortasse correctione egent, praecedentes operationes multo accuratius instituere atque ad ordinem terminorum vlteriorem progredi poterimus. Obtinebimus ergo:

$$\frac{d\Phi}{d\omega} = m\,(1+\tfrac{1}{2}gg-C) - 2mg\cos v$$

$$-m(\tfrac{3}{2}\mathfrak{A}+A+B)\cos 2\eta - mA\cos(2\eta-2v) - mB\cos(2\eta+2v)+m(\tfrac{1}{2}gg-C)\cos 2v$$

hincque $\frac{d\eta}{d\omega} = \frac{d\Phi}{d\omega} - 1$.

Porro

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Porro ob $\frac{1}{q} = \frac{1}{g} - \frac{A}{gg}\cos(2\eta - v) - \frac{B}{gg}\cos(2\eta + v) - \frac{C}{gg}\cos v$ erit

$$\frac{d\Phi - dv}{d\omega} = \frac{9}{4mg}(1 + \tfrac{7}{12}gg)\cos(2\eta - v) - \frac{3}{4mg}(1 + \tfrac{1}{4}gg)\cos(2\eta + v)$$
$$+ \frac{1}{2mg}(1 + \tfrac{3}{4}gg)\cos v + \frac{i}{g}\sin v + \frac{1}{4m}\left(1 - \frac{9A + 3B - 2C}{2gg}\right)$$
$$+ \frac{1}{4m}\left(3 - \frac{A - B - 3C}{gg}\right)\sin 2\eta + \frac{1}{4m}\left(6 - \frac{2A - 9C}{2gg}\right)\sin(2\eta - 2v)$$
$$- \frac{1}{4m}\left(3 + \frac{2B - 3C}{2gg}\right)\sin(2\eta + 2v) + \frac{1}{4m}\left(1 + \frac{3A - 9B - 2C}{2gg}\right)\sin 2v$$
$$+ \frac{3A - 9B}{8mgg}\cos 4\eta - \frac{9A}{8mgg}\cos(4\eta - 2v) + \frac{3B}{8mgg}\cos(4\eta + 2v)$$

XXVII.

Ponatur ad abbreuiandum:

$$m(1 + \tfrac{1}{2}gg - C) - 1 = \alpha \quad ; \quad 2mg = \gamma$$
$$m(\tfrac{3}{2}\mathfrak{A} + A + B) = \varepsilon \quad ; \text{ vt fit}$$

$$\frac{d\eta}{d\omega} = \alpha - \gamma\cos v - \varepsilon\cos 2\eta$$
$$- mA\cos(2\eta - 2v) - mB\cos(2\eta + 2v) + m(\tfrac{1}{2}gg - C)\cos 2v$$

Porro fit

$$m(1 + \tfrac{1}{2}gg - C) - \frac{1}{4m}\left(1 - \frac{9A + 3B - 2C}{2gg}\right) = \zeta$$

$$2mg + \frac{1}{2mg} + \frac{3g}{8m} + \frac{i}{g} = \delta$$

$$+ m(\tfrac{1}{2}\mathfrak{A} + A + B) + \frac{1}{4m}\left(3 - \frac{A - B - 3C}{gg}\right) = \zeta$$

mA

$$mA + \frac{1}{4m}\left(6 - \frac{2A-9C}{2gg}\right) = \eta$$

$$mB - \frac{1}{4m}\left(3 + \frac{2B-3C}{2gg}\right) = \theta$$

$$m(C - \tfrac{1}{2}gg) + \frac{1}{4m}\left(1 + \frac{3A-9B-2C}{2gg}\right) = \varkappa$$

vt habeatur

$$\frac{dv}{d\omega} = \mathfrak{C} - \delta\cos v - \frac{9}{4mg}\cos(2\eta - v) + \frac{3}{4mg}\cos(2\eta + v)$$
$$- \zeta\cos 2\eta - \eta\cos(2\eta - 2v) - \theta\cos(2\eta + 2v) - \varkappa\cos 2v$$
$$- \frac{3A+9B}{8mgg}\cos 4\eta + \frac{9A}{8mgg}\cos(4\eta - 2v) - \frac{3B}{8mgg}\cos(4\eta + 2v)$$

vbi caueatur, ne coefficientes η, θ, cum angulis cognominibus confundantur.

XXVIII.

Opus plane non eſt, vt valores litterarum ξ et φ accuratius determinemus, atque ad plures terminos, quam ante inuenimus, expediamus; verum hos ipſos terminos, quos ante inuenimus, accuratius obtinebimus, ſi litteris α et $\mathfrak{C}$ eos valores tribuemus, quos nunc eis conuenire collegimus. Pluribus autem terminis non indigebimus tam ad longitudinem lunae Φ, quam ad eius anomaliam veram v ſatis exacte definiendam. Verum ad hoc ipſum negotium valores differentiales $\frac{d\Phi}{d\omega}$ et $\frac{d\Phi - dv}{d\omega}$, ac praecipue hunc poſteriorem, quo motus apogei continetur, accuratius euolui oportet, quoniam imprimis in motu medio apogei minimae particulae ingentis momenti eſſe poſſunt.

XXIX.

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XXIX.

Cum igitur accuratius quam adhuc assumsimus sit

$$\xi = \mathfrak{A}\cos 2\eta + \mathfrak{B}\cos(2\eta - v) + \mathfrak{C}\cos(2\eta + v) \quad \text{et}$$

$$q = g + A\cos(2\eta - v) + B\cos(2\eta + v) + C\cos v$$
$$+ D\cos 2\eta + E\cos(2\eta - 2v) + F\cos(2\eta + 2v) + G\cos 2v$$
$$+ H\cos 4\eta + J\cos(4\eta - 2v) + K\cos(4\eta + 2v)$$

erit terminus ad quartum vsque ordinem extensis

I. II.

$$\frac{d\Phi}{d\omega} = m(1 + \tfrac{1}{2}gg - C) - (2mg - \tfrac{3}{2}mgC + mG)\cos v$$

III.

$$-m(\tfrac{3}{2}\mathfrak{A} + A + B)\cos 2\eta - mA\cos(2\eta - 2v) - mB\cos(2\eta + 2v)$$
$$- m(C - \tfrac{1}{2}gg)\cos 2v$$

IV.

$$+ m(\tfrac{3}{2}g\mathfrak{A} - \tfrac{3}{2}\mathfrak{B} + gA + \tfrac{1}{2}gB - D - E)\cos(2\eta - v)$$
$$+ m(\tfrac{3}{2}g\mathfrak{A} - \tfrac{3}{2}\mathfrak{C} + gB + \tfrac{1}{2}gA - D - F)\cos(2\eta + v)$$
$$+ m(\tfrac{1}{2}gA - E)\cos(2\eta - 3v) + m(\tfrac{1}{2}gB - F)\cos(2\eta + 3v)$$
$$+ m(\tfrac{1}{2}gC - G)\cos 3v$$
$$- m(H + J)\cos(4\eta - v) - m(H + K)\cos(4\eta + v) - mJ\cos(4\eta - 3v)$$
$$- mK\cos(4\eta + 3v)$$

vnde cum esset ante $\gamma = 2mg$, nunc accuratius erit

$$\gamma = 2mg - \tfrac{3}{2}mgC + mG$$

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XXX.

Deinde cum nunc quoque ſit accuratius:

II.

$$\frac{1}{q} = \left(\frac{1}{g} + \frac{AA+BB+CC}{2g^3}\right)$$

III.

$$-\frac{A}{gg}\cos(2\eta - v) - \frac{B}{gg}\cos(2\eta + v) - \frac{C}{gg}\cos v$$

IV.

$$+\left(\frac{(A+B)C}{g^3} - \frac{D}{gg}\right)\cos 2\eta + \left(\frac{AC}{g^3} - \frac{E}{gg}\right)\cos(2\eta - 2v)$$
$$+\left(\frac{BC}{g^3} - \frac{F}{gg}\right)\cos(2\eta + 2v) + \left(\frac{2AB+CC}{2g^3} - \frac{G}{gg}\right)\cos 2v$$
$$+\left(\frac{AB}{g^3} - \frac{H}{gg}\right)\cos 4\eta + \left(\frac{AA}{2g^3} - \frac{J}{gg}\right)\cos(4\eta - 2v)$$
$$+\left(\frac{BB}{g2^3} - \frac{K}{gg}\right)\cos(4\eta + 2v)$$

Hinc quoque ad terminos quarti ordinis vsque valor formulae $\frac{d\Phi - dv}{d\omega}$ definiri poſſet, ſed expreſſio prodiret tantopere complicata, vt eius euolutio ſummam requireret patientiam; neque tamen hic labor vllius foret vſus, niſi forte in motu apogei exactius eruendo: ipſae enim inaequalitates nullius forent momenti; propterea quod error in anomalia commiſſus multo minorem errorem in longitudine producit.

XXXI.

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XXXI.

Ponatur ergo longitudo apogei:

$$\Phi - v = \text{Conſt.}$$
$$+ A' \sin(2\eta - v) + B' \sin(2\eta + v) + C' \sin 2v$$
$$+ \Delta\omega + D' \sin 2\eta + E' \sin(2\eta - 2v) + F' \sin(2\eta + 2v) + G' \sin 2v$$
$$+ H' \sin 4\eta + J \sin(4\eta - 2v) + K \sin(4\eta + 2v)$$

et erit differentiando:

$$\frac{d\Phi - dv}{d\omega} =$$
$$(2\alpha - \epsilon) A' \cos(2\eta - v) + (2\alpha + \epsilon) B' \cos(2\eta + v) + \epsilon C' \cos v$$
$$+ \Delta - \tfrac{1}{2}\delta C' + \frac{9A'}{8mg} + \frac{3B'}{8mg} - \varepsilon D' - (mA - \eta) E' .$$
$$- (mB + \theta) F' - \varkappa G' - \frac{9AJ'}{8mgg} - \frac{3BK'}{8mgg}$$
$$\cos 2\eta \left(- \tfrac{1}{2}(2\gamma - \delta) A' - \tfrac{1}{2}(2\gamma + \delta) B' - \frac{3C'}{4mg} + 2\alpha D' \right.$$
$$\cos(2\eta - 2v) \left(- \tfrac{1}{2}(2\gamma - \delta) A' - \frac{9C'}{8mg} + 2(\alpha - \epsilon) E' \right.$$
$$\cos(2\eta + 2v) \left(- \tfrac{1}{2}(2\gamma + \delta) B' + \frac{3C'}{8mg} + 2(\alpha + \epsilon) F' \right.$$
$$\cos 2v \left(- \tfrac{1}{2}\delta C' - \frac{3A'}{8mg} - \frac{9B'}{8mg} + 2\epsilon G' \right.$$
$$\cos 4\eta \left(- \frac{3A'}{8mg} - \frac{9B'}{8mg} + 4\alpha H' \right.$$
$$\cos(4\eta - 2v) \left(\frac{9A'}{8mg} + 2(2\alpha - \epsilon) J' \right.$$
$$\cos(4\eta + 2v) \left(\frac{3B'}{8mg} + 2(2\alpha + \epsilon) K' \right.$$

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XXXIII.

Calculo autem praecipue in primis terminis accuratius expedito est:

$$\frac{d\Phi - dv}{dw} =$$

$$+\frac{1}{4mg}\cos(2\eta - v)\left(9 + \tfrac{21}{4}gg + \frac{27AA + 18BB + 15CC}{4gg} - \frac{3AB + 2AC + BC}{gg} - \frac{2D - 2E + 3G + 3H - 9J}{2g}\right)$$

$$+\frac{1}{4mg}\cos(2\eta + v)\left(-3 - \tfrac{3}{4}gg - \frac{6AA - 9BB + 15CC}{4gg} + \frac{9AB + AC + 2BC}{gg} - \frac{2D - 2F - 9G - 9H + 3K}{2g}\right)$$

$$+\frac{1}{2mg}\cos v\left(1 + \tfrac{3}{4}gg + \frac{2AA + 2BB + 3CC}{4gg} + \frac{AB + 6AC + 6BC}{2gg} - \frac{3D - 3E - 3F - G}{2g} + 2mi\right)$$

$$+\frac{1}{4m}\left(1 - \frac{9A + 3B - 2C}{2gg}\right)$$

$$+\frac{1}{4m}\left(3 - \frac{A - B - 3C}{gg}\right)\cos 2\eta + \frac{1}{4m}\left(1 + \frac{3A - 9B - 2C}{2gg}\right)\cos 2v$$

$$+\frac{1}{4m}\left(6 - \frac{2A - 9C}{2gg}\right)\cos(2\eta - 2v) - \frac{1}{4m}\left(3 + \frac{2B - 3C}{2gg}\right)\cos(2\eta + 2v)$$

$$+\frac{3A - 9B}{8mgg}\cos 4\eta - \frac{9A}{8mgg}\cos(4\eta - 2v) + \frac{3B}{8mgg}\cos(4\eta + 2v)$$

Simili

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Simili autem modo ex valore ipsius $\frac{dq}{d\omega}$ accuratius erit

$$(2\alpha-\mathfrak{G})\,A = -\frac{1}{4m}(9+\tfrac{15}{4}gg+\tfrac{9}{2}C)$$

$$(2\alpha+\mathfrak{G})\,B = -\frac{1}{4m}(3+\tfrac{9}{4}gg+3C)$$

$$\mathfrak{G}\,C = \frac{1}{2m}(1+\tfrac{3}{4}gg+\tfrac{3}{4}A+2mi)$$

XXXIII.

Comparatione autem instituta reperitur:

$$(2\alpha-\mathfrak{G})\,A' = \frac{1}{4mg}\left(9+\tfrac{21}{2}gg+\frac{27AA+18BB+15CC}{4gg}-\frac{3AB+2AC+BC}{gg}-\frac{2D-2E+3G+3H-9J}{2g}\right)$$

seu

$$(2\alpha-\mathfrak{G})A' = -\frac{1}{g}(2\alpha-\mathfrak{G})A+\frac{1}{4mg}\left(\tfrac{3}{2}gg-\tfrac{9}{2}C+\frac{27AA+18BB+15CC}{4gg}-\frac{3AB+2AC+BC}{gg}-\frac{2D-2E+3G+3H-9J}{2g}\right)$$

$$(2\alpha+\mathfrak{G})B' = +\frac{1}{g}(2\alpha+\mathfrak{G})B+\frac{1}{4mg}\left(\tfrac{3}{2}gg+3C-\frac{6AA-9BB+15CC}{4gg}+\frac{9AB+AC+2BC}{gg}-\frac{2D-2F-9G-9H+3K}{2g}\right)$$

$$\mathfrak{G}\,C' = +\frac{1}{g}\mathfrak{G}\,C+\frac{1}{2mg}\left(\tfrac{3}{2}gg-\tfrac{3}{4}A+\frac{2AA+2BB+3CC}{4gg}+\frac{AB+6AC+6BC}{2gg}-\frac{3D-3E-3F-G}{2g}\right)$$

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Quibus valoribus ſubſtitutis obtinebitur pro apogei motu medio, qui in termino $\Delta\omega$ continetur:

$$\Delta = \frac{1}{4m} + \frac{2mg\delta - 1}{4mgg} C$$
$$+ \frac{\delta}{46mg}\left(\tfrac{1}{2}gg - \tfrac{3}{4}A + \frac{2AA + 2BB + 3CC}{4gg} + \frac{AB + 6AC + 6BC}{2gg} - \frac{3D - 3E - 3F - G}{2g}\right)$$
$$- \frac{9}{32(2\alpha - 6)mmgg}\left(\tfrac{3}{2}gg - \tfrac{9}{2}C + \frac{27AA + 18BB + 15CC}{4gg} - \frac{3AB + 2AC + BC}{gg} - \frac{2D - 2E + 3G + 3H - 9J}{2g}\right)$$
$$- \frac{3}{32(2\alpha + 6)mmgg}\left(\tfrac{3}{2}gg + 3C - \frac{6AA - 9BB + 15CC}{4gg} + \frac{9AB + AC + 2BC}{gg} - \frac{2D - 2F - 9G - 9H + 3K}{2g}\right)$$
$$+ \varepsilon D' + (m A - \eta) E' + (mB + \vartheta) F' + \varkappa G' + \frac{9AJ'}{8mgg} + \frac{3BK'}{8mgg}$$

Quae expreſſio, cum omnino ſit ſimilis illi, quae methodo praecedente eſt inuenta, nullum etiam dubium relinquit, quin et hinc motus apogei proditurus ſit obſeruationibus conformis; ideoque littera illa i omitti poterit.

XXXIV.

Hinc igitur patet ad motum apogei definiendum valores litterarum A, B, C et A′, B′, C′ ſumma accuratione inueſtigari debere, qui cum conſtent partibus duplicis ordinis, etiam ſi partes poſterioris ordinis prae primo

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primo admodum videantur paruae, eas tamen omni cura euolui oportet, propterea quod pro motu apogei partes primi ordinis ſe deſtruunt. Quod cum in determinatione reliquorum coefficientium vſu non eueniat, in his quoque non erit opus, vt partes iſtae minores in computum ducantur, ſed ſufficiet partibus principalibus vti. Scilicet etſi determinatio litterae Δ maxime eſt lubrica, atque a reliquorum coefficientium exactiſſimis valoribus pendet, reliqui tamen coefficientes tantam ſollertiam minime requirunt, ſed ſatis exacte ſine tanta opera definiri poſſunt.

XXXV.

Valores ergo reliquorum coefficientium ſequenti modo neglectis exiguis particulis ita ſe habebunt,

$$(2\alpha-\epsilon)\,A' = \frac{9}{4mg}\,;\quad (2\alpha+\epsilon)\,B' = -\frac{3}{4mg}\,;\quad \epsilon\,C' = \frac{1}{2mg}.$$

$$2\alpha D' = \tfrac{1}{2}(2\gamma-\delta)A' + \tfrac{1}{2}(2\gamma+\delta)B' + \frac{3C'}{4mg} + \frac{1}{4m}\left(3-\frac{A-B-3C}{gg}\right)$$

$$2(\alpha-\epsilon)\,E' = \tfrac{1}{2}(2\gamma-\delta)\,A' + \frac{9C'}{8mg} + \frac{1}{4m}\left(6-\frac{2A-9C}{2gg}\right)$$

$$2(\alpha+\epsilon)\,F' = \tfrac{1}{2}(2\gamma+\delta)\,B' - \frac{3C'}{8mg} - \frac{1}{4m}\left(3+\frac{2B-3C}{2gg}\right)$$

$$2\epsilon\,G' = \tfrac{1}{2}\delta C' + \frac{3A'+9B'}{8mg} + \frac{1}{4m}\left(1+\frac{3A-9B-2C}{2gg}\right)$$

$$4\alpha\,H' = \frac{3A'+9B'}{8mg} + \frac{8A-9B}{8mgg} = 0$$

$$2(2\alpha-\epsilon)\,J' = -\frac{9A'}{8mg} - \frac{9A}{8mgg} = 0 \quad \Big| \quad 2(2\alpha+\epsilon)\,K' = -\frac{3B'}{8mg} + \frac{3B}{8mgg} = 0$$

ob $A' = -\frac{A}{g}$; $B' = \frac{B}{g}$ et $C' = \frac{C}{g}$ proxime.

XXXVI.

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XXXVI.

Cum autem sit proxime: $\gamma = 2mg$; $\vartheta = 2mg + \frac{1}{2mg}$; his valoribus quoque substitutis fiet:

$$(2\alpha-\mathfrak{G})A' = \frac{9}{4mg}; \quad (2\alpha+\mathfrak{G})B' = -\frac{3}{4mg}; \quad \mathfrak{E}C' = \frac{1}{2mg}; \text{ sive:}$$

$$A' = -\frac{A}{g}; \quad B' = \frac{B}{g}; \quad C' = \frac{C}{g};$$

$$2\alpha D' = \frac{3}{4m} - mA + 3mB;$$

$$2(\alpha-\mathfrak{G})E' = \frac{3}{2m} - mA$$

$$2(\alpha+\mathfrak{G})F' = -\frac{3}{4m} + 3mB$$

$$2\mathfrak{G}G' = \frac{1}{4m} + mC$$

et reliqui coefficientes H', J', K' pro euanescentibus sunt habendi. Valores autem litterarum A, B, C, etc. §. 25. sunt exhibiti.

XXXVII.

Quaeramus nunc quoque longitudinem lunae Φ, huncque in finem fingamus:

$$\Phi = \text{Const.}$$
$$+\mathfrak{A}'\omega + \mathfrak{B}'\sin v + \mathfrak{C}'\sin 2\eta + \mathfrak{D}'\sin(2\eta-2v) + \mathfrak{E}'\sin(2\eta+2v) + \mathfrak{F}'\sin 2v$$
$$+\mathfrak{G}'\sin(2\eta-v) + \mathfrak{H}'\sin(2\eta+v) + \mathfrak{J}'\sin(2\eta-3v) + \mathfrak{K}'\sin(2\eta+3v) + \mathfrak{L}'\sin 3v$$
$$+\mathfrak{M}'\sin(4\eta-v) + \mathfrak{N}'\sin(4\eta+v) + \mathfrak{P}'\sin(4\eta-3v) + \mathfrak{Q}'\sin(4\eta+3v)$$

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ac differentiatione inſtituta obtinebimus:

$$\frac{d\Phi}{d\omega} = + \mathfrak{A}' - \tfrac{1}{2}\delta\mathfrak{B}'$$

$$+ \quad \text{cof } v\left(\mathfrak{C}\mathfrak{B}' - \tfrac{1}{2}\varkappa\mathfrak{B}' + \frac{9\mathfrak{D}'}{4mg} + \frac{3\mathfrak{E}'}{4mg} - \delta\mathfrak{F}'\right.$$

$$+ \quad \text{cof } 2\eta\left(-\frac{3\mathfrak{B}'}{4mg} + 2\alpha\mathfrak{C}'\right.$$

$$+ \text{cof}(2\eta-2v)\left(-\frac{9\mathfrak{B}'}{8mg} + 2(\alpha-\mathfrak{C})\mathfrak{D}'\right.$$

$$+ \text{cof}(2\eta+2v)\left(+\frac{3\mathfrak{B}'}{8mg} + 2(\alpha+\mathfrak{C})\mathfrak{E}'\right.$$

$$+ \quad \text{cof } 2v\left(-\tfrac{1}{2}\delta\mathfrak{B}' + 2\mathfrak{C}\mathfrak{F}'\right.$$

$$+ \text{cof}(2\eta-v)\left(-\tfrac{1}{2}\zeta\mathfrak{B}' - \tfrac{1}{2}\eta\mathfrak{B}' - \gamma\mathfrak{C}' - (\gamma-\delta)\mathfrak{D}' + \frac{3\mathfrak{F}'}{4mg} + (2\alpha-\mathfrak{C})\mathfrak{G}'\right.$$

$$+ \text{cof}(2\eta+v)\left(-\tfrac{1}{2}\zeta\mathfrak{B}' - \tfrac{1}{2}\theta\mathfrak{B}' - \gamma\mathfrak{C}' - (\gamma+\delta)\mathfrak{E}' - \frac{9\mathfrak{F}'}{4mg} + (2\alpha+\mathfrak{C})\mathfrak{H}'\right.$$

$$+ \text{cof}(2\eta-3v)\left(-\tfrac{1}{2}\eta\mathfrak{B}' - (\gamma-\delta)\mathfrak{D}' - \frac{9\mathfrak{F}'}{4mg} + (2\alpha-3\mathfrak{C})\mathfrak{J}'\right.$$

$$+ \text{cof}(2\eta+3v)\left(-\tfrac{1}{2}\theta\mathfrak{B}' - (\gamma+\delta)\mathfrak{E}' + \frac{3\mathfrak{F}'}{4mg} + (2\alpha+3\mathfrak{C})\mathfrak{K}'\right.$$

$$+ \quad \text{cof } 8v\left(-\tfrac{1}{2}\varkappa\mathfrak{B}' - \frac{3\mathfrak{D}'}{4mg} - \frac{9\mathfrak{E}'}{4mg} - \delta\mathfrak{F}' + 3\mathfrak{C}\mathfrak{L}'\right.$$

$$+ \text{cof}(4\eta-v)\left(-\frac{3A+9B}{16mgg}\mathfrak{B}' + \frac{9A}{16mgg}\mathfrak{B}' - \frac{3\mathfrak{D}'}{4mg} + (4\alpha-\mathfrak{C})\mathfrak{M}'\right.$$

$$+ \text{cof}(4\eta+v)\left(-\frac{3A+9B}{16mgg}\mathfrak{B}' - \frac{3B}{16mgg}\mathfrak{B}' - \frac{9\mathfrak{E}'}{4mg} + (4\alpha+\mathfrak{C})\mathfrak{N}'\right.$$

$$+ \text{cof}(4\eta-3v)\left(+\frac{9A}{16mgg}\mathfrak{B}' + \frac{9\mathfrak{D}'}{4mg} + (4\alpha-3\mathfrak{C})\mathfrak{P}'\right.$$

$$+ \text{cof}(4\eta+3v)\left(-\frac{3B}{16mgg}\mathfrak{B}' + \frac{3\mathfrak{E}'}{4mg} + (4\alpha+3\mathfrak{C})\mathfrak{Q}'\right.$$

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XXXVIII.

Comparata iam hac forma cum valore ipsius $\frac{d\Phi}{d\omega}$ in §. 29. exhibito, obtinebitur

$$\mathfrak{A}' = \tfrac{1}{2}\delta\mathfrak{B}' + m(1 + \tfrac{1}{2}gg - C)$$

$$\mathfrak{C}\mathfrak{B}' = \tfrac{1}{2}\nu\mathfrak{B}' - \frac{9\mathfrak{D}' - 3\mathfrak{E}'}{4mg} + \delta\mathfrak{F}' - 2mg + \tfrac{3}{4}mgC - mG$$

$$2\alpha\mathfrak{E}' = \frac{3\mathfrak{B}'}{4mg} - m(\tfrac{3}{2}\mathfrak{A} + A + B)$$

$$2(\alpha - \mathfrak{C})\mathfrak{D}' = \frac{9\mathfrak{B}'}{8mg} - mA$$

$$2(\alpha + \mathfrak{C})\mathfrak{E}' = -\frac{3\mathfrak{B}'}{8mg} - mB$$

$$2\mathfrak{C}\mathfrak{F}' = \tfrac{1}{2}\delta\mathfrak{B}' - m(C - \tfrac{1}{2}gg)$$

$$(2\alpha - \mathfrak{C})\mathfrak{G}' = \tfrac{1}{2}(\zeta + \eta)\mathfrak{B}' + \gamma\mathfrak{C}' + (\gamma - \delta)\mathfrak{D}' - \frac{3\mathfrak{F}'}{4mg} + m(\tfrac{3}{2}g\mathfrak{A} - \tfrac{3}{2}\mathfrak{B} + gA + \tfrac{1}{2}gB - D - E)$$

$$(2\alpha + \mathfrak{C})\mathfrak{H}' = \tfrac{1}{2}(\zeta + \theta)\mathfrak{B}' + \gamma\mathfrak{C}' + (\gamma + \delta)\mathfrak{E}' + \frac{9\mathfrak{F}'}{4mg} + m(\tfrac{3}{2}g\mathfrak{A} - \tfrac{3}{2}\mathfrak{C} + gB + \tfrac{1}{2}gA - D - F)$$

$$(2\alpha - 3\mathfrak{C})\mathfrak{J}' = \tfrac{1}{2}\eta\mathfrak{B}' + (\gamma - \delta)\mathfrak{D}' + \frac{9\mathfrak{F}'}{4mg} + m(\tfrac{1}{2}gA - E)$$

$$(2\alpha + 3\mathfrak{C})\mathfrak{K}' = \tfrac{1}{2}\theta\mathfrak{B}' + (\gamma + \delta)\mathfrak{E}' - \frac{3\mathfrak{F}'}{4mg} + m(\tfrac{1}{2}gB - F)$$

$$3\mathfrak{C}\mathfrak{L}' = \tfrac{1}{2}\nu\mathfrak{B}' + \frac{3\mathfrak{D}' + 9\mathfrak{E}'}{4mg} + \delta\mathfrak{F}' + m(\tfrac{1}{2}gC - G)$$

$$(4\alpha - \mathfrak{C})\mathfrak{M}' = -\frac{6A - 9B}{16mgg}\mathfrak{B}' + \frac{3\mathfrak{D}'}{4mg} - m(H + J)$$

$$(4\alpha + \mathfrak{C})\mathfrak{N}' = +\frac{3A - 6B}{16mgg}\mathfrak{B}' + \frac{9\mathfrak{E}'}{4mg} - m(H + K)$$

$$(4\alpha - 3\mathfrak{C})\mathfrak{P}' = -\frac{9A}{16mgg}\mathfrak{B}' - \frac{9\mathfrak{D}'}{4mg} - mJ$$

$$(4\alpha + 3\mathfrak{C})\mathfrak{Q}' = +\frac{3B}{16mgg}\mathfrak{B}' - \frac{3\mathfrak{E}'}{4mg} - mK$$

XXXIX.

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XXXIX.

Inuentis iam valoribus litterarum $p = b(1+\xi)$ et q vna cum anomalia vera v, distantia curtata lunae a terra $x = \frac{p}{1-q\cos v}$ cognoscetur: ac si deinceps latitudinis lunae ψ ratio habebitur, erit distantia vera $= \frac{p}{(1-q\cos v)\cos\psi}$. In Astronomia autem non tam distantia lunae, quam eius diameter apparens et parallaxis requiri solet; quarum vtraque cum sit distantiae lunae a terra reciproce proportionalis, erit tam diameter apparens quam parallaxis vt $\frac{(1-q\cos v)\cos\psi}{p}$; vnde si vtriusque valor medius ex obseruationibus fuerit definitus, ad quoduis tempus valor verus assignari poterit. Sit igitur siue diametri apparentis siue parallaxis horizontalis valor medius $= \sigma$, eritque is pro tempore dato $= \frac{b\sigma}{p}(1-q\cos v)\cos\psi$. Est autem proxime $\cos\psi = 1 - \frac{1}{4}\tan g\varrho^2 - \frac{1}{4}\operatorname{tg}\varrho^2\cos 2(\Phi-\pi) = \frac{2}{3} + \frac{\lambda+\Pi}{3}$, et $\frac{b}{p} = 1 - \xi + \xi\xi$: vnde fit diameter seu parallaxis $= \frac{1}{3}\sigma(2+\lambda+\Pi)(1-\xi+\xi\xi)(1-q\cos v)$, quae euoluitur in hanc expressionem: ob $\frac{2+\lambda}{3} = 1$ proxime:

$$\tfrac{1}{3}(2+\lambda)\sigma\,[1-q\cos v-\xi+q\xi\cos v+\xi\xi+\tfrac{1}{3}\Pi-\tfrac{1}{3}q\Pi\cos v]$$

XL.

Pro praesenti ergo casu, quo parallaxin solis, eiusque excentricitatem vna cum inclinatione orbitae lunaris

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ris negligimus, erit lunae diameter apparens vel parallaxis horizontalis

$$= \tfrac{1}{3}(2+\lambda)\sigma \text{ inult. per}$$

$$1-\tfrac{1}{2}C-(g+\tfrac{1}{2}G)\cos v$$
$$-\tfrac{1}{2}(2\mathfrak{A}+A+B)\cos 2\eta-\tfrac{1}{2}A\cos(2\eta-2v)-\tfrac{1}{2}B\cos(2\eta+2v)-\tfrac{1}{2}C\cos 2v$$
$$-\tfrac{1}{2}(2\mathfrak{B}+D+E)\cos(2\eta-v)-\tfrac{1}{2}(2\mathfrak{C}+D+F)\cos(2\eta+v)$$
$$-\tfrac{1}{2}E\cos(2\eta-3v)-\tfrac{1}{2}F\cos(2\eta+3v)-\tfrac{1}{2}G\cos 3v$$
$$-\tfrac{1}{2}(H+J)\cos(4\eta-v)-\tfrac{1}{2}(H+K)\cos(4\eta+v)-\tfrac{1}{2}J\cos(4\eta-3v)-\tfrac{1}{2}K\cos(4\eta+3v)$$

vbi quidem factor constans $\frac{1}{3}(2+\lambda)\sigma$ omitti potest, siquidem tantum proportio vel diametri apparentis vel parallaxis horizontalis desideretur.

XLI.

Si nunc hos valores in numeris euoluere velimus, ex obseruationibus primum colligimus has determinationes:

$\mathfrak{A}' = 13,3682$; $\Delta = 0,1123$ et proxime $g = 0,05445$

ac postremo quidem valore ipsius g tantum in terminis minimis vtar, in maioribus ipsam litteram g relicturus, vt deinceps ex collatione calculi cum obseruationibus accuratius fortasse determinari possit. Habemus ergo

$$13,3682 = \tfrac{1}{2}\delta\mathfrak{B}' + m(1+\tfrac{1}{2}gg-C)$$

vnde ob $\mathfrak{B}'$, gg et C numeros admodum paruos, statim prope colligitur $m = 13,3682$. Tum vero est prope

$$\mathfrak{B}' = -\frac{2mg}{\mathfrak{C}};\ \mathfrak{C} = m \text{ et } \delta = 2mg + \frac{1}{2mg} \text{ seu } \mathfrak{C} = 13,3682;$$

$\delta = 2,$

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$\delta = 2,1419$; hinc $\frac{1}{2}\delta\mathfrak{B}' = -0,1165$, ergo accuratius
$m(1+\frac{1}{2}gg-C) = 13,4847 = \alpha+1$ et $\alpha = 12,4847$
Porro ob $C = -\frac{1}{2mm}$ erit satis exacte . $m = 13,5039$
vnde ex valoribus AetB proxime collectis fit $\mathfrak{C} = 13,0644$
$\gamma = 26,9524g$.
At valor ipsius δ duabus constat partibus, altera per g multiplicata altera diuisa, quibus separatim expressis erit
$\delta = 27,0355\,g + 0,0370.\frac{1}{g} = 2,1521$ proxime.

XLII.

Hinc iam computo instituto sequentes supra assumtorum coefficientium eruuntur valores numerici:
$\mathfrak{A} = 0,008931; \mathfrak{B} = 0,03895g = 0,00209; \mathfrak{C} = 0,01219g = 0,00064$
ideoque
$\xi = 0,008931\,cf2\eta + 0,03895\,g\,cf(2\eta-v) + 0,01219\,g\,cf(2\eta+v)$
Deinde reperitur:
$A = -0,013995$; $B = -0,001460$; $C = +0,002834$
$D = -0,001213.g + 0,00002198.\frac{1}{g} = -0,000280$ proxime
$E = +0,25834.g - 0,00001889.\frac{1}{g} = +0,014012$ proxime
$F = -0,00225.g - 0,00000207.\frac{1}{g} = -0,000161$ proxime
$G = +0,00218.g + 0,00001221.\frac{1}{g} = +0,000340$ proxime
$H = -0,00001022.\frac{1}{g} = -0,000184$
$J = +0,00004897.\frac{1}{g} = +0,000882$
$K = +0,00000053.\frac{1}{g} = +0,000009$

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XLIII.

Hinc porro pro motu apogei eiusque inaequalitatibus colligitur: $\Delta = 0,1123$; qui quidem valor ex obseruationibus est desumtus

$$A' = -0,013995.\frac{1}{g} = -0,25703 \text{ proxime}$$
$$B' = -0,001460.\frac{1}{g} = -0,02682 \text{ proxime}$$
$$C' = +0,002834.\frac{1}{g} = +0,05205 \text{ proxime}$$
$$D' = +0,007432 \qquad F' = -0,002249$$
$$E' = -0,259170 \qquad G' = -0,002176$$

in minutis secundis

$$A' = -53018'' = -14^\circ, 43', 38''$$
$$B' = -5532 = -1^\circ, 32, 12$$
$$C' = +10736 = +2, 58, 56$$
$$D' = +1533 = +0, 25, 33$$
$$E' = -53459 = -14, 50, 59$$
$$F' = -464 = -0, 7, 44$$
$$G' = +449 = +0, 7, 29$$

Ergo longitudo apogei in minutis secundis

$$\Phi - v = \text{Const.}$$
$$+0,1123\omega - 53018'' \sin(2\eta - v) + 1533'' \sin 2\eta$$
$$- 5532 \sin(2+v) - 53459 \sin(2\eta - 2v)$$
$$+ 10736 \sin v - 464 \sin(2\eta + 2v)$$
$$+ 449 \sin 2v$$

XLIV.

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XLIV.

Iam pro longitudine ipſa inuenienda habentur primo ex §. 27. valores: $\gamma = 1,46756$

$\delta = +2,15210$; $\varepsilon = -0,027791$; $\zeta = +0,07138$

$\eta = -0,06974$; $\theta = -0,039809$; $\varkappa = -0,070920$

Deinde cum ſit proxime

$$\mathfrak{C}\mathfrak{B}' = -2mg \text{ ſeu } \mathfrak{B}' = -0,11256$$

erit quoque proxime

$$\mathfrak{C}' = -0,003485 \; ; \; \mathfrak{D}' = -0,014456$$
$$\mathfrak{E}' = +0,001509 \; ; \; \mathfrak{F}' = -0,005334$$

Hinc ergo accuratius elicietur $\mathfrak{B}' = -0,11019$, ideoque hic et reliqui conficientes tam abſolute quam in numeris ſecundis erunt:

abſolute	in minutis ſecundis
$\mathfrak{B}' = -0,11019$ . .	$\mathfrak{B}' = -22728'' = -6°, 18', 4''$
$\mathfrak{C}' = -0,00339$ . .	$\mathfrak{C}' = -700 = -0, 11, 40$
$\mathfrak{D}' = -0,01742$ . .	$\mathfrak{D}' = -3594 = -0, 59, 54$
$\mathfrak{E}' = +0,00149$ . .	$\mathfrak{E}' = +306 = +0, 5, 6$
$\mathfrak{F}' = -0,00524$ . .	$\mathfrak{F}' = -1081 = -0, 18, 1$
$\mathfrak{G}' = -0,01824$ . .	$\mathfrak{G}' = -3762 = -1, 2, 42$
$\mathfrak{H}' = -0,00056$ . .	$\mathfrak{H}' = -115 = -0, 1, 55$
$\mathfrak{J}' = +0,01368$ . .	$\mathfrak{J}' = +2823 = +0, 47, 3$
$\mathfrak{K}' = +0,00023$ . .	$\mathfrak{K}' = +47 = +0, 0, 47$
$\mathfrak{L}' = -0,00062$ . .	$\mathfrak{L}' = -128 = -0, 2, 8$
$\mathfrak{M}' = -0,00119$ . .	$\mathfrak{M}' = -246 = -0, 4, 6$
$\mathfrak{N}' = +0,00020$ . .	$\mathfrak{N}' = +41 = +0, 0, 41$
$\mathfrak{P}' = +0,00184$ . .	$\mathfrak{P}' = +379 = +0, 6, 19$
$\mathfrak{Q}' = -0,00001$ . .	$\mathfrak{Q}' = -2 = -0, 0, 2$

XLV.

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XLV.

Hinc ergo si ad datum tempus iam cognita sit anomalia lunae vera v cum angulo η, longitudo lunae per aequationes in minutis secundis expressas erit

$$\varphi = \text{Const.}$$

$$\begin{array}{llll}
+13{,}3682\omega & -22728'' \sin v & - 700'' \sin 2\eta & \\
& - 1081 \sin 2v & - 3594 \sin (2\eta - 2v) & \\
& - 128 \sin 3v & + 306 \sin (2\eta + 2v) & \\
& - 3762'' \sin (2\eta - v) & - 246'' \sin (4\eta - v) & \\
& - 115 \sin (2\eta + v) & + 41 \sin (4\eta + v) & \\
& + 2823 \sin (2\eta - 3v) & + 379 \sin (4\eta - 3v) & \\
& + 47 \sin (2\eta + 3v) & - 2 \sin (4\eta + 3v) &
\end{array}$$

vbi Const. $+ 13{,}3682\omega$ denotat longitudinem mediam; in reliquis autem terminis continentur inaequalitates periodicae pro hac hypothesi.

XLVI.

Inde iam vicissim anomalia vera lunae v colligitur, vt sit

$$v = \text{}$$

$$\begin{array}{llll}
13{,}2559\omega & -33464'' \sin v & - 2233'' \sin 2\eta & \\
& - 1530 \sin 2v & + 49864 \sin (2\eta - 2v) & \\
& - 128 \sin 3v & + 770 \sin (2\eta + 2v) & \\
& + 49256 \sin (2\eta - v) & - 246 \sin (4\eta - v) & \\
& + 5417 \sin (2\eta + v) & + 41 \sin (4\eta + v) & \\
& + 2823 \sin (2\eta - 3v) & + 379 \sin (4\eta - 3v) & \\
& + 47 \sin (2\eta + 3v) & - 2 \sin (4\eta + 3v) &
\end{array}$$

vbi primus terminus $13{,}2559\omega$ designat anomaliam mediam lunae, quae sit $= \zeta$: tum ex ea primum quaeratur ano-

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anomalia Keplerіana, quae ſcilicet a ſola excentricitate pendet, ſitque ea $= u$, vt ſit

$$u = \zeta - 33464'' \sin u - 1530'' \sin 2u - 128'' \sin 3u$$

vnde quidem facile tabulae conſtruentur. Tum ſtatuatur $v = u + z$, et quia angulus z eſt modicus, inde is ſatis prope poterit definiri. Interim tamen expedire videtur aliquot operationibus iterandis iſtam anomaliam veram v determinari; dum ſcilicet primum valor non nimis a veritate abhorrens pro v aeſtimando aſſumitur, ex eoque deinceps exactior colligitur; qui ſi nimis ab aſſumto diſcrepare reperiatur, ex hoc denuo exactior quaeratur, donec nulla amplius correctione fuerit opus.

XLVII.

Formula denique, cui tam diameter lunae apparens geocentrica quam parallaxis horizontalis eſt proportionalis, ex §. 40. reperitur

$$\begin{array}{ll}
1 - 0{,}05470 \cos v & - 0{,}00120 \cos 2\eta \\
- 0{,}00142 \cos 2v & + 0{,}00700 \cos(2\eta - 2v) \\
- 0{,}00017 \cos 3v & + 0{,}00073 \cos(2\eta + 2v) \\
- 0{,}00898 \cos(2\eta - v) & - 0{,}00035 \cos(4\eta - v) \\
- 0{,}00042 \cos(2\eta + v) & + 0{,}00009 \cos(4\eta + v) \\
- 0{,}00701 \cos(2\eta - 3v) & - 0{,}00044 \cos(4\eta - 3v) \\
+ 0{,}00008 \cos(2\eta + 3v) & - 0{,}00001 \cos(4\eta + 3v)
\end{array}$$

quorum quidem terminorum plures, qui pro parallaxi infra aliquot minuta ſecunda ſubſiſtunt, tuto omitti poterunt. His igitur tribus formulis pro anomalia vera v, longitudine $\odot$ et parallaxi ſeu diametro apparente inuentis motus lunae continerētur, ſi quidem tam ſolis paral-

laxis quam eius excentricitas et inclinatio orbitae lunaris ad ecliptícam negligatur. Hae autem sunt inaequalitates praecipuae, quae etiam ad reliquas eruendas adhiberi debent; vnde nunc ad inaequalitates ab excentricitate solis oriundas progrediamur.

INUESTIGATIO INAEQUALITATUM LUNAE SECUNDI GENERIS SEU AB EXCENTRICITATE SOLIS PENDENTIUM.

XLVIII.

Formulae nostrae differentiales, quatenus ab excentricitate orbitae solaris pendent, omissis terminis, quos iam constat esse minimos, erunt

$$\frac{d\xi}{d\omega} = \text{Praec.} + \frac{9e}{2m}\sin(2\eta-u) + \frac{9e}{2m}\sin(2\eta+u)$$

$$\frac{dq}{d\omega} = \text{Praec.} + \frac{3e}{4m}\sin(v-u) + \frac{3e}{4m}\sin(v+u)$$
$$- \frac{27e}{8m}\sin(2\eta-v-u) - \frac{27e}{8m}\sin(2\eta-v+u)$$
$$- \frac{9e}{8m}\sin(2\eta+v-u) - \frac{9e}{8m}\sin(2\eta+v+u)$$

$$\frac{q(d\Phi-dv)}{d\omega} = \text{Pr.} - \frac{3e}{4m}\cos(v-u) - \frac{3e}{4m}\cos(v+u)$$
$$- \frac{27e}{8m}\cos(2\eta-v-u) - \frac{27e}{8m}\cos(2\eta-v+u)$$
$$+ \frac{9e}{8m}\cos(2\eta+v-u) + \frac{9e}{8m}\cos(2\eta+v+u)$$

Quan-

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Quanquam enim nunc tam ξ quam q etiam ab excentricitate e pendeant, tamen in his formulis, in quas hae quantitates ingrediuntur, haec mutatio earum sine errore pro nihilo haberi potest; quoniam hi termini per se sunt minimi, et quia iam terminos ab e et q simul pendentes omisimus. Tum vero erit

$$\frac{d\Phi}{d\omega} = m(1+\tfrac{1}{2}qq) - 2mq\cos v + \tfrac{1}{2}mqq\cos 2v - \tfrac{3}{2}m\xi + 3m\xi\cos v$$

$$\text{et } \frac{du}{d\omega} = \frac{d\theta}{d\omega} = 1 + 2ee - 2e\cos u$$

XLIX.

Ad formulas has integrandas seu tantum ad eas integralium partes inueniendas, quae ab excentricitate solis e pendent, opus est vt formularum $\frac{d\eta}{d\omega}$, $\frac{dv}{d\omega}$ et $\frac{d\pi}{d\omega}$ primum habeamus partes principales, tum vero etiam eas quae a simplici solis excentricitate e pendent; habebimus ergo primo

$$\frac{d\eta}{d\omega} = m(1+\tfrac{1}{2}gg) - 1 - 2ee - 2mg\cos v + 2e\cos u$$

$$\frac{dv}{d\omega} = m(1+\tfrac{1}{2}gg) - 2mg\cos v + \frac{3e}{4mg}\cos(v-u) + \frac{3e}{4mg}\cos(v+u)$$
$$- \frac{1}{4m}\left(1 - \frac{9A+3B-2C}{2gg}\right) + \frac{27e}{8mg}\cos(2\eta - v - u) + \frac{27e}{8mg}\cos(2\eta - v + u)$$
$$- \frac{9e}{8mg}\cos(2\eta + v - u) - \frac{9e}{8mg}\cos(2\eta + v + u)$$

seu introducendis, vt supra §. 27. breuitatis gratia, literis

$$\alpha = m(1+\tfrac{1}{2}gg - C) - 2ee \quad ; \quad \gamma = 2mg$$

$$6 = m(1+\tfrac{1}{2}gg - C) - \frac{1}{4m}\left(1 - \frac{9A+3B-2C}{2gg}\right) ; \quad \delta = 2mg + \frac{1}{2mg} + \frac{3g}{8m}$$

erit: $\frac{d\eta}{d\omega}\alpha - \gamma \cos v + 2e\cos u$

$$\frac{dv}{d\omega} = 6 - \delta\cos v - \frac{9}{4mg}\cos(2\eta - v) + \frac{3}{4mg}\cos(2\eta + v)$$
$$+ \frac{3e}{4mg}\cos(v-u) + \frac{3e}{4mg}\cos(v+u) + \frac{27e}{8mg}\cos(2\eta - v - u)$$
$$+ \frac{27e}{8mg}\cos(2\eta - v + u) - \frac{9e}{8mg}\cos(2\eta + v - u) - \frac{9e}{8mg}\cos(2\eta + v + u)$$

et $\frac{du}{d\omega} = 1 - 2e\cos u.$

L.

Fingamus nunc primo:

$\xi = \mathfrak{A}\cos 2\eta + \mathfrak{B}\cos(2\eta - v) + \mathfrak{C}\cos(2\eta + v) + \mathfrak{P}\cos(2\eta - u) + \mathfrak{Q}\cos(2\eta + u)$

&c differentiando eos tantum sumamus terminos, qui formulae differentiali respondent, quandoquidem reliquos iam inuenimus: eritque

$$\frac{d\xi}{d\omega} = -2e\mathfrak{A}\sin(2\eta - u) - 2e\mathfrak{A}\sin(2\eta + u)$$
$$- (2\alpha - 1)\mathfrak{P} - (2\alpha + 1)\mathfrak{Q}$$

vnde colligitur:

$$(2\alpha - 1)\mathfrak{P} = -\frac{9e}{2m} - 2e\mathfrak{A}\,;\quad (2\alpha + 1)\mathfrak{Q} = -\frac{9e}{2m} - 2e\mathfrak{A}$$

Cum igitur sit $e = 0,0168$, erit in numeris:

$$\mathfrak{P} = -0,000247 \quad \text{et} \quad \mathfrak{Q} = -0,000227.$$

LI.

Fingatur porro: $q = g$
$+ A\cos(2\eta - v) + B\cos(2\eta + v) + C\cos v + M\cos(v - u) + N\cos(v + u)$
$+ P\cos(2\eta - v - u) + Q\cos(2\eta - v + u) + R\cos(2\eta + v - u) + S\cos(2\eta + v + u)$ &c

ac differentiando obtinebitur: $\frac{dq}{dw} =$

$-2eA\sin(2\eta-v-u)-2eA\sin(2\eta-v+u)-2eB\sin(2\eta+v-u)-2eB\sin(2\eta+v+u)$
$-(2\alpha-\delta-1)P-(2\alpha-\delta+1)Q-(2\alpha+\delta-1)R-(2\alpha+\delta+1)S$
$-(\delta-1)M\sin(v-u)-(\delta+1)H\sin(v+u)$

Comparatione ergo instituta reperietur:

$(\delta-1)M=-\frac{3e}{4m}$; $(\delta+1)N=-\frac{3e}{4m}$

$(2\alpha-\delta-1)P=\frac{27e}{8m}-2eA$; $(2\alpha-\delta+1)Q=\frac{27e}{8m}-2eA$

$(2\alpha+\delta-1)R=\frac{9e}{8m}-2eB$; $(2\alpha+\delta+1)S=\frac{9e}{8m}-2eB$

et in numeris

$M=-0,00008$; $P=+0,00042$; $R=+0,00004$
$N=-0,00006$; $Q=+0,00036$; $S=+0,00004$

LII.

Hic autem in differentiatione negleximus partes ipsius $\frac{dv}{dw}$ ab excentricitate e pendentes, quarum tamen eodem iure ratio haberi debuisset, atque partis in differentiali $\frac{d\eta}{dw}$; inde autem multo plures termini accedent ad valorum ipsius q, ponatur ergo ob hos terminos:

$$q = g$$
$$+A\cos(2\eta-v)+B\cos(2\eta+v)+C\cos v+M\cos(v-u)$$
$$+N\cos(v+u)+D\cos(2\eta-u)+E\cos(2\eta+u)$$
$$+P\cos(2\eta-v-u)+Q\cos(2\eta-v+u)+R\cos(2\eta+v-u)$$
$$+S\cos(2\eta+v+u)+K\cos(2v-u)+L\cos(2v+u)$$

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$$+F\cos(2\eta-2v-u)+G\cos(2\eta-2v+u)+H\cos(2\eta+2v-u)$$
$$+J\cos(2\eta+2v+u)+T\cos(4\eta-u)+V\cos(4\eta+u)$$
$$+W\cos(4\eta-2v-u)+X\cos(4\eta-2v+u)+Y\cos(4\eta+2v-u)$$
$$+Z\cos(4\eta+2v+u)$$

et sumto differentiali pleno reperiotur:

$$\frac{dq}{d\omega}=+\sin(2\eta-v-u)\,[-2eA-(2\alpha-\mathfrak{G}-1)P]$$
$$+\sin(2\eta-v+u)\,[-2eA-(2\alpha-\mathfrak{G}+1)Q]-(\mathfrak{G}-1)M\sin(v-u)$$
$$+\sin(2\eta\mp v-u)\,[-2eB-(2\alpha+\mathfrak{G}-1)R]$$
$$+\sin(2\eta+v+u)\,[-2eB-(2\alpha+\mathfrak{G}+1)S]-(\mathfrak{G}+1)N\sin(v+u)$$
$$+\sin(2\eta-u)\left(+\frac{3eA}{8mg}-\frac{3eB}{8mg}-\frac{27eC}{16mg}-\frac{9eC}{16mg}-(2\alpha-1)D\right)$$
$$+\sin(2\eta+u)\left(+\frac{3eA}{8mg}-\frac{3eB}{8mg}-\frac{27eC}{16mg}-\frac{9eC}{16mg}-(2\alpha+1)E\right)$$
$$+\sin(2\eta-2v+u)\left(+\frac{3eA}{8mg}+\frac{27eC}{16mg}-(2\alpha-2\mathfrak{G}+1)G\right)$$
$$+\sin(2\eta-2v-u)\left(+\frac{3eA}{8mg}+\frac{27eC}{16mg}-(2\alpha-2\mathfrak{G}-1)F\right)$$
$$+\sin(2\eta+2v-u)\left(-\frac{3eB}{8mg}+\frac{9eC}{16mg}-(2\alpha+2\mathfrak{G}-1)H\right)$$
$$+\sin(2\eta+2v+u)\left(-\frac{3eB}{8mg}+\frac{9eC}{16mg}-(2\alpha+2\mathfrak{G}+1)J\right)$$
$$+\sin u\left(+\frac{27eA}{16mg}-\frac{27eA}{16mg}+\frac{9eB}{16mg}-\frac{9eB}{16mg}-\frac{3eC}{8mg}+\frac{3eC}{8mg}\right)$$
$$+\sin(4\eta-2v-u)\left(+\frac{27eA}{16mg}-(4\alpha-2\mathfrak{G}-1)W\right)$$
$$+\sin(4\eta-2v+u)\left(+\frac{27eA}{16mg}-(4\alpha-2\mathfrak{G}+1)X\right)$$

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$$+\sin(4\eta+2v-u)\left(+\frac{9eB}{16mg}-(4a+2c-1)Y\right)$$
$$+\sin(4\eta+2v+u)\left(+\frac{9eB}{16mg}-(4a+2c+1)Z\right)$$
$$+\sin(2v-u)\left(+\frac{9eA}{16mg}-\frac{27eB}{16mg}-\frac{3eC}{8mg}-(2c-1)K\right)$$
$$+\sin(2v+u)\left(+\frac{9eA}{16mg}-\frac{27eB}{16mg}-\frac{3eC}{8mg}-(2c+1)L\right)$$
$$+\sin(4\eta-u)\left(-\frac{9eA}{16mg}-\frac{27eB}{16mg}-(4a-1)T\right)$$
$$+\sin(4\eta+u)\left(-\frac{9eA}{16mg}-\frac{27eB}{16mg}-(4a+1)V\right)$$

vnde reperitur:

$D = -0{,}000010$	;	$H = +0{,}000001$
$E = -0{,}000010$	;	$J = +0{,}000001$
$F = +0{,}000005$	;	$K = -0{,}000006$
$G = +0{,}000065$	;	$L = -0{,}000006$
$T = +0{,}000004$	;	$X = -0{,}000022$
$V = +0{,}000004$	;	$Y = -0{,}000000$
$W = -0{,}000023$	;	$Z = -0{,}000000$

LIII.

Ponamus nunc etiam pro motu apogei

$$\Phi - v = \text{Conſt.} + \Delta\omega$$

$$+A'\sin(2\eta-v)+B'\sin(2\eta+v)+C'\sin v+M'\sin(v-u)+N'\sin(v+u)$$
$$+P'\sin(2\eta-v-u)+Q'\sin(2\eta-v+u)+R'\sin(2\eta+v-u)+S'\sin(2\eta+v+u)$$
$$+D'\sin(2\eta-u)+E'\sin(2\eta+u)+K'\sin(2v-u)+L'\sin(2v+u)+O'\sin u$$
$$+F'\sin(2\eta-2v-u)+G'\sin(2\eta-2v+u)+H'\sin(2\eta+2v-u)+J'\sin(2\eta+2v+u)$$
$$+W'\sin(4\eta-2v-u)+X'\sin(4\eta-2v+u)+Y'\sin(4\eta+2v-u)+Z'\sin(4\eta+2v+u)$$
$$+T'\sin(4\eta-u)+V'\sin(4\eta+u)$$

et

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et sumto differentiali pleno reperietur:

$$\frac{d\Phi - dv}{d\omega} = \Delta + \cos(2\eta - v - u)\,[2eA' + (2\alpha - \mathfrak{G} - 1)P']$$
$$+ \cos(2\eta - v + u)\,[2eA' + (2\alpha - \mathfrak{G} + 1)Q'] + (\mathfrak{G} - 1)M'\cos(v - u)$$
$$+ \cos(2\eta + v - u)\,[2eB' + (2\alpha + \mathfrak{G} - 1)R']$$
$$+ \cos(2\eta + v + u)\,[2eB' + (2\alpha + \mathfrak{G} + 1)S'] + (\mathfrak{G} + 1)N'\cos(v + u)$$
$$+ \cos(2\eta - u)\left(-\frac{3e}{8mg}A' + \frac{3e}{8mg}B' + \frac{27e}{16mg}C' - \frac{9e}{16mg}C' + (2\alpha - 1)D'\right)$$
$$+ \cos(2\eta + u)\left(-\frac{3e}{8mg}A' + \frac{3e}{8mg}B' + \frac{27e}{16mg}C' - \frac{9e}{16mg}C' + (2\alpha + 1)E'\right)$$
$$+ \cos(2\eta - 2v - u)\left(-\frac{3e}{8mg}A' + \frac{27e}{16mg}C' + (2\alpha - 2\mathfrak{G} - 1)F'\right)$$
$$+ \cos(2\eta - 2v + u)\left(-\frac{3e}{8mg}A' + \frac{27e}{16mg}C' + (2\alpha - 2\mathfrak{G} + 1)G'\right)$$
$$+ \cos(2\eta + 2v - u)\left(+\frac{3e}{8mg}B' + \frac{9e}{16mg}C' + (2\alpha + 2\mathfrak{G} - 1)H'\right)$$
$$+ \cos(2\eta + 2v + u)\left(+\frac{3e}{8mg}B' + \frac{9e}{16mg}C' + (2\alpha + 2\mathfrak{G} + 1)J'\right)$$
$$+ \cos u\left(-\frac{27e}{16mg}A' - \frac{27e}{16mg}A' - \frac{9e}{16mg}B' - \frac{9e}{16mg}B' + \frac{3e}{8mg}C' + \frac{3e}{8mg}C' + O'\right)$$
$$+ \cos(4\eta - 2v - u)\left(-\frac{27e}{16mg}A' + (4\alpha - 2\mathfrak{G} - 1)W'\right)$$
$$+ \cos(4\eta - 2v + u)\left(-\frac{27e}{16mg}A' + (4\alpha - 2\mathfrak{G} + 1)X'\right)$$
$$+ \cos(4\eta + 2v - u)\left(-\frac{9e}{16mg}B' + (4\alpha + 2\mathfrak{G} - 1)Y'\right)$$
$$+ \cos(4\eta + 2v + u)\left(-\frac{9e}{16mg}B' + (4\alpha + 2\mathfrak{G} + 1)Z'\right)$$
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$$+\cos(2v-u)\left(+\frac{9e}{16mg}A'+\frac{27e}{16mg}B'+\frac{3e}{8mg}C'+(2\mathfrak{C}-1)K'\right)$$

$$+\cos(2v+u)\left(+\frac{9e}{16mg}A'+\frac{27e}{16mg}B'+\frac{3e}{8mg}C'+(2\mathfrak{C}+1)L'\right)$$

$$+\cos(4\eta-u)\left(+\frac{9e}{16mg}A'+\frac{27e}{16mg}B'+(2\alpha-1)T'\right)$$

$$+\cos(4\eta+u)\left(+\frac{9e}{16mg}A'+\frac{27e}{16mg}B'+(2\alpha+1)V'\right)$$

LIV.

Singuli iam hi termini multiplicentur per q, cuius valor quidem erit $=g$, quoniam hi termini in suo genere iam sunt minimi: sed quoniam valor $\frac{dQ-dv}{d\omega}$ adhuc hos terminos praecipuos continet:

$$(2\alpha-\mathfrak{C})A'\cos(2\eta-v)+(2\alpha+\mathfrak{C})B'\cos(2\eta+v)+\mathfrak{C}C'\cos v$$

si et hi per q multiplicentur, inde nascentur quoque termini angulum u involventem, erit autem pro his, sumtis partibus tantum praecipuis:

$$q=\text{Praec.}+P\cos(2\eta-v-u)+Q\cos(2\eta-v+u)$$

Ergo ad illos terminos per q multiplicatos insuper accedent isti:

$\cos u\ [\frac{1}{2}(2\alpha-\mathfrak{C})PA'+\frac{1}{2}(2\alpha-\mathfrak{C})QA']\quad+\frac{1}{2}\mathfrak{C}PC'\cos(2\eta-u)$

$\cos(4\eta-2v-u)\ [\frac{1}{2}(2\alpha-\mathfrak{C})PA'+\frac{1}{2}\mathfrak{C}PC']\quad+\frac{1}{2}\mathfrak{C}QC'\cos(2\eta+u)$

$\cos(4\eta-2v+u)\ [\frac{1}{2}(2\alpha-\mathfrak{C})QA'+\frac{1}{2}\mathfrak{C}QC']$

$\cos(2v+u)\ [\frac{1}{2}(2\alpha+\mathfrak{C})PB'\quad+\cos(4\eta-u)\ [\frac{1}{2}(2\alpha+\mathfrak{C})PB'$

$\cos(2v-u)\ [\frac{1}{2}(2\alpha+\mathfrak{C})QB'\quad+\cos(4\eta+u)\ [\frac{1}{2}(2\alpha+\mathfrak{C})QB'$

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LV.

Hinc ergo obtinentur ſequentes determinationes:

$$2eg\,A' + (2\alpha - \mathfrak{G} - 1)\,g\,P' = -\frac{27e}{8m}$$

$$2eg\,A' + (2\alpha - \mathfrak{G} + 1)\,g\,Q' = -\frac{27e}{8m}$$

$$2eg\,B' + (2\alpha + \mathfrak{G} - 1)\,g\,R' = +\frac{9e}{8m}$$

$$2eg\,B' + (2\alpha + \mathfrak{G} + 1)\,g\,S' = +\frac{9e}{8m}$$

$$(\mathfrak{G} - 1)\,g\,M' = -\frac{3e}{4m} \quad ; \quad (\mathfrak{G} + 1)\,g\,N' = -\frac{3e}{4m}$$

$$-\frac{3e}{8m}(A' - B') + \frac{9e}{8m}C' + (2\alpha - 1)\,g\,D' + \tfrac{1}{2}\mathfrak{G}\,PC' = 0$$

$$-\frac{3e}{8m}(A' - B') + \frac{9e}{8m}C' + (2\alpha + 1)\,g\,E' + \tfrac{1}{2}\mathfrak{G}\,QC' = 0$$

$$-\frac{3e}{8m}A' + \frac{27e}{16m}C' + (2\alpha - 2\mathfrak{G} - 1)\,g\,F' = 0$$

$$-\frac{3e}{8m}A' + \frac{27e}{16m}C' + (2\alpha - 2\mathfrak{G} + 1)\,g\,G' = 0$$

$$+\frac{3e}{8m}B' + \frac{9e}{16m}C' + (2\alpha + 2\mathfrak{G} - 1)\,g\,H' = 0$$

$$+\frac{3e}{8m}B' + \frac{9e}{16m}C' + (2\alpha + 2\mathfrak{G} + 1)\,g\,J' = 0$$

$$-\frac{27e}{8m}A' - \frac{9e}{8m}B' + \frac{3e}{8m}C' + g\,O' + \tfrac{1}{2}(2\alpha - \mathfrak{G})\,(P + Q)\,A' = 0$$

$$-\frac{27e}{8m}A' + (4\alpha - 2\mathfrak{G} - 1)\,g\,W' + \tfrac{1}{2}(2\alpha - \mathfrak{G})\,PA' + \tfrac{1}{2}\mathfrak{G}\,PC' = 0$$

$$-\frac{27e}{8m}A' + (4\alpha - 2\mathfrak{G} + 1)\,g\,X' + \tfrac{1}{2}(2\alpha - \mathfrak{G})\,QA' + \tfrac{1}{2}\mathfrak{G}\,QC' = 0$$

$$-\frac{9e}{16m}B' + (4\alpha + 2\mathfrak{G} - 1)\,gY' = 0 \quad ; \quad -\frac{9e}{16m}B' + (4\alpha + 2\mathfrak{G} + 1)\,gZ' = 0$$

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$$+\frac{9e}{16m}A'+\frac{27e}{16m}B'+\frac{3e}{8m}C'+(2\mathfrak{C}-1)gK'+\tfrac{1}{2}(2\alpha+\mathfrak{C})QB'=0$$

$$+\frac{9e}{16m}A'+\frac{27e}{16m}B'+\frac{3e}{8m}C'+(2\mathfrak{C}+1)gL'+\tfrac{1}{2}(2\alpha+\mathfrak{C})PB'=0$$

$$+\frac{9e}{16m}A'+\frac{27e}{16m}B'+(4\alpha-1)gT'+\tfrac{1}{2}(2\alpha+\mathfrak{C})PB'=0$$

$$+\frac{9e}{16m}A'+\frac{27e}{16m}B'+(4\alpha+1)gV'+\tfrac{1}{2}(2\alpha+\mathfrak{C})QB'=0$$

LVI.

Valores ergo horum coefficientium iam ad minuta secunda reductorum erunt: $O' = +310''$

$P' = -1285''$; $M' = -293''$; $F' = +401''$
$Q' = -1087$; $N' = -251$; $G' = +5412$
$R' = +148$; $D' = -52$; $H' = -2$
$S' = +141$; $E' = -45$; $J' = -2$
$W' = -91''$; $K' = +61''$
$X' = -95$; $L' = +61$
$Y' = -1$; $T' = +35$
$Z' = -1$; $V' = +31$

Vnde ob excentricitatem orbitae solaris erit:

$$\xi = +0{,}008931\cos 2\eta - 0{,}000247\cos(2\eta-u)$$
$$+0{,}002090\cos(2\eta-v) - 0{,}000227\cos(2\eta+u)$$
$$+0{,}000640\cos(2\eta+v)$$

$$q = -0{,}013795\cos(2\eta-v) - 0{,}000280\cos 2\eta$$
$$-0{,}001460\cos(2\eta+v) + 0{,}014012\cos(2\eta-2v)$$
$$+0{,}002834\cos v - 0{,}000162\cos(2\eta+2v)$$
$$+0{,}000340\cos 2v$$
$$-0{,}000184\cos 4\eta + 0{,}000420\cos(2\eta-v-u)$$
$$+0{,}000882\cos(4\eta-2v) + 0{,}000360\cos(2\eta-v+u)$$
$$+0{,}000069\cos(4\eta+2v)$$

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$$\Phi - v = \text{Const.} + 0,1123\,\omega$$

$$\begin{array}{lll}
-53018''\sin(2\eta-v) + 1533''\sin 2\eta & & -1285''\sin(2\eta-v-u)\\
-5532\sin(2\eta+v) & -53549\sin(2\eta-2v) & -1087\sin(2\eta-v-u)\\
+10736\sin v & -464\sin(2\eta+2v) & +148\sin(2\eta+v-u)\\
 & +449\sin 2v & +141\sin(2\eta+v+u)\\
-293''\sin(v-u) & +401''\sin(2\eta+2v-u) & +61''\sin(2v-u)\\
-251\sin(v+u) & +5412\sin(2\eta-2v+u) & +61\sin(2v+u)\\
-52\sin(2\eta-u) & -91\sin(4\eta-2v-u) & +35\sin(4\eta-u)\\
-45\sin(2\eta+u) & -95\sin(4\eta-2v+u) & +31\sin(4\eta+u)\\
+310\sin u & &
\end{array}$$

neglectis scilicet terminis minimis.

LVII.

Denique pro longitudine lunae vera Φ inuenienda, cum sit $\frac{d\Phi}{d\omega} = \text{Praec.}$

$$\begin{array}{ll}
+mg.\ 0,00005\cos(2\eta-v-u) & -m.\ 0,00005\cos(2\eta-u)\\
+mg.\ 0,00002\cos(2\eta-v+u) & -m.\ 0,00002\cos(2\eta+u)\\
+mg.\ 0,00005\cos(2\eta+v-u) & -m.\ 0,00042\cos(2\eta-2v-u)\\
+mg.\ 0,00002\cos(2\eta+v+u) & -m.\ 0,00036\cos(2\eta-2v+u)\\
+mg.\ 0,00042\cos(2\eta-3v-u) & \\
+mg.\ 0,00036\cos(2\eta-3v+u) &
\end{array}$$

ponatur $\Phi = \text{Const.}$

$$+\mathfrak{A}'\omega + \mathfrak{B}'\sin v + \mathfrak{D}'\sin(2\eta-2v) + \mathfrak{G}'\sin(2\eta-v) + \mathfrak{J}'\sin(2\eta-3v)$$

vna cum nouis terminis

$$\begin{array}{lll}
+a'\sin(2\eta-v-u) & +e'\sin(2\eta-u) & +g'\sin(2\eta-2v-u)+l'\sin u\\
+b'\sin(2\eta-v+u) & +f'\sin(2\eta+u) & +h'\sin(2\eta-2v+u)+m'\sin(2v-u)\\
+c'\sin(2\eta+v-u) & & +j'\sin(2\eta-3v-u)+n'\sin(2v+u)\\
+d'\sin(2\eta+v+u) & & +k'\sin(2\eta-3v+u)
\end{array}$$

+ o'

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$+\mathfrak{o}'\sin(2\eta+2v-u) + \mathfrak{q}'\sin(v-u) + \mathfrak{s}'\sin(4v-u)$
$+\mathfrak{p}'\sin(2\eta+2v+u) + \mathfrak{r}'\sin(v+u) + \mathfrak{t}'\sin(4v+u)$

pro reliquorum terminorum, quos forma differentialis requirit, coefficientibus ponamus litteram I.

LVIII.

Differentiatione iam per regulas praecedentes instituta erit: $\frac{d\Phi}{d\omega} = \text{Praec.}$

$$+\cos u\left(\frac{3e}{8mg}\mathfrak{B}'+\frac{3e}{8mg}\mathfrak{B}'-\frac{27e}{16mg}\mathfrak{G}'-\frac{27e}{16mg}\mathfrak{G}'+\mathfrak{l}'\right)$$
$$+(2\mathfrak{a}+\mathfrak{E}-1)\mathfrak{c}'\cos(2\eta+v-u)$$
$$+\cos(2v-u)\left(\frac{3e}{8mg}\mathfrak{B}'+\frac{9e}{16mg}\mathfrak{G}'-\frac{81e}{16mg}\mathfrak{J}'+(2\mathfrak{E}-1)\mathfrak{m}'\right)$$
$$+(2\mathfrak{a}+\mathfrak{E}+1)\mathfrak{d}'\cos(2\eta+v+u)$$
$$+\cos(2v+u)\left(\frac{3e}{8mg}\mathfrak{B}'+\frac{9e}{16mg}\mathfrak{G}'-\frac{18e}{16mg}\mathfrak{J}'+(2\mathfrak{E}+1)\mathfrak{n}'\right)$$
$$+\cos(2\eta-u)\left(+\frac{27e}{16mg}\mathfrak{B}'-\frac{9e}{16mg}\mathfrak{B}'-\frac{3e}{8mg}\mathfrak{G}'+(2\mathfrak{a}-1)\mathfrak{e}'\right)$$
$$+\cos(2\eta+u)\left(+\frac{27e}{16mg}\mathfrak{B}'-\frac{9e}{16mg}\mathfrak{B}'-\frac{3e}{8mg}\mathfrak{G}'+(2\mathfrak{a}+1)\mathfrak{f}'\right)$$
$$+\cos(2\eta-2v-u)\left(+\frac{27e}{16mg}\mathfrak{B}'+2e\mathfrak{D}'-\frac{9e}{8mg}\mathfrak{G}'-\frac{9e}{8mg}\mathfrak{J}'\right.$$
$$\left.+(2\mathfrak{a}-2\mathfrak{E}-1)\mathfrak{g}'\right)$$
$$+\cos(2\eta-2v+u)\left(+\frac{27e}{16mg}\mathfrak{B}'+2e\mathfrak{D}'-\frac{3e}{8mg}\mathfrak{G}'-\frac{9e}{8mg}\mathfrak{J}'\right.$$
$$\left.+(2\mathfrak{a}-2\mathfrak{E}+1)\mathfrak{h}'\right)$$

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$$+\cos(4v-u)\left(+\frac{27e}{16mg}\mathfrak{Z}'+(4\mathfrak{E}-1)\,s'\right)$$

$$+\cos(4v+u)\left(+\frac{27e}{16mg}\mathfrak{Z}'+(4\mathfrak{E}+1)\,t'\right)$$

$$+\cos(2\eta+2v-u)\left(-\frac{9e}{16mg}\mathfrak{B}'+(2\alpha+2\mathfrak{E}-1)\,o'\right)$$

$$+\cos(2\eta+2v+u)\left(-\frac{9e}{16mg}\mathfrak{B}'+(2\alpha+2\mathfrak{E}+1)\,p'\right)$$

$$+\cos(2\eta-v-u)\left(-\frac{3e}{4mg}\mathfrak{D}'+2e\mathfrak{G}'+(2\alpha-\mathfrak{E}-1)\,a'\right)$$

$$+\cos(2\eta-3v+u)\left(-\frac{3e}{8mg}\mathfrak{D}'+2e\mathfrak{Z}'+(2\alpha-3\mathfrak{E}+1)\,f'\right)$$

$$+\cos(2\eta-v+u)\left(-\frac{3e}{4mg}\mathfrak{D}'+2e\mathfrak{G}'+(2\alpha-\mathfrak{E}+1)\,b'\right)$$

$$+\cos(2\eta-3v-u)\left(-\frac{3e}{4mg}\mathfrak{D}'+2e\mathfrak{Z}'+(2\alpha-3\mathfrak{E}-1)\,j'\right)$$

$$+\cos(v-u)\left(-\frac{27e}{8mg}\mathfrak{D}'+(\mathfrak{E}-1)\,q'\right)$$

$$+\cos(v+u)\left(-\frac{27e}{8mg}\mathfrak{D}'+(\mathfrak{E}+1)\,r'\right)$$

$$+\cos(4\eta-3v-u)\left(-\frac{27e}{8mg}\mathfrak{D}'+(4\alpha-3\mathfrak{E}-1)\,l\right)$$

$$+\cos(4\eta-3v+u)\left(-\frac{27e}{8mg}\mathfrak{D}'+(4\alpha-3\mathfrak{E}+1)\,l\right)$$

$$+\cos(3v-u)\left(+\frac{9e}{8mg}\mathfrak{D}'+(3\mathfrak{E}-1)\,l\right)$$

$$+\cos(3v+u)\left(+\frac{9e}{8mg}\mathfrak{D}'+(3\mathfrak{E}+1)\,l\right)$$

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$$+\cos(4\eta-v-u)\left(+\frac{9e}{8mg}\mathfrak{D}'+(4\alpha-6-1)\,\mathfrak{l}\right)$$

$$+\cos(4\eta-v+u)\left(+\frac{9e}{8mg}\mathfrak{D}'+(4\alpha-6+1)\,\mathfrak{l}\right)$$

$$+\cos(4\eta-2v-u)\left(-\frac{27e}{16mg}\mathfrak{G}'+\frac{27e}{16mg}\mathfrak{J}'+(4\alpha-2\mathfrak{C}-1)\,\mathfrak{l}\right)$$

$$+\cos(4\eta-2v+u)\left(-\frac{27e}{16mg}\mathfrak{G}'+\frac{27e}{16mg}\mathfrak{J}'+(4\alpha-2\mathfrak{C}+1)\,\mathfrak{l}\right)$$

$$+\cos(4\eta-u)\left(+\frac{9e}{16mg}\mathfrak{G}'+(4\alpha-1)\,\mathfrak{l}\right)$$

$$+\cos(4\eta+u)\left(+\frac{9e}{16mg}\mathfrak{G}'+(4\alpha+1)\,\mathfrak{l}\right)$$

$$+\cos(2\eta-4v-u)\left(-\frac{9e}{8mg}\mathfrak{J}'+(2\alpha-4\mathfrak{C}-1)\,\mathfrak{l}\right)$$

$$+\cos(2\eta-4v+u)\left(-\frac{9e}{8mg}\mathfrak{J}'+(2\alpha-4\mathfrak{C}+1)\,\mathfrak{l}\right)$$

$$+\cos(4\eta-4v-u)\left(-\frac{81e}{3mg}\mathfrak{J}'+(4\alpha-4\mathfrak{C}-1)\,\mathfrak{l}\right)$$

$$+\cos(4\eta-4v+u)\left(-\frac{81e}{8mg}\mathfrak{J}'+(4\alpha-4\mathfrak{C}+1)\,\mathfrak{l}\right)$$

LIX.

Collectis hinc valoribus coefficientium assumtorum, obtinebitur longitudo lunae vt sequitur:

$$\varphi =$$

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$$\varphi = C + 13,3682\,\omega$$

$$\begin{array}{lll}
-22728''\sin v & +2823''\sin(2\eta-3v) & +100''\sin u \\
-\ 1081\ \sin 2v & +\ 47\ \sin(2\eta+3v) & -\ 23\ \sin(v-u) \\
-\ 128\ \sin 3v & -\ 246\ \sin(4\eta-v) & -\ 20\ \sin(v+u) \\
-\ 700\ \sin 2\eta & +\ 41\ \sin(4\eta+v) & +\ 22\ \sin(2v-u) \\
-\ 3594\ \sin(2\eta-2v) & +\ 379\ \sin(4\eta-3v) & +\ 21\ \sin(2v+u) \\
+\ 306\ \sin(2\eta+2v) & -\ 2\ \sin(4\eta+3v) & +\ 2\ \sin(3v-u) \\
-\ 3762\ \sin(2\eta-v) & & +\ 2\ \sin(3v+u) \\
-\ 115\ \sin(2\eta+v) & & -\ 2\ \sin(4v-u) \\
 & & -\ 2\ \sin(4v+u)
\end{array}$$

$$\begin{array}{ll}
+\ 17''\sin(2\eta-u) & -\ 1''\sin(2\eta-4v-u) \\
+\ 19\ \sin(2\eta+u) & -\ 1\ \sin(2\eta-4v+u) \\
+\ 1\ \sin(4\eta-u) & -\ 11\ \sin(4\eta-2v-u) \\
+\ 1\ \sin(4\eta+u) & -\ 10\ \sin(4\eta-2v+u) \\
+\ 6\ \sin(2\eta-v-u) & -\ 28\ \sin(4\eta-3v-u) \\
+\ 5\ \sin(2\eta-v+v) & -\ 23\ \sin(4\eta-3v+u) \\
+\ 60\ \sin(2\eta-2v-u) & -\ 98\ \sin(4\eta-4v-u) \\
-\ 194\ \sin(2\eta-2v+u) & -\ 245\ \sin(4\eta-4v+u) \\
-\ 6\ \sin(2\eta+2v-u) & +\ 2\ \sin(4\eta-v-u) \\
-\ 6\ \sin(2\eta+2v+u) & +\ 2\ \sin(4\eta-v+u) \\
+\ 6\ \sin(2\eta-3v-u) & \\
+\ 8\ \sin(2\eta-3v+u) &
\end{array}$$

LX.

Plurimae igitur prodeunt inaequalitates ab excentricitate solis pendentes, quarum nonnullae ita sunt magnae,

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gnae, vt sine notabili errore omitti nequeant; cuiusmodi sunt imprimis, quae ab angulis $2\eta - 2v \mp u$ et $4\eta - 4v \mp u$ pendent. Sed in his fere idem incommodum vsu venit, quo methodus praecedens premebatur, quod magnitudo harum inaequalitatum per Theoriam non satis accurate definiri queat. Cum enim pro his terminis inueniendis diuisores $2\alpha - 2\zeta \mp 1$ et $4\alpha - 4\zeta \mp 1$ fiant perquam exigui, manifestum est in diuidendis terminos minimos neglectos non exigui fore momenti: praecipue cum pro litteris g' et h' termini maiores fere se mutuo destruxissent. Vnde cum ex valore Φ tantum termini maiores $\mathfrak{B}'$, $\mathfrak{D}'$, $\mathfrak{G}'$ et $\mathfrak{J}'$ essent adhibiti, perspicuum est si etiam reliqui minores fuissent introducti, ex iis insignem mutationem in valore coefficientium g' et h' orituram fuisse.

LXI.

De his autem inaequalitatibus tenendum est, eas per satis notabile temporis spatium vix immutari; nam inaequalitates ab angulo $2\eta - 2v + u$, ob quantitatem $2\alpha - 2\zeta + 1 = -0{,}1594$ periodum habent annorum circiter $6\frac{1}{3}$ annorum, et interuallo 19 annorum ter tantum reuoluuntur: et inaequalitas ab angulo $4\eta - 4v + u$ pendens spatio 29 annorum 19 periodos absoluit. Ex quo cum istae inaequalitates per theoriam saltem propemodum fuerint definitae, eas deinceps per obseruationes accuratius definiri conueniet: nisi forte quis laborem in se suscipere voluerit, calculum hic adumbratum multo accuratius instituendi terminorumque hic omissorum rationem habendi; tum vero etiam valores ξ, q et $\Phi - b$

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multo maiori ſtudio, quam hic feci, euolui oporteret, quoniam in horum determinatione multa neglexi, quae in calculo tandem ad nōtabilem quantitatem excreſcere potuiſſent.

LXII.

Interim tamen hic notari conuenit, hac methodo eas tantum inaequalitates prodire incertas, quae ſatis longis periodis abſoluuntur; quae incertitudo minus officit, cum per obſeruationes facilius emendari poſſit: praecedente vero methodo etiam aliae inaequalitates minoribus periodis circumſcriptae aliquantum incertae prodierunt, quod ſane ingens erat incommodum. Vnde ex hac parte haec methodus poſterior priori anteferenda videtur: verum ſi ingentem inaequalitatum numerum ſpectemus, quibus non ſolum lunae longitudo afficitur, ſed etiam longitudo apogei, calculus tantopere fit operoſus, vt etiamſi has formulas accuratiſſime euolverem, tamen in praxi difficillimi foret vſus. Quin etiam plurimae inaequalitates in motum apogei ingredi videntur, quarum effectum deinceps per alias longitudinis inaequalitates iterum deſtrui oportet, ita vt ſatius fuiſſet illas penitus omittere.

LXIII.

Multitudo autem harum inaequalitatum, quibus tam apogei, quam ipſius lunae longitudo turbatur, inde potiſſimum originem trahit, quod inaequalitates excentricitatis prae eius quantitate media admodum ſint notabiles, atque adeo quadrantem mediae quantitatis ſuperent; ita vt prae ea negligi minime queant. Multo plures autem

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tem adhuc inaequalitates essent accessurae, si excentricitas lunae media adhuc esset minor, quo certe casu calculi difficultates insuperabiles euasissent: hoc vero ipso casu methodus prior multo tractabilior redderetur, tum enim pleraeque inaequalitates ibi multo minores prodirent. Atque ob hanc causam minus expedire videtur, anomaliam lunae ita constituere, vt eius sinus tam pro maximis quam pro minimis distantiis lunae a terra plane euanescat, etiamsi haec ratio naturae rei maxime consentanea videatur.

LXIV.

Cum igitur numerus inaequalitatum iam tantopere increuerit, facile perspicitur eum adhuc multo magis auctum iri, si eas inaequalitates, quae cum a parallaxi solis, tum ab eius inclinatione ad eclipticam essem euoluturus, quo labore propterea, cum eius vsus fere nullus futurus esset, supersedebo. Interim tamen hinc tantum colligere licet, inaequalitates ab angulis $2\eta - 2v \mp \varkappa$ et $4\eta - 4v \pm \varkappa$ ortas, minime esse contemnendas; quae cum methodo praecedente sint vel omissae vel non satis accurate determinatae, sine dubio causam in se continent, quod etiam accuratissimae tabulae per obseruationes emendatae adhuc vltra $4'$ saepe a veritate aberrent.

LXV.

Sufficiat igitur methodum exposuisse, cuius ope inaequalitates lunae tam ratione apogei, quam longitudinis ac latitudinis verae ex anomalia hic adhibita determinari queant; neque propterea laborem calculi reli-

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quarum inaequalitatum, quae vel ex ſolis parallaxi vel ex inclinatione orbitae lunaris ad eclipticam oriuntur, ſuſcipio; quippe quarum numerus, ſiquidem omnes, quae alicuius momenti eſſent futurae, perſequi vellem, in immenſum excreſceret. Non ſolum autem multitudo inaequalitatum hanc methodum omni vtilitate in praxi priuabit, ſed etiam ingentes aequationes, quas determinatio apogei, atque anomaliae inde pendentis requirit, ita ſunt comparatae, vt ipſae iam ſatis exactam tam longitudinis quam anomaliae cognitionem requirant; quae res etſi initio ſupponi poſſent, deinceps iterata eadem operatione accuratius definiendae, tamen quia correctio apogei vltra 30° gradus aſſurgere poteſt, calculus ob inaequalitatum multitudinem per ſe taedioſus, nimis crebro repeti deberet, antequam de concluſione certi eſſe poſſemus.

APPLICATIO FORMULARUM INUENTARUM AD ALIOS CALCULOS LUNARES.

LXVI.

Cum igitur calculus inaequalitatum motus lunae hactenus duplici modo ſit inſtitutus, dum priori anomalia vera regulis Keplerianis conformis eſt aſſumta, poſteriori vero ita conſtituta, vt eius ſinus tam pro maximis lunae a terra diſtantiis quam pro minimis prorſus euaneſceret, quorum vterque vti vidimus incommodis non caret: ita etiam infinitis aliis modis lunae inaequalitates repraeſentari poterunt, quos breuiter expoſuiſſe haud abs

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re fore arbitror. Nullum enim est dubium, quin inter hos infinitos modos quidam reperiantur, qui ipsi naturae rei magis sint consentanei, neque iis incommodis laborent, quibus vtrumque expositum non mediocriter impediri comperimus; etiamsi adhuc difficillimum videatur, inter hanc infinitam multitudinem modum conuenientissimum eligere.

LXVII.

Postquam autem inuestigationem ab aequationibus differentio-differentialibus ad aequationes simpliciter differentiales produximus, etiamsi ad hoc anomalia vera v cuius sinus in maximis ac minimis lunae a terra distantiis euanescat, simus vsi, tamen haec conditio iam iterum exui potest. Cum enim tam sinus quam cosinus ipsius v vbique per quantitatem q sit multiplicatus, loco harum duarum variabilium q et v iam alias duas variabiles in calculum introducere poterimus, quod commodissime fiet ponendo $q \cos v = r$ et $q \sin v = s$, vt sit $qq = rr + ss$ et $\tang v = \frac{s}{r}$, tum enim vi formularum §. IX. exhibitarum habebimus istas aequationes:

$$dr = -s\,d\Phi + \frac{2M}{A}\,du\,V\!A\,p;$$

$$ds = r\,d\Phi + \left(\frac{N}{A} - \frac{Ms}{A(1-r)}\right) du\,V\!A\,p.$$

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LXVIII.

Hinc autem porro erit: $x = \frac{p}{1-r}$;

$dp = -\frac{2Mp\,d\omega}{A(1-r)}\sqrt{Ap}$ et $d\Phi = \frac{d\omega(1-r)^2}{pp}\sqrt{Ap}$, tum vero vt ante $du = d\vartheta = \frac{d\omega(1-e\cos u)^2}{(1-ee)\sqrt{(1-ee)}}$.

Deinde vero si statuamus $p = b(1+\xi)$, reliquasque denominationes in §§. 11, 12, 13. factas adhibeamus, obtinebimus:

$$\frac{M}{A}\sqrt{Ap} = \frac{3(1+3ee)}{2m}\cdot\frac{(1-e\cos u)^3}{1-r}(1+\tfrac{3}{2}\xi)\sin 2\eta$$
$$+\frac{3n}{8m}\cdot\frac{(1-e\cos u)^4}{(1-r)^2}(1+\tfrac{5}{2}\xi)(\sin\eta+5\sin 3\eta)$$

$$\frac{N}{A}\sqrt{Ap} = -\frac{(1+3ee)}{2m}\cdot\frac{(1-e\cos u)^3}{1-r}(1+\tfrac{3}{2}\xi)(1+3\cos 2\eta)$$
$$-\frac{3n}{8m}\cdot\frac{(1-e\cos u)^4}{(1-r)^2}(1+\tfrac{5}{2}\xi)(3\cos\eta+5\cos 3\eta)$$
$$+m(1-r)^3(1-\tfrac{3}{2}\xi)\,\Pi-i$$

atque

$$x = \frac{b(1+\xi)}{1-r};\quad d\xi = \frac{2(1+\xi)\,d\omega}{1-r}\cdot\frac{M}{A}\sqrt{Ap};$$

ac tandem:

$$d\Phi = m\,d\omega(1-\tfrac{3}{4}\xi+\tfrac{15}{8}\xi\xi)(1-r)^3$$

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LXIX.

Hinc iam omnium differentialium rationes ad $d\omega$ habentur, erit enim

$$\frac{d\xi}{d\omega} = -\frac{2(1+\xi)}{1-r}\cdot\frac{M}{A}\sqrt{A}p :$$

$$\frac{d\Phi}{d\omega} = m\,(1-\tfrac{3}{2}\xi+\tfrac{15}{8}\xi\xi)\,(1-r)^2$$

$$\frac{du}{d\omega} = \frac{d\theta}{d\omega} = \frac{(1-e\cos u)^2}{(1-ee)\sqrt{(1-ee)}} \quad \text{et} \quad \frac{d\eta}{d\omega} = \frac{d\Phi-d\theta}{d\omega}.$$

$$\frac{dr}{d\omega} = -ms\,(1-\tfrac{3}{2}\xi+\tfrac{15}{8}\xi\xi)\,(1-r)^2 + 2.\frac{M}{A}\sqrt{A}p$$

$$\frac{ds}{d\omega} = mr\,(1-\tfrac{3}{2}\xi+\tfrac{15}{8}\xi\xi)\,(1-r)^2 + \frac{N}{A}\sqrt{A}p - \frac{s}{1-r}\cdot\frac{M}{A}\sqrt{A}p$$

$$\frac{d\pi}{d\omega} = -\frac{1}{m}\sin(\theta-\pi)\sin(\Phi-\pi)\left(\frac{3\,(1+3ee)\,(1+\tfrac{3}{2}\xi)}{(1-r)^2}\cos\eta + \tfrac{3}{4}n\,(3+5\cos 2\eta)\right)$$

$$\frac{d.\mathrm{ltg}\,\varrho}{d\omega} = -\frac{1}{m}\sin(\theta-\pi)\cos(\Phi-\pi)\left(\frac{3\,(1+3ee)\,(1+\tfrac{3}{2}\xi)}{(1-r)^2}\cos\eta + \tfrac{3}{4}n\,(3+5\cos 2\eta)\right)$$

at est $\Pi = 1-\lambda-\frac{3}{4}\tan\varrho^2-\frac{3}{4}\tan\varrho^2\cos 2(\Phi-\pi)$, vbi pro λ assumi potest $1-\frac{3}{4}\tan e^2$, denotante e inclinationem mediam orbitae lunaris ad eclipticam, vt sit $\Pi = \frac{3}{4}\tan e^2 - \frac{3}{4}\tan\varrho^2\,[1+\cos 2(\Phi-\pi)]$

LXX.

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LXX.

Quodsi iam pro r statueretur iste valor $k \cos v$, ita vt k esset quantitas constans, oriretur modus initio traditus inaequalitates lunae repraesentandi, foret enim tum v anomalia vera Kepleri et k denotaret excentricitatem orbitae lunaris. Vnde patet etiam inaequalitates lunae per methodum primam erutas, ex his formulis inueniri posse, neque ad hoc aequationibus secundi gradus esse opus. Reduceretur autem hoc casu indoles differentio-differentialium ad inuentionem quantitatis s, quem in finem pro s assumi deberet series quaedam sinuum angulorum η, v, u et $\Phi - \pi$ formatorum cum indefinitis coefficientibus, quos deinceps determinare liceret: hoc autem modo solutio primum tradita esset proditura.

LXXI.

Cum sit $p = b(1+\xi)$ et $\sqrt{A} = m\sqrt{b^3}$, ob $dx = -\frac{s d\omega}{p}\sqrt{Ap}$, fiat $\frac{dx}{d\omega} = -\frac{mbs}{\sqrt{(1+\xi)}} = -mbs(1-\frac{1}{2}\xi+\frac{3}{8}\xi\xi)$, vnde patet si eiusmodi anomalia v introducatur, vt sit $s = q \sin v$, siue q sit quantitas constans siue variabilis, tum hanc anomaliam tam in maximis quam minimis distantiis sinum euanescentem esse habituram. Ac si pro q quantitas vel constans vel ex angulis cognitis composita assumatur, tum inde coefficientes assumti ac praeterea valor litterae r determinabitur. Sin autem pro q eiusmodi quantitas incognita assumatur, vt sit praeterea $r = q \cos v$, tum solutio ante exposita resultabit.

LXXII.

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LXXII.

Semper autem vſus aſtronomicus exigit, vt anomalia vera quaedam angulo v contenta introducatur, id quod infinitis modis fieri poteſt. Quo autem quantitas r variabilitatem diſtantiarum lunae a terra accuratius exprimat, et valor ipſius ξ quam minimas mutationes ſubeat, neceſſe eſt, vt quantitas r huiusmodi contineat terminum $k \cos v$, vbi k excentricitatem deſignet, qui ſit quaſi eius pars praecipua; hocque etiam locum habet, ſi pro r ſumatur $q \cos v$, denotante q quantitatem variabilem, quippe cuius pars potior excentricitatem k praebere debet. Verum praeterea quantitas r alios terminos continere poteſt, qui ab angulo v vel pendeant vel non pendeant: ita poni poſſet: $r = k \cos v + A \cos 2\eta + B \cos 4\eta + C \cos (2\eta - v)$ etc. quo valore aſſumto litterae quoque s, ξ cum reliquis ſuos valores debitos obtinerent.

LXXIII.

Hoc modo illud incommodum euitari poteſt, quo methodum in hoc additamento traditam laborare vidimus, ſi excentricitas orbitae lunaris eſſet nimis parua, vel adeo euaneſcens; tum enim diſtantiae maximae et minimae non amplius ab anomalia penderent, ſed potius ab angulo η, atque imprimis quidem a coſinu dupli anguli 2η. Caſu ergo quo excentricitas plane euaneſcit, pro variabili r, cuius loco vtique noua variabilis introduci debet, non conueniet anomaliam v introducere, ſed praeſtabit aſſumi ſeriem coſinuum ex ſolis angulis 2η, 4

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et $\Phi - \pi$ conſtantem, quorum coefficientes etſi ſunt cónſtantes, tamen quia terminorum numerus in infinitum excurrit, vicem novae variabilis ſuſtinebunt. Tum autem valor ipſius s ex ſimili ſerie ſinuum eorundem angulorum conſtabit.

LXXIV.

Quodſi ergo rem generatim pro quacunque excentricitate expedire velimus, poterimus ad hos terminos, qui ex hypotheſi excentricitatis euaneſcentis prodeunt, adhuc adiungere terminos ex anomalia v formatos. Ita neglectis tam inaequalitatibus parallacticis, quam iis quae tum ab excentricitate orbitae ſolaris, tum ab inclinatione orbitae lunaris ad eclipticam pendent, poni conueniet:

$$r = k \cos v + A \cos 2\eta + B \cos(2\eta - v) + C \cos(2\eta + v) + D \cos 4\eta + E \cos(4\eta - v) \text{ etc.}$$
$$s = \Delta k \sin v + \mathfrak{A} \sin 2\eta + \mathfrak{B} \sin(2\eta - v) + \mathfrak{C} \sin(2\eta + v) + \mathfrak{D} \sin 4\eta + \mathfrak{E} \sin(4\eta - v) \text{ etc.}$$
$$\xi = .. \cos v + .. \cos 2\eta + .. \cos(2\eta - v) + .. \cos(2\eta + v) + .. \cos 4\eta + .. \cos(4\eta - v) \text{ etc.}$$

Atque ſi hoc modo omnes angulorum 2η et v combinationes adhibeantur, hique valores in aequationibus ſupra datis ſubſtituantur, primo inde elicietur ratio $dv : d\omega$, ac deinceps coefficientes determinationes ſuas nanciſcentur.

LXXV.

Manebunt autem coefficientes vnius ſeriei veluti ipſius r indeterminati, propterea quod ipſa ſeries haec

ab

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ab arbitrio noſtro pendet, dum pro r vel ſolum primum terminum $k \operatorname{cof} v$, vel quotquot lubuerit, aſſumere potuiſſemus. Hinc autem id commodi conſequemur, vt iſtos coefficientes ad ſcopum quam conuenientiſſime definire valeamus: ſcilicet eos ita definiri conueniet, vt primo nullius reliquorum coefficientium determinatio lubrica et incerta euadat, vti in vtraque methodo expoſita vſu venit: deinde vero vt nulli coefficientes fiant nimis magni praeter neceſſitatem, ita vt eorum effectus per alios terminos iterum deſtrui neceſſe ſit. Fateri quidem cogor calculum hoc modo inſtituendum admodum futurum eſſe prolixum, verum fortaſſe in ipſa operatione non contemnenda ſe offerent compendia; vnde confido hanc ſpeculationem, etiamſi mihi ipſi eam ſuſcipere non vacet, vſu non eſſe carituram.

BEROLINI, EX OFFICINA MICHAELIS.

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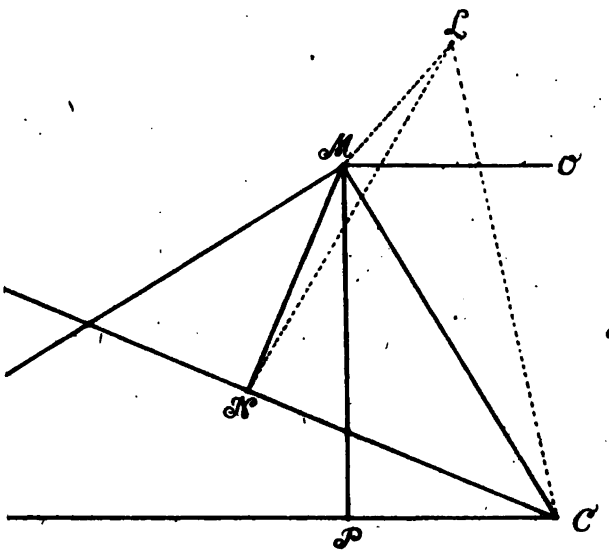
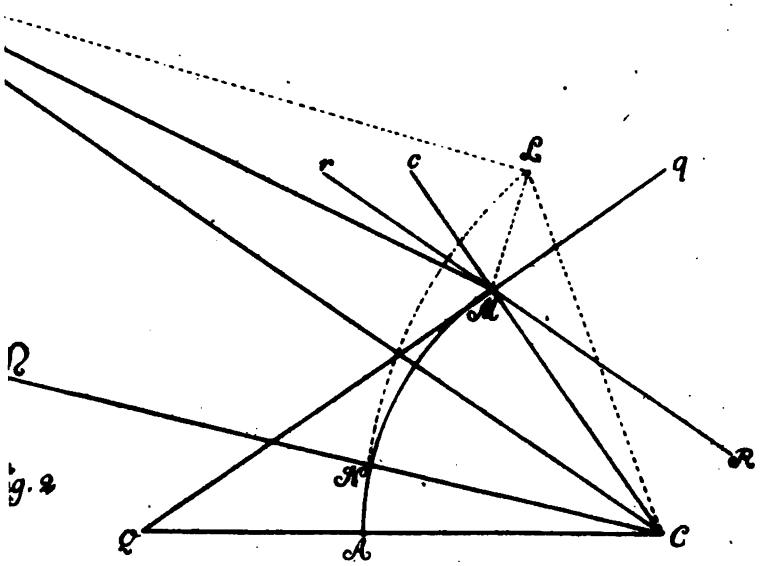


Fig. 1.



g. 2

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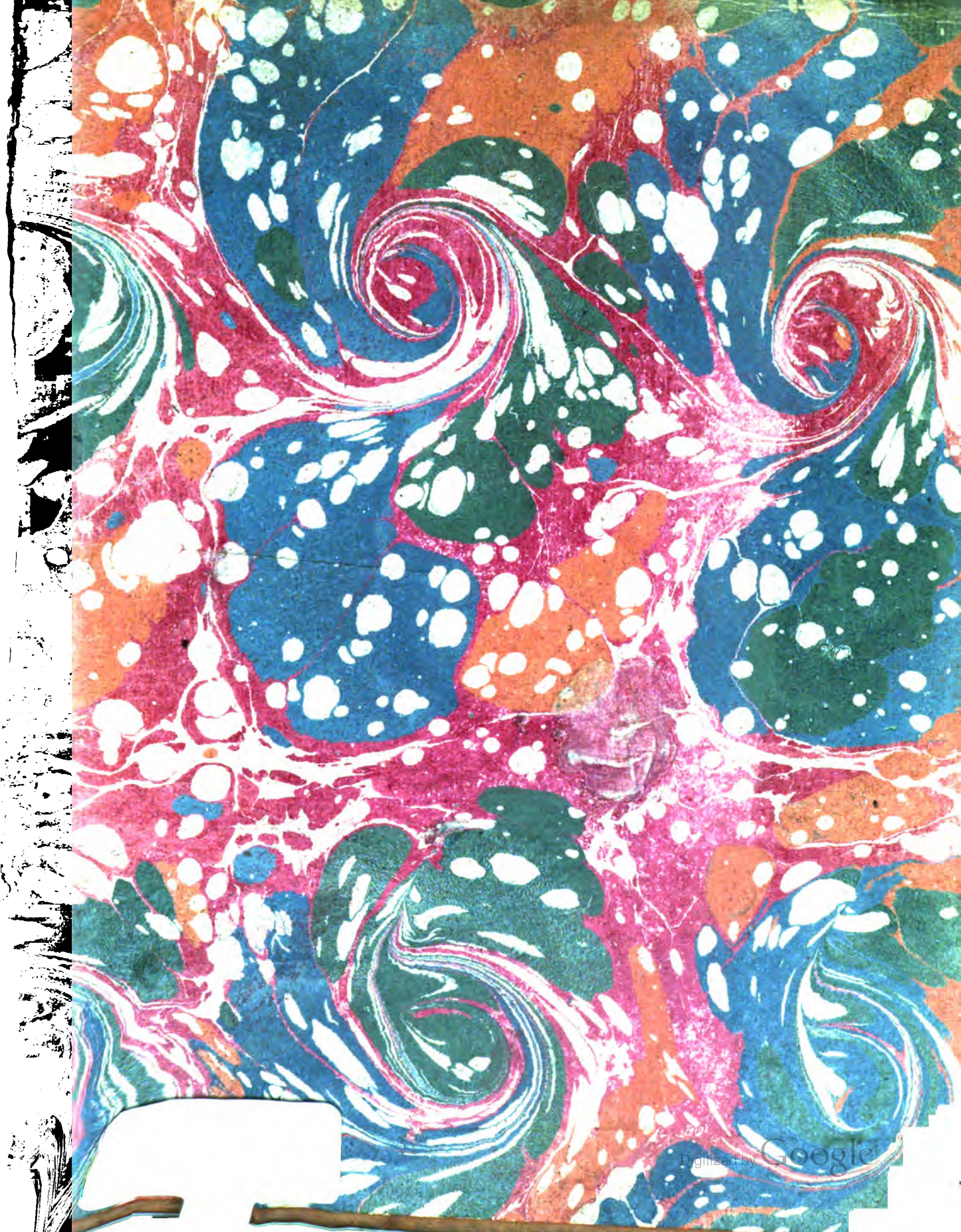
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